HIGH SCHOOL ARITHMETIC;

CONTAINING

THE ELEMENTARY AND THE HIGHER PRINCIPLES
AND APPLICATIONS OF THE SCIENCE,

WITH THE MOST USEFUL

ABBREVIATED METHODS OF CALCULATION;

PRACTICAL MENSURATION;

AND

APPENDICES

ON EXCHANGE, AND MATHEMATICAL PROBABILITIES,

WITH APPLICATIONS OF THE LATTER TO

LIFE ANNUITIES AND LIFE INSURANCE.

BY

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FOURTH EDITION.

NEW-YORK:
PUBLISHED BY PRATT, WOODFORD & CO.
1853.

PREFACE.

The greater portion of the following work, under the title of "Elementary and Practical Arithmetic," was published about two years ago, and has met with the most gratifying reception from eminent Teachers wherever it has become known.

In enlarging the work with a view to meet the wants of Schools in which a more extensive course of arithmetical study is pursued, the object has been to add whatever seemed wanting to a complete theoretical and practical treatise; to present the supplemental principles and processes in the clearest light of which they are susceptible; to furnish Exercises which shall be found well adapted to the expanding capacity of the Student—prove a means and a test of his intellectual development and discipline—and inure him to some of the more recondite, as well as to the more obvious applications of the science.

But in the completeness aimed at, a regard has been entertained only for what appeared to be really useful. One or two subjects, therefore, of merely theoretical interest, may be found in some of the Higher Arithmetics, which have been omitted, or partially presented in this; while others have been pursued to an unusual extent; and two or three, of much theoretical and practical interest, have been introduced; which, so far as is known to the author, have not hitherto been treated in any work designed for educational purposes.

To place in the hands of his fellow laborers the best instrumental aid towards an able performance of their duty, in a department of their labors which is peculiarly liable to suffer from indolence or incompetency in Authors, as well as in Teachers,—has been the earnest endeavor of the writer, in the composition of every part of this work.

To those who, in various, even the remotest parts of the country, have so strongly commended his former publication—who, with comparatively few exceptions, are personally unknown to him, the author will do himself the pleasure, on this occasion, to offer the expression of his grateful acknowledgments.

TRANSYLVANIA UNIVERSITY,
September 24th, 1851.

REMARKS

ON THE METHOD OF USING THIS WORK, AND CONDUCTING EXAMINATIONS IN ARITHMETIC.

1. The Analysis of Contents may be used in the manner explained under that head. (See the next page.)

2. The Student should be much exercised in *elementary oral solutions*, as exemplified under various questions, for instance, the 11th on page 36.

3. A very effectual method of Examination on the practical Exercises, and a very useful discipline to the learner, will be, for the Teacher to read to him a few of the more difficult questions under each Rule, in sentences or parts of sentences,—the learner *repeating*, and setting down the *numbers*, and, with his book closed, working for, and announcing the *Answer*.

ANALYSIS OF CONTENTS.

This Analysis is designed to be used in an oral examination, in review. The Teacher will name the *topic* as presented in this table: the Learner will respond according to his knowledge of the subject.

For example; the Teacher will say "Arithmetic;" the Learner will respond "Arithmetic is the science of Numbers; or, when practically applied, the art of Calculation."

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ARITHMETIC.

CHAPTER I.

PRELIMINARY DEFINITIONS.—NUMERATION.—NOTATION.

Science and Art.

§ 1. SCIENCE is knowledge reduced to a *system*, so as to be conveniently taught, and readily applied.—ART is knowledge applied to practical purposes.—The Rules of art are founded on the Principles of science.

Unity and Numbers.—Quantity.

§ 2. A *Unit* is any thing regarded simply as *one;* and *Numbers* are *repetitions* of a unit.

The numbers *two, three,* &c., are repetitions of a *unit* or one.

§ 3. *Quantity* is any thing which admits of being *measured.*

Thus a *line* is a quantity, and we express its *measure* when we say it is so many *feet* or *inches* long.

Is *time* a quantity? Is *industry* a quantity? Is *weight* a quantity?
Is *hope* a quantity? Is *distance* a quantity? Is *virtue* a quantity?
Are *length, breadth,* and *height* quantities?

Numbers are quantities, since every number is necessarily measured by a *unit,* (§ 2); and numbers are necessary to express the measures of all other quantities.

Thus we express the measure of a quantity of water by saying it is *five gallons;* and the measure or weight of a *quantity of iron* by saying it is *ten pounds.*

PRELIMINARY DEFINITIONS.

§ 4. MATHEMATICS is the science of Quantity. Its most general divisions are Arithmetic and Geometry.

ARITHMETIC is the science of Numbers; or, when practically applied, the art of Calculation.—GEOMETRY is the science which treats of Extension, including *length*, *breadth*, and *height* or *depth*.

Of these two divisions of mathematical science, Arithmetic comes first in order; and we begin by distinguishing different kinds of numbers.

Abstract and Concrete Numbers.

§ 5. An *abstract* number is a number without any *kind of units* expressed; as the numbers *one, five, ten*, a *hundred.*

§ 6. A *concrete* number is a number of *some kind* of units expressed; as the numbers *one* book, *five* men, a *hundred* dollars.

Is *twenty* an abstract or a concrete number? Is *nine* pounds an abstract or a concrete number? *One hundred?* *Two hundred* miles?

Give two other examples of abstract, and two of concrete numbers.

Similar and Dissimilar Numbers.

§ 7. *Similar* concrete numbers are such as express the *same kind* of units; as *three* dollars and *five* dollars.

§ 8. *Dissimilar* concrete numbers are such as express *different kinds* of units; as *three* dollars and *five* miles.

Are *four* inches and *seven* inches, similar or dissimilar concrete numbers? *Nine* pounds and *twelve* yards? One cent and *ten* pints?

Give another example of *similar* concrete numbers. Of *dissimilar* concrete numbers. Give another example of each kind.

The groundwork of a thorough knowledge of Arithmetic must be laid in the principles of Numeration and Notation; for on these principles depend the four fundamental operations in Arithmetic—Addition, Subtraction, Multiplication, and Division.

NUMERATION.

§ **9.** NUMERATION is the method of *naming* abstract numbers. Numbers are named as so many *units, tens, hundreds, &c.*

Thus *eleven* is *one* and *ten;*
twelve is *two* and *ten;*
thirteen is *three* and *ten;*
seventeen is *seven* and *ten.*

Twenty is *two tens;*
twenty-five is *two tens* and *five,*
thirty-nine is *three tens* and *nine.*

A *hundred* is *ten tens;*
A *thousand* is ten hundred.

A *million* is ten hundred thousand, or a *thousand* thousands.
A *billion* is ten hundred millions, or a *thousand* millions.
A *trillion* is ten hundred billions, or a *thousand* billions.

In like manner Numeration proceeds through *quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions,* &c.

What two numbers are implied in the name *fourteen?* In the name *fifteen?* In *eighteen?* In fifteen? In seventeen?

What is implied in the name *twenty?* In the name twenty-one? In thirty? In forty? In fifty? Sixty? Seventy? Ninety?

A quadrillion is how many? A quintillion? A sextillion? A septillion? An octillion? A nonillion? A duodecillion?

Different Orders of Units.

§ **10.** The naming of numbers by *tens, hundreds,* &c., introduces different *orders of units* in Numeration.

The numbers *two, three, four,* &c., are repetitions of the simple unit *one,* which is called a *unit* of the *first order.*

Twenty, thirty, forty, &c., are, respectively, two *tens,* three *tens,* four *tens,* &c.; and in these repetitions of *ten, ten* is regarded as a *unit of the second order.*

So in repetitions of a *hundred,* as two *hundred,* three *hundred,* &c., *one hundred* is regarded as a *unit* of the *third order.*

In like manner, *one thousand* is made a *unit* of the *fourth order;* and so on.

In the number *Three hundred and forty-seven*, for example, we find *seven units* of the *first* order, *four* of the *second*, and *three* of the *third*.

In the number *Five thousand, nine hundred and four*, we find *four* units of the *first* order, *none* of the *second*, *nine* of the *third*. and *five* of the *fourth*.

How many *units* of the first and second *orders*, respectively, do you find in the number Twenty-five? In the number Thirty-one? In the number Seventy-six? In the number Nineteen? In the number Ninety-nine?

How many *units* of *distinct orders* are found in the number Five hundred and thirty-four? In the number Nine hundred and fifty-one? In Three thousand seven hundred and sixteen? In Eight thousand one hundred and seventy?

Scale of Numeration.

§ **11.** *Ten* of any *lower order* of units make *one* of the next higher order; or, *one* of a higher order makes *ten* of the next lower order.

Thus *ten units* (of the first order) make *one* ten.

How many Tens make One hundred? How many Hundreds make One thousand? How many Hundreds make Two thousand? How many Hundreds make Three thousand?

One million is how many Hundred thousand? One billion is how many Hundred millions? One trillion is how many Hundred billions?

Numeration Table.

§ **12.** The *ascending orders* of units are given from right to left, in the following Table:

Billions, &c.	*Hund. of mill.,*	*Tens of mill.,*	*Millions,*	*Hund. of thous.,*	*Tens of thous.,*	*Thousands,*	*Hundreds,*	*Tens,*	*Units,*

Recite the orders of units, *ascending*, from Units to Decillions. Recite them, *descending*, beginning with Hundreds. Beginning with Thousands. Beginning with Tens of thousands. Beginning with Hundreds of thousands. Beginning with Millions. Beginning with Billions.

What are the *relative values* of these different orders of units (§ 11)?

NOTATION.

§ 13. NOTATION is the method of *denoting* numbers by numeral characters or *figures*.

These Figures—sometimes called the *digits* of numbers—are 1 *one*, 2 *two*, 3 *three*, 4 *four*, 5 *five*, 6 *six*, 7 *seven*, 8 *eight*, 9 *nine*, and the 0 *zero* or *cipher*, which denotes *nothing*.

The figures from 1 to 9 inclusive, are *significant;* the 0 is *insignificant*.

Tens, Hundreds, &c.—how denoted.

§ 14. *Tens, hundreds*, &c., are denoted by *two, three*, &c., *figures* in a row, right and left,—the *first* on the *right* being *units*,—the *second, tens*,—the *third, hundreds*, and so on, according to the ascending orders of units. (§ 12.)

Thus 12, 1 *ten* and 2, that is, *twelve;*
123, 1 *hundred*, 2 *tens*, and 3; or *one hundred and twenty-three*.

What would be the value of 4 in the *first place* on the right? In the second place? In the third place? In the fourth place?

Simple and Local Values.

§ 15. The *simple* value of a figure is that which is expressed by the simple *name* of the figure—being its value when used *alone*, or in the units' place.

Thus the simple value of 5 is *five;*
and 5 has its simple value in 25, 2 *tens* and 5, or *twenty-five*.

§ 16. The *local* value of a figure is that which arises from its *place* in a row of figures—being its simple value increased *tenfold* for each place it is removed from *units* toward the left.

In 25, 2 has the local value 2 *tens*, which is 10 *times* the simple 2; and in 125, 1 has the local value 1 *hundred*.

3 in the second place from the right would be how many times the *simple* 3? In the third place? In the fourth place? In the fifth?

Use of 0 Zero or Cipher.

§ 17. The 0 *zero* or cipher, having no value, is used to occupy *vacant* places in notation; and thus to assign a required *local value* to another figure, by removing it the proper distance from the units' place.

Thus 10, 1 with a 0 occupying the *units'* place, denotes *Ten*.
so, 100, 1 with two 0s for *tens* and *units*, is One hundred.

How may the figure 2 be made to denote 2 *tens*, or *twenty?* How may it be made to denote 2 *hundred?* 2 *thousand?* 2 *millions?*

How may 3 be made to denote 3 hundred? 3 thousand? 30 thousand? 3 hundred thousand? 3 millions? 3 hundred millions?

RULES OF ARITHMETIC.

§ 18. A *Rule*, in Arithmetic, is a prescribed mode of performing an Arithmetical operation.

RULE I.

§ 19. *To Numerate or read a row of Figures.*

Call the successive figures *units, tens, hundreds*, &c., from *right to left* (§ 12), and then read them according to their respective names or values from *left to right*.

EXAMPLE.

To read the number

704360521

Calling the figures, one after another, *units, tens, hundreds*, &c., from right to left, we find the last figure 7 to be *hundreds of millions*. Then reading from left to right, we say,

Seven hundred and four millions, Three hundred and sixty thousand, Five hundred and twenty one.

EXERCISES.

Read each of the following numbers,—from the top to the bottom of each column.

1st,	11	16th,	510	31st,	1000000
2nd,	12	17th,	701	32nd,	5400375
3rd,	13	18th,	900	33rd,	7347531
4th,	16	19th,	1000	34th,	20000000
5th,	19	20th,	3800	35th,	34000000
6th,	20	21st,	24000	36th,	75603740
7th,	21	22nd,	37220	37th,	80730783
8th,	30	23rd,	40079	38th,	300000000
9th,	34	24th,	71003	39th,	473830731
10th,	40	25th,	89050	40th,	380707349
11th,	49	26th,	94376	41st,	701803909
12th,	100	27th,	100000	42nd,	890760540
13th,	205	28th,	270270	43rd,	4000000000
14th,	300	29th,	341143	44th,	5738146839
15th,	473	30th,	404404	45th,	9030707301

RULE II.

§ **20.** *To write in Figures any given Number.*

Place proper figures from *left to right*, to denote the number in each of the *descending orders* of units, from the highest in the given number down to simple units; observing to fill each *vacant* place with a 0. (§ 17.)

EXAMPLE.

To write in figures the number

Three millions, twenty-five thousand, and thirty.

The descending orders of units in this number, are

3 *millions*, 2 *tens* of thous., 5 *thousands*, and 3 *tens*.

Hence we write it,

3025030;

in which the vacant places of *hundreds* of thousands, *hundreds*, and *units*, are filled with 0s.

EXERCISES.

Write in figures each of the following numbers:

1. One hundred.
2. Two hundred and one.
3. Three hundred and ten.
4. Four hundred and five.
5. Five hundred and fifteen.
6. Six hundred and twenty.
7. Seven hund. and thirty-four.
8. Eight hundred and eleven.
9. Nine hund. and ninety-nine.

10. One thousand.
11. Two thousand and nine.
12. Five thousand and ten.
13. Seven thous. one hundred.
14. Three thousand and five.
15. Eight thous. and nineteen.
16. Nine thous. and eleven.
17. Four thousand, five hundred and seventy-eight.

18. Ten thousand.
19. Twelve thousand and ten.
20. Twenty thousand and nine.
21. Fifty-four thousand, one hundred and twenty-three.
22. Eighty-seven thousand, five hundred and seventy-eight.
23. Seventy-one thousand, two hundred and one.
24. Forty thousand, three hundred and two.

25. One hundred thousand.
26. Two hundred and thirty thousand, one hundred.
27. Five hundred and one thousand, two hundred and three.
28. Seven hundred and thirteen thous., four hundred and fifty.
29. Nine hundred and ninety-nine thousand, and one.
30. Eight hundred thousand, and seven hundred.
31. Nine hundred and one thousand, one hundred and nine.

32. One million.
33. Five millions, five hundred thousand.
34. Nineteen millions, two hundred and forty-seven thous.
35. Thirty millions, one hundred and fifty thousand, seven hundred.
36. Seventy-five millions, eight hundred and sixty-four thousand, nine hund. and twelve.
37. Eleven millions, seven hundred and fourteen thousand.
38. Twenty-nine millions, four hundred and one thousand, two hundred and ten.
39. Thirty millions, nine hundred and twenty thousand.
40. Seven millions, eighty-five thousand, six hundred and forty-nine.
41. Eighty-five millions, eighty-seven thousand, four hundred and ninety seven.
42. Ninety-nine millions, one hundred and eleven thousand, one hundred and one.
43. One hundred millions.
44. Three hundred millions, one hundred thousand.
45. Five hundred and thirty-four millions, nine hundred.
46. Six hundred and nine millions, fifty thousand, one hundred and twenty-five.
47. Nine hundred and seventeen millions, five hundred thous., four hundred and sixty.
48. Five hundred millions, sixty thousand, three hundred and four.
49. Seven hundred and ten millions, one hundred thousand, five hundred and ninety-one.
50. Three hundred and one millions, seven hundred and ten.
51. Eight hundred and six millions, nine hundred and nineteen thousand, one hundred.
52. Nine hundred and ninety-nine millions, nine hundred and ninety-nine thousand, nine hund. and ninety-nine.

French and English Numeration.

§ **21.** In the French system of Numeration, which prevails in continental Europe, and in America, a *thousand millions* make *one billion*, a *thousand* billions make *one trillion*, and so on (§ 9 and § 12).

In the English system, a *million millions* make *one billion*, a *million* billions make one *trillion*, and so on.

Hence, in this system, after *hundreds of millions*, the ascending order of units are, *thousands* of millions, *tens of thousands* of millions, *hundreds of thousands* of millions, *billions;* and in like manner after *hundreds of billions*, &c.

For example, the number 3 840 930 670 820, in the French system, is 3 *trillions*, 840 *billions*, 930 *millions*, 670 *thousand*, eight hundred and twenty.

In the English, it is 3 billions, 840930 *millions*, 670820.

CHAPTER II.

ADDITION. — SUBTRACTION. — MULTIPLICATION. — DIVISION.

ADDITION.

§ **22.** Addition consists in finding the *sum* or *aggregate* amount of two or more numbers.

Thus the *sum* of 4 and 5 is 9; or 5 *added to* 4 makes 9.

What is the *sum* of 7 and 3? Of 6 and 4? Of 8 and 5?

The Sum found may be regarded as a *whole*, of which the given numbers are the *parts*.

What is the *sum* of 4, 6 and 8? Then what is the *whole*, and what are its *parts*? What is the sum of 10, 4, and 3? Then what is the whole, and what are its parts?

§ **23.** The *whole* is equal to the *sum of all its parts*.

Recite the *elementary sums* of numbers; thus 1 and 1 are 2; 1 and 2 are 3; &c.—2 and 1 are 3; 2 and 2 are 4; &c., as given from left to right in the

Addition Table.

1 *and*	1 *are* 2	2 *are* 3	3 *are* 4	4 *are* 5	5 *are* 6	6 *are* 7	7 *are* 8	8 *are* 9	9 *are* 10
2 *and*	1 *are* 3	2 *are* 4	3 *are* 5	4 *are* 6	5 *are* 7	6 *are* 8	7 *are* 9	8 *are* 10	9 *are* 11
3 *and*	1 *are* 4	2 *are* 5	3 *are* 6	4 *are* 7	5 *are* 8	6 *are* 9	7 *are* 10	8 *are* 11	9 *are* 12
4 *and*	1 *are* 5	2 *are* 6	3 *are* 7	4 *are* 8	5 *are* 9	6 *are* 10	7 *are* 11	8 *are* 12	9 *are* 13
5 *and*	1 *are* 6	2 *are* 7	3 *are* 8	4 *are* 9	5 *are* 10	6 *are* 11	7 *are* 12	8 *are* 13	9 *are* 14
6 *and*	1 *are* 7	2 *are* 8	3 *are* 9	4 *are* 10	5 *are* 11	6 *are* 12	7 *are* 13	8 *are* 14	9 *are* 15
7 *and*	1 *are* 8	2 *are* 9	3 *are* 10	4 *are* 11	5 *are* 12	6 *are* 13	7 *are* 14	8 *are* 15	9 *are* 16
8 *and*	1 *are* 9	2 *are* 10	3 *are* 11	4 *are* 12	5 *are* 13	6 *are* 14	7 *are* 15	8 *are* 16	9 *are* 17
9 *and*	1 *are* 10	2 *are* 11	3 *are* 12	4 *are* 13	5 *are* 14	6 *are* 15	7 *are* 16	8 *are* 17	9 *are* 18
10 *and*	1 *are* 11	2 *are* 12	3 *are* 13	4 *are* 14	5 *are* 15	6 *are* 16	7 *are* 17	8 *are* 18	9 *are* 19
11 *and*	1 *are* 12	2 *are* 13	3 *are* 14	4 *are* 15	5 *are* 16	6 *are* 17	7 *are* 18	8 *are* 19	9 *are* 20
12 *and*	1 *are* 13	2 *are* 14	3 *are* 15	4 *are* 16	5 *are* 17	6 *are* 18	7 *are* 19	8 *are* 20	9 *are* 21

Sign of Addition.

§ **24.** The *sign* +, called *plus*, placed between numbers, signifies that the numbers are to be *added together*.

Thus 5+4, 5 *plus* 4, signifies 5 and 4 added together.

What is the sum of 4+3+2? Of 6+4+8? Of 7+3+4+6?

Sum of Concrete Numbers.

§ **25.** The sum of similar concrete numbers, is a concrete number of the *same kind*.

Thus, the sum of 5 *cents* and 4 *cents* is 9 *cents*.

What is the sum of 6 pounds + 5 pounds? Of 7 days + 3 days + 5 days? Of 10 miles + 9 miles? Of 12 pints + 6 pints + 2 pints?

§ **26.** *Dissimilar* concrete numbers cannot be united in one number.

Thus we cannot add 5 *cents* and 3 *days* together.

RULE III.

§ **27.** *To Add two or more Numbers together.*

1. Set the numbers one under another, with *units* under *units*, tens under tens, &c.

2. Proceeding from *right* to *left*, add up each column of figures, and under each set its amount, if *less than* 10.

3. If the amount be 10 or more, set down its *right hand figure*, and add the left figure or figures to the next column.

Set down the whole amount of the last column.

EXAMPLE.

To find the Sum of 930+6754+8621

$$\begin{array}{r} 930 \\ 6754 \\ \underline{8621} \\ 16305 \end{array}$$

Having set *units* under *units*, tens under tens, &c., we say 1 and 4 are 5, and set 5 under the units' column.

Then, 2 and 5 are 7, and 3 are 10. We set the 0 under the tens' column, and add or *carry* the 1 to the next column; thus 1 and 6 are 7, and 7 are 14, and 9 are 23. We set 3 under that column, and say 2 and 8 are 10, and 6 are 16.

☞ The *left hand figure* in the amount of any column, denotes the number of *tens* in that amount; and these *tens* are so many *units* when carried to the next column on the left. (§ 11.)

In the preceding example, the amount of the *tens'* column is 10 *tens*. But 10 *tens* being 1 *hundred*, we set down *no tens*, and add 1 to the hundreds' column,—which makes 23 *hundreds*, or 2 *thousands* and 3 *hundreds*. The 3 is put in the hundreds' place, and the 2 is added to the thousands' column, making 16 thousands.

By thus carrying *one* for every *ten*, from right to left, we find the number belonging to each distinct order of units in the sum of the several numbers. The whole is equal to the sum of all its parts. ☜

The Operation Proved.

§ 28. Addition may be *verified* or *proved*, by adding the several columns of figures *downwards;* the Sum must be the same as when they are added upwards.

Thus to prove the operation in the preceding example, we begin at the top, and say 4 and 1 are 5; then 3 and 5 are 8, and 2 are 10. Setting down the 0, and carrying the 1, we say 1 and 9 are 10, and 7 are 17, and 6 are 23; then 2 and 6 are 8, and 8 are 16.

The Sum found is the same as before.

EXERCISES.

1. John has 95 chestnuts, Thomas has 180, and Charles 270; what number have they all together?

The whole number of chestnuts will be found by adding together 95, 180, and 270. *Answer*, 545 chestnuts.

2. A farmer being asked how many sheep he had, replied: "in one field I have 410, in another 500, in another 602." How many had he? *Ans.* 1512 sheep.

3. A merchant bought cloth for 375 dollars, linen for 83 dollars, silk for 234 dollars, and calico for 75 dollars. What sum did he expend for the whole? *Ans.* 767 dollars.

4. A gentleman bought a carriage for 350 dollars, a pair of horses for 240 dollars, and a set of harness for 100 dollars. What did the whole amount to? *Ans.* 690 dollars.

5. Going out to collect money, I received from one person 13 dollars, from another 124 dollars, from another 89 dollars, and from another 20 dollars. What was the whole sum collected? *Ans.* 246 dollars.

6. Suppose a man should raise on the several fields of his farm the following quantities of grain, viz: 685 bushels, 97 bushels, 330 bushels, and 1000 bushels,—how many bushels would he raise altogether? *Ans.* 2112 bushels.

7. Allowing a person's estate to be estimated as follows, viz: real estate 9000 dollars, personal property 1375 dollars, recoverable debts 875 dollars, and cash 300 dollars,—what would be the value of his estate? *Ans.* 11550 dollars.

8. Admitting I bought of A 500 bushels of wheat, of B 934 bushels, of C 83 bushels, and of D 125 bushels,—how many bushels did I buy in all? *Ans.* 1642 bushels.

9. Three persons deposite flour in the same warehouse; the first, 43 barrels; the second 150 barrels; and the third, 89 barrels. What quantity do they all deposite? *Ans.* 282 barrels.

10. A merchant bought four pieces of cloth; the first for 225 dollars, the second for 310 dollars, the third for 279 dollars, and the fourth for 95 dollars. What did the whole cost him? *Ans.* 909 dollars.

11. If a merchant buy a stock of goods for 5000 dollars, for what sum must he sell the goods so as to gain by them 475 dollars? *Ans.* 5475 dollars.

12. Bought a barrel of sugar for 15 dollars, a barrel of molasses for 13 dollars, and a sack of coffee for 20 dollars. For what sum must the whole be sold to gain 10 dollars? *Ans.* 58 dollars.

13. A person on a journey travels, the first week 255 miles; the second, 240 miles; the third and fourth, each 200 miles. How far did he travel in the four weeks? *Ans.* 895 miles.

14. Four persons, A, B, C, and D, engage in speculation. A gains 75 dollars, B 100 dollars, and C and D each 235 dollars; what sum was gained by all together? *Ans.* 645 dollars.

15. A farmer bought three plantations for 3750 dollars each, and sold them again so as to make 1000 dollars on the whole. For what sum did he sell the three. *Ans.* 12250 dollars.

16. A draper sold four bales of linen; the first and second contained each 480 yards; the third and fourth each 542 yards. How many yards did he sell? *Ans.* 2044 yards.

17. Bought of A 325 bushels of wheat; of B, 280 bushels; of C as much as from A; and of D as much as from B. What quantity of wheat did I buy in all? *Ans.* 1210 bushels.

18. A merchant bought at one time 375 barrels of flour, for 1875 dollars; and at another 400 barrels for 2000 dollars. What quantity of flour did the merchant buy, and for what sum? *Ans.* 775 barrels, for 3875 dollars.

19. Find the Sum 345+480+2346+7864. *Ans.* 11035.
20. Find the Sum 201+342+4000+1009. *Ans.* 5552.
21. Find the Sum 804+4364+25+1231. *Ans.* 6424.
22. Find the Sum 58603+75+49+2400. *Ans.* 61127.
23. Find the Sum 73846+37+63+9000. *Ans.* 82946.
24. Find the Sum 7+43+479+8+3703. *Ans.* 4240.
25. Find the Sum 9+60+340+4+8264. *Ans.* 8677.
26. Find the Sum 5+50+600+8+8900. *Ans.* 9563.
27. Find the Sum 6+10+100+9+1000. *Ans.* 1125.

28. A lends to B 2500 dollars; to C 3000; and has 5325 dollars left. What sum had A at first? *Ans.* 10825 dollars.

29. A speculator bought stock at one time for 325 dollars; and at another time for 705 dollars. In selling the whole, he realized a gain of 175 dollars; for what sum did he sell?
Ans. 1205 dollars.

30. A gentleman is 15 years older than his wife, and she is 20 years older than their eldest son, who is 29 years of age. Required the gentleman's age, and the age of his wife?
Ans. His age is 64 years; hers 49.

31. A, B, and C form a partnership in trade. A puts in 4250 dollars; B, 2000 dollars; and C as much as A and B together; what is their whole stock in trade? *Ans.* 12500 dollars.

32. Bought at one time 75 yards of cotton, and 100 yards of linsey; at another, 37 yards of cotton, and 87 yards of linsey; and at another, 125 yards of cotton, and 9 yards of linsey. What quantity of each kind did I buy?
Ans. 237 yards of cotton, and 196 of linsey.

33. A merchant bought cloth for 375 dollars, and silk for 95 dollars. In selling, he gained 50 dollars on the cloth, and 45 dollars on the silk; for what sum did he sell the whole?
Ans. 565 dollars.

34. A grocer paid 300 dollars for sugar, 174 dollars for coffee, 85 dollars for rice, and 56 dollars for tobacco. He sold the sugar at a profit of 25 dollars, and the other articles at cost; what did he get for the whole? *Ans.* 640 dollars.

35. A father bequeaths to his only daughter 2500 dollars, and to each of his two sons, 500 dollars more than to his daughter. What sum did each son receive, and what was the amount of the several bequests?
Ans. { Each son, 3000 dollars,
Amount, 8500 dollars.

36. Several persons contributed towards the establishment of a library. A gave 200 dollars, and B 50 dollars more than A; C gave 300 dollars, and D 25 dollars more than C. What was the whole amount contributed? *Ans.* 1075 dollars.

37 The produce of two farms was as follows, viz: of the first, 785 bushels of wheat, and 250 of rye; of the second, 1000 bushels of wheat, and 113 of rye. What was the entire produce of the farms (§ 26)?

Ans. 1785 bushels of wheat, and 363 of rye.

38. A merchant bought 4 bales of cotton; the first and second contained 470 yards each; and the third and fourth, 532 yards each. What was the number of yards purchased?

Ans. 2004 yards.

39. Bought live stock as follows, viz: of A 13 cows, 16 oxen, and 120 sheep; of B 24 cows, 30 oxen, and 153 sheep: and of C 100 cows, and 425 sheep. It is required to find the amount of stock purchased (§ 26).

Ans. 137 cows; 46 oxen; 698 sheep.

40. A has 2375 dollars; B 150 dollars more than A; and C as much as A and B together. What sum is possessed by C, and what by the three together?

Ans. { C has 4900 dollars.
The three 9800 dollars.

41. A farmer has in store at one place 500 bushels of wheat, 325 of oats, and 50 of corn; and at another, 475 bushels of wheat, 75 of oats, and 83 of corn. What amount of produce has the farmer in store?

Ans. 975 bushels of wheat; 400 of oats; 133 of corn.

42. Bought a quantity of cloth for 386 dollars, of cotton for 200 dollars, and of silk for 150 dollars. The cloth was sold at a profit of 73 dollars, the cotton at a profit of 35 dollars, and the silk at cost. What was the whole sold for?

Ans. 844 dollars.

43. The population of each of the grand divisions of the Earth, is estimated as follows, viz: of Europe, at 238 millions, 473 thousand, nine hundred and fifty-seven; of Asia, at 390 millions; of Africa, at 65 millions; of North America, at 35 millions; of South America, at 15 millions, 240 thousand; and of Oceanica, at 20 millions. What then is the whole population of the several grand divisions of the Earth?

Ans. 763713957 inhabitants.

SUBTRACTION.

§ **29.** SUBTRACTION consists in finding the *difference* between two numbers; that is, the *remainder* when the less number is taken from the greater.

The less number is called the *subtrahend*, and the greater the *minuend*.

Thus 4 from 9 leaves 5; then 4 is the *subtrahend*, 9 the *minuend*, and 5 the *difference* or remainder.

What is the difference between 5 and 8? Between 6 and 10? Between 9 and 15? Between 8 and 17? Between 10 and 19?

Addition and Subtraction.

§ **30.** Addition and Subtraction are the *reverse of each other.*

In Addition, the *parts* are given, to find the *sum* or *whole;* in Subtraction, the *sum* or whole, and one of its parts, are given, to find the *other part.*

The sum being 10, and one of its parts 7, what is the other part? The sum being 13, and one part 6, what is the other part? The sum being 19, and one part 12, what is the other part?

Sign of Subtraction.

§ **31.** The *sign* —, called *minus,* placed between two numbers, signifies that the one before which it stands, is to be *subtracted from the other.*

Thus 9—4, 9 *minus* 4, signifies 4 subtracted from 9.

How many is 8—5? 13—7? 15—4? 18—11? 20—10?
How many is 10—3? 14—9? 17—8? 21—10? 25—15?

Difference of Concrete Numbers.

§ **32.** The difference of two similar concrete numbers, is a concrete number of the *same kind.*

Thus the difference of 5 *cents* and 8 *cents* is 3 *cents.*

What is the difference between 12 pounds and 3 pounds? Between 18 days and 9 days? Between 20 dollars and 12 dollars?

§ **33.** Two *dissimilar* concrete numbers cannot be subtracted, the one from the other.

Thus we cannot subtract 5 *pounds* from 8 *days.*

Constant Difference.

§ **34.** The Difference of two numbers remains *the same*, when those numbers are *equally increased* or *diminished*.

What is the difference between 4 and 7? Between 4+1 and 7+1? Between 5 and 9? Between 5+2 and 9+2? Between 7 and 12? Between 7+10 and 12+10? Between 20—9 and 15—9?

RULE IV.

§ **35.** *To Subtract one Number from another.*

1. Set the less number under the greater, with *units* under *units*, tens under tens, &c.

2. Proceeding from *right to left*, take each lower figure from the one above it, and underneath set the remainder.

3. If the lower figure *exceed the upper*, add 10 to the upper figure; from the *sum* subtract the lower, and then add 1 to the next lower figure before subtracting it.

EXAMPLE.

To find the Difference between 654739 and 80657.

$$\begin{array}{r} 654739 \\ 80657 \\ \hline 574082 \end{array}$$

Having set the less number under the greater, with units under units, tens under tens, &c., and drawn a line below them,

We begin at the right, and say 7 from 9 leaves two; the 5 being greater than the 3 above it, we say 10 and 3 are 13, and 5 from 13 leaves 8; then adding 1 to the 6, making 7, we say 7 from 7 leaves 0; 0 from 4 leaves 4; 10 and 5 are 15, and 8 from 15 leaves 7; there being no figure under the 6, the 1 to be added there makes 1 for that place,—then 1 from 6 leaves 5. The Difference required is 574082.

☞ The 10 added to any upper figure is always equal to the 1 added to the next lower figure (§ 11); so that these additions preserve *the same difference* between the two given numbers (§ 34).

In the preceding example, the 10 *tens* added to the 3 *tens*, are equal to the 1 *hundred* added to the 6 *hundreds;* in like manner the 10 added to the 5 in the upper number is equal to the 1 added to the lower number, and subtracted from the 6.

The several figures in the *remainder* express the differences between the corresponding units, in their several orders, in the two given numbers or those numbers equally increased in the process of subtracting. ☜

The Operation Proved.

§ 36. Subtraction may be *verified* or *proved*, by adding the *difference* to the less number; the sum must be equal to the greater number.

EXERCISES.

1. William had 325 chestnuts, but gave James 148 of them. How many chestnuts had William left?

The number left is the difference between 325 and 148.

Ans. 177 chestnuts.

2. A person who undertook a journey of 735 miles, has traveled 93 miles of the distance. What distance has he yet to travel? *Ans.* 642 miles.

3. From a farm which contained 2350 acres, 1234 acres were sold. How many acres remained of the original farm?

Ans. 1116 acres.

4. A young man received from his father 5325 dollars, of which he paid 2500 dollars for a house. How many dollars of the first sum had he remaining? *Ans.* 2825 dollars.

5. A merchant deposited 5800 dollars in bank, but afterwards made a draft upon it for 3270 dollars. What sum remained?

Ans. 2530 dollars.

6. Suppose a farmer who has 4000 bushels of wheat in his granary, should take out 2100 bushels to be sent to market; how many bushels would remain? *Ans.* 1900 bushels.

7. A vintner bought 4036 gallons of wine, and afterwards sold at different times to the amount of 2373 gallons. How many gallons had he remaining? *Ans.* 1663 gallons.

8. Suppose I should borrow of my neighbor 1000 dollars, and three months afterwards should pay him 385 dollars of the debt; what balance would still be owing? *Ans.* 615 dollars.

9. A drover bought cattle for 1495 dollars, and sold the same at a loss of 270 dollars. For what sum did he sell them?

Ans. 1225 dollars.

10. A gentleman sold a farm for 6700 dollars, which sum was 530 dollars more than he gave for it. What did he pay for the farm? *Ans.* 6170 dollars.

11. A weaver made 30 pieces of cotton, containing 1200 yards; of which he has sold 17 pieces, containing 875 yards. How many pieces, and how many yards remain?

Ans. 13 pieces; and 325 yards.

12. Bought of A 385 barrels of flour, and 2805 bushels of corn; of which I sold to B 109 barrels of flour, and 936 bushels of corn. What quantity of each remains unsold?

Ans. 276 barrels of flour; 1869 bushels of corn.

13. A salter bought 35850 pounds of beef, and 150000 pounds of pork. Having exported 20500 pounds of the beef, and 75900 pounds of the pork, what quantity of each has he still on hand? *Ans.* 15350 pounds of beef; 74100 of pork.

14. A farmer raised 1200 bushels of wheat, and 213 bushels of oats. He sold to A 835 bushels of wheat, and 179 of oats, and the remainder of the crop to B. What amount of produce did the farmer sell to B?

Ans. 365 bushels of wheat; and 34 of oats.

15. A merchant bought 375 yards of cloth, for 1645 dollars; of which he has sold 103 yards, for 685 dollars. How many yards of the cloth remain on hand, and for what sum must the remainder be sold, to lose nothing?

Ans. 272 yards; and 960 dollars.

16. A grocer bought coffee for 420 dollars, and sugar for 545 dollars. He sold the coffee for 500 dollars, and the sugar for 603 dollars; what did he gain on each?

Ans. 80 dollars on the coffee; and 58 on the sugar.

17. Sold a lot of hams for 275 dollars—which made a profit of 43 dollars; and a lot of cheese for 305 dollars—which made a profit of 39 dollars. What did each kind cost me?

Ans. The hams, 232 dollars; the cheese, 266 dollars.

18. Four persons contribute towards the founding of a literary institution; A gave 2500 dollars, B 3300, C 375 less than A, and D 283 less than B. What were the sums contributed by C and D respectively?

Ans. 2125 dollars by C; and 3017 by D.

19. Find the Difference 230469—85340. *Ans.* 145129.
20. Find the Difference 349130—94131. *Ans.* 254999.
21. Find the Difference 400500—80973. *Ans.* 319527.
22. Find the Difference 739745—76378. *Ans.* 663367.
23. Find the Difference 511839—84674. *Ans.* 427165.
24. Find the Difference 601813—13834. *Ans.* 587979.
25. Find the Difference 803460—45009. *Ans.* 758451.
26. Find the Difference 910311—87300. *Ans.* 823011.
27. Find the Difference 999830—99001. *Ans.* 900829.

28. A borrowed of B 8794 dollars; of which he paid to B at one time 2340 dollars, and at another time 1375 dollars. What balance remains to be paid?

From the whole debt subtract the first payment, and from the remainder subtract the second payment. *Ans.* 5079 dollars.

29. A person who set out on a journey of 900 miles, traveled the first week 255 miles, and the second 233 miles. How many miles then remained to be traveled? *Ans.* 412 miles.

30. A testator provided in his will, that his estate, valued at 25479 dollars, should be divided as follows, viz: his only son to have 13500 dollars, his only daughter 5375 dollars, and his widow the remainder. What sum did the widow receive?

Ans. 6604 dollars.

31. A gentleman who owned a tract of land containing 15735 acres, sold from it, at different times, 3050 acres, 521 acres, 200 acres, and 2370 acres. How many acres of the tract had he then remaining? *Ans.* 9594 acres.

32. Borrowed of my neighbor at one time 175 dollars, at another 340 dollars, and at another 520 dollars. Having paid him 685 dollars, what balance have I yet to pay?

From the sum 175+340+520, *subtract* 685.

Ans. 350 dollars.

33. Put in store at one time 500 pounds of hemp; at another time 3800 pounds; and at another 2005 pounds. Having withdrawn 3473 pounds, what quantity remains in store?

Ans. 2832 pounds

34. A merchant bought flour at one time for 325 dollars, and at another time for 460 dollars. Having become damaged, the whole was sold at a loss of 184 dollars; for what sum was it sold? *Ans.* 601 dollars.

35. A grocer purchased brandy for 195 dollars, and wine for 370 dollars. He sold the former at a loss of 35 dollars, and the latter at a loss of 80 dollars. What did he get for the whole? *Ans.* 450 dollars.

36. A manufacturer sold two bales of cotton, which together contained 2000 yards. The first bale containing 985 yards, how many yards were in the second bale? *Ans.* 1015 yards.

37. A bought of B 875 acres of land, for 23400 dollars. For 500 acres of the tract, he paid 11379 dollars; how many acres were in the remainder of the tract, and for what sum was it purchased? *Ans.* 375 acres; and 12021 dollars.

38. One farmer had two fields of corn, each producing 230 barrels; and another had three fields, each producing 195 barrels. Which of the two raised the larger crop, and by how many barrels? *Ans.* The second, by 125 barrels.

39. Three persons, A, B, and C, propose to purchase a manufactory, valued at 25850 dollars. A agrees to pay 5000 dollars, B twice as much as A, and C the remainder; what sum will C have to pay? *Ans.* 10850 dollars.

40. A merchant exchanges a stock of goods worth 6725 dollars, and a house worth 3120 dollars, with a farmer, for a tract of land valued at 5900 dollars,—the deficiency on the part of the land to be made up in money. What sum will the merchant receive? *Ans.* 3945 dollars.

41. A has 2070 dollars more than B, and 375 dollars less than C, who has 4000 dollars; what sum then has B ?

Ans. 1555 dollars.

42. Two travelers start together from the same place, and journey in the same direction, for three successive weeks,—the first at the rate of 240, the second 275 miles, per week; now far are the travelers apart at the end of the time named ?

Ans. 105 miles.

43. Having 4800 dollars on hand, I wish to add to this a sum which will enable me to purchase a farm at 5390 dollars, and leave 300 dollars for other purposes. Required the sum to be added to the first one. *Ans.* 890 dollars.

44. A planter sold cotton amounting to 3460 dollars, and out of these proceeds purchased groceries for 150 dollars, and other provisions for 375 dollars. How much of the first sum had he remaining ? *Ans.* 2935 dollars.

45. A cabinet maker sold furniture to the amount of 4000 dollars, and received, in part payment, at different times, 100 dollars, 200 dollars, 475 dollars and 904 dollars. How much of the debt remains to be paid ? *Ans.* 2321 dollars.

46. A says to B, "in my fruitful pastures, I have 300 fat oxen." B replies, "then I have 120 more than you." Says C, "I lack but 35 of having as many as you both." How many had C ? *Ans.* 685 oxen.

47. In 1820, the population of the United States was 9 millions, 638 thousand, one hundred and sixty-three; in 1830, it was 12 millions, 856 thousand, one hundred and sixty-five; and in 1840, 17 millions, 63 thousand, three hundred and fifty-three. What was the increase from 1820 to 1830, and what from 1830 to 1840 ?

Ans. $\begin{cases} \text{3218002, from 1820 to 1830;} \\ \text{4207188, from 1830 to 1840.} \end{cases}$

48. The population of Europe being estimated at 238 millions, 473 thousands, 957; of Asia at 390 millions; of Africa at 65 millions; of Oceanica at 20 millions; of N. America at 35 millions; and of S. America at 15 millions, 240 thousand;—how much does the population of Asia exceed that of all the other grand divisions of the Earth ?

Ans. 16286043 inhabitants.

MULTIPLICATION.

§ **37.** MULTIPLICATION consists in finding the sum or *product* of a number *taken any given number of times.*

The number *multiplied* is called the *multiplicand;* and the multiplying number, the *multiplier;* the two together are also called the *factors* of the product.

Thus, 3 *times* 5 is 15; then 3 is the *multiplier*, 5 the *multiplicand*, and 15 the *product*. Also 3 and 5 are the *factors* of 15.

How many is 3 times 2? Which is the multiplicand? The multiplier? The product? The *factors?*
How many is 3 times 3? 4 times 3? 5 times 3? 6 times 3?

Addition and Multiplication.

§ **38.** The addition of the *same number to itself*, repeatedly, is a *multiplication* of that number.

Thus 5+5+5 is 3 *times* 5; the *sum* or *product* being 15.

How many is 6+6, or *twice* 6? 6+6+6, or 3 times 6?
How many is 7+7, or twice 7? 7+7+7, or 3 times 7?
How many is 8+8+8, or 3 times 8? 8+8+8+8, or 4 times 8?
How many is 9+9+9, or 3 times 9? 9+9+9+9, or 4 times 9?

Recite the *elementary products, once* 1 is 1, *once* 2 is 2, &c., *twice* 1 is 2, *twice* 2 is 4, &c., 3 *times* 1 is 3, 3 *times* 2 is 6 &c., as given from left to right in the

Multiplication Table.

Once	1	2	3	4	5	6	7	8	9	10	11	12
Twice	*is* 2	*is* 4	*is* 6	*is* 8	*is* 10	*is* 12	*is* 14	*is* 16	*is* 18	*is* 20	*is* 22	*is* 24
3 *times*	*is* 3	*is* 6	*is* 9	*is* 12	*is* 15	*is* 18	*is* 21	*is* 24	*is* 27	*is* 30	*is* 33	*is* 36
4 *times*	*is* 4	*is* 8	*is* 12	*is* 16	*is* 20	*is* 24	*is* 28	*is* 32	*is* 36	*is* 40	*is* 44	*is* 48
5 *times*	*is* 5	*is* 10	*is* 15	*is* 20	*is* 25	*is* 30	*is* 35	*is* 40	*is* 45	*is* 50	*is* 55	*is* 60
6 *times*	*is* 6	*is* 12	*is* 18	*is* 24	*is* 30	*is* 36	*is* 42	*is* 48	*is* 54	*is* 60	*is* 66	*is* 72
7 *times*	*is* 7	*is* 14	*is* 21	*is* 28	*is* 35	*is* 42	*is* 49	*is* 56	*is* 63	*is* 70	*is* 77	*is* 84
8 *times*	*is* 8	*is* 16	*is* 24	*is* 32	*is* 40	*is* 48	*is* 56	*is* 64	*is* 72	*is* 80	*is* 88	*is* 96
9 *times*	*is* 9	*is* 18	*is* 27	*is* 36	*is* 45	*is* 54	*is* 63	*is* 72	*is* 81	*is* 90	*is* 99	*is* 108
10 *times*	*is* 10	*is* 20	*is* 30	*is* 40	*is* 50	*is* 60	*is* 70	*is* 80	*is* 90	*is* 100	*is* 110	*is* 120
11 *times*	*is* 11	*is* 22	*is* 33	*is* 44	*is* 55	*is* 66	*is* 77	*is* 88	*is* 99	*is* 110	*is* 121	*is* 132
12 *times*	*is* 12	*is* 24	*is* 36	*is* 48	*is* 60	*is* 72	*is* 84	*is* 96	*is* 108	*is* 120	*is* 132	*is* 144

Constant Product.

§ **39.** The Product of two numbers remains *the same*, when the multiplicand and multiplier are taken *the one for the other.*

Thus 3 *times* 15 gives the *same product* as 15 *times* 3.

For, 3 times 15 must be 15 times as many as 3 times 1, which is 3; that is, 3 times 15 is just as many as 15 times 3.

Prove that 4 times 23 is equal to 23 times 4. Prove that 5 times 17 is equal to 17 times 5. Prove that 6 times 39 is equal to 39 times 6.

Sign of Multiplication.

§ **40.** The *sign* $\times$, called *into*, placed between two numbers, signifies that they are to be *multiplied together.*

Thus 9×4, 9 *into* 4, signifies 9 *times* 4, or 4 *times* 9.

How many is 6×7?	5×9?	8×3?	4×11?	8×9?	3×7?
How many is 9×9?	7×8?	8×6?	4×12?	9×6?	8×4?
How many is 3×7?	5×8?	3×9?	5×10?	6×7?	2×11?
How many is 8×7?	6×5?	7×6?	6×11?	4×7?	3×12?

Product of Concrete Numbers.

§ **41.** The product of a concrete number taken any number of times, is a *similar* concrete number.

Thus 3 times 5 *cents* is 15 *cents.*

What is 3 times 9 pounds? 4 times 7 yards? 3 times 8 dollars? 6 times 9 miles? 7 times 10 acres? 8 times 11 furlongs?

§ **42.** A concrete number cannot be taken *concretely* as a *multiplier;* for, as a multiplier, a number can express nothing but *repetitions* of the multiplicand.

For example, 3 *hats* at 5 dollars apiece would amount to 3 times 5 dollars, which is 15 dollars; the multiplier 3 expressing only *repetitions* of 5 dollars. We multiply by 3,—not by 3 *hats.*

What would 4 slates amount to at 10 cents apiece? What would 5 books amount to at 8 cents apiece? What would 6 pounds of butter amount to at 12 cents a pound? What would 7 cords of wood amount to at 3 dollars a cord? What would 8 pair of boots amount to at 5 dollars a pair? What would 9 tons of hay amount to at 8 dollars a ton?

If a man can walk 4 miles an hour, how far could he walk in 10 hours? If a yard of cloth sells for 6 dollars, what should be paid for 11 yards at the same rate? Allowing 12 men to do a piece of work in 5 days, in what time ought one man to do the same?

RULE V.

§ 43. *To Multiply by a Number not exceeding* 12; *or by such Number with* 0s *annexed.*

1. Proceeding from *right to left,* multiply each figure of the multiplicand, and under each set its product, when *less than* 10.

2. When the product is 10 or more, set down its *right hand figure,* and add the left figure or figures to the next product. Set down the whole of the last product.

3. *Ciphers in the right of either or both of the factors,* are omitted in multiplying; but as many 0s must be placed in the *right of the product.*

4. When the multiplier is 10, or 100, &c., the product will be formed by merely *annexing to the multiplicand* as many 0s as there are 0s in the right of the Multiplier.

EXAMPLES.

1. To Multiply 5070 by 3; that is, to find 3 *times* 5070.

$$\begin{array}{r} 5070 \\ 3 \\ \hline 15210 \end{array}$$

We place a 0 in the right of the product, for the 0 in the right of the multiplicand; and then say,

3 times 7 is 21; setting down the 1, we say 3 times 0 is 0, and the 2, from 21, added, makes 2; 3 times 5 is 15.

The required product is 15210.

2. To Multiply 243 by 30; or to find 30 *times* 243.

$$\begin{array}{r} 243 \\ 30 \\ \hline 7290 \end{array}$$

Having placed a 0 in the right of the product, for the 0 in the right of the multiplier, we say

3 times 3 is 9; 3 times 4 is 12; 3 times 2 is 6, and 1 makes 7.

3. To multiply 359 by 100, we merely *annex two* 0s to the multiplicand, and thus find the *product* 35900.

☞ The adding of the *left hand figure* in the product of any figure, to the next product, is carrying *one* for every *ten*, as in Addition.

In the first example, we might have begun by saying 3 times 0 is 0, and then said 3 times 7 is 21, &c.* But this amounts to the same thing as merely placing a 0 on the right, and beginning the multiplication with 3 times 7.

In the second example, 243 multiplied by 3 produces 729. But since the true multiplier is 30, which is 10 times 3, the true product must be 10 times 729; and this *tenfold* value is assigned to 729 by the 0 on the right, which removes each of its figures one place farther toward the left from units (§ 16).

We thus find that for each 0 omitted in the right of either factor, a 0 must be placed in the right of the product. ☜

The Operation Proved.

§ 44. Multiplication may be *verified* or *proved*, by multiplying the multiplicand by the multiplier *minus* 1, and adding the multiplicand to the product thus obtained; the sum must be equal to the product found with the *entire multiplier*.

EXERCISES.

1. Mary bought two books, at 31 cents apiece. How many cents did she pay for them?

The 2 *books cost twice* 31 *cents;* or 31 *cents*+31 *cents* (§ 38).

Ans. 62 cents.

2. A farmer sold 3 horses at 125 dollars apiece. What sum did he receive for them? *Ans.* 375 dollars.

3. What would be the value of 20 shares of road stock, at 95 dollars for each share? *Ans.* 1900 dollars.

4. What would be the weight of 30 bales of cotton, allowing 450 pounds to each bale? *Ans.* 13500 pounds.

5. How many pounds of flour are there in 10 barrels of flour, there being 196 pounds in each barrel? *Ans.* 1960 pounds.

6. If a steamboat can run 305 miles in a day, how many miles could it run in 4 days? *Ans.* 1220 miles.

7. There being 1760 yards in a mile, what number of yards is there in 5 miles? *Ans.* 8800 yards.

8. A farmer sold 100 acres of land, at 43 dollars per acre. What sum did he receive for the land? *Ans.* 4300 dollars.

9. There being 660 feet in a furlong, how many feet are there in a mile, which is 8 furlongs? *Ans.* 5280 feet.

10. A merchant bought 60 sacks of coffee, at 13 dollars a sack. What did the whole amount to? *Ans.* 780 dollars.

11. A butcher bought 7 fat oxen, at an average of 32 dollars a head. What sum did he pay for them? *Ans.* 224 dollars.

12. Find the sum that should be paid for 9 horses at 130 dollars apiece, and 10 cows at 21 dollars apiece.
Ans. 1380 dollars.

13. Bought at one time 5 coils of rope, each containing 139 yards; and at another, 7 coils, each containing 150 yards. What was the whole number of yards bought?
Ans. 1745 yards.

14. A farmer bought 5 horses, at 95 dollars each; 12 mules, at 60 dollars each; and 40 head of cattle, at 27 dollars a head. What did he pay for all the stock purchased?
Ans. 2275 dollars.

15. Sold to A 7 pieces of cotton, to B 11 pieces, and to C 12 pieces,—each piece containing 31 yards. What was the number of pieces, and what the number of yards that was sold?
Ans. 30 pieces; and 930 yards.

16. A merchant bought 213 beaver hats, at 4 dollars apiece. What did the whole amount to?

The hats amounted to 213 *times* 4 *dollars; but* 4 *times* 213 *is the same number as* 213 *times* 4. (§ 39). *Ans.* 852 dollars.

17. Bought 175 cords of wood, at 3 dollars per cord. What did the whole amount to? *Ans.* 525 dollars.

18. A planter sold 325 bales of cotton, at the average sum of 30 dollars per bale. What did the sale amount to?
Ans. 9750 dollars.

19. If a steam ship can run at the rate of 300 miles per day, how far would she run in 31 days. *Ans.* 9300 miles.

20. A drover sold 573 head of cattle, at the average price of 20 dollars a head. What did the sale amount to?
Ans. 11460 dollars.

21. A farmer filled, at one time, 83 sacks of corn; and at another time 112 sacks;—each sack containing 5 bushels. What quantity of corn was put into all the sacks?
Ans. 975 bushels.

22. Bought of A, 137 acres of land; of B, 89 acres; and of C, 384 acres;—at the average price of 40 dollars per acre. What quantity of land was bought, and what sum was paid for it? *Ans.* 610 acres; and 24400 dollars.

23. Shipped to New Orleans, at one time, 120 barrels of apples; at another, 75 barrels; at another, 100 barrels; and at another, 9 barrels;—each barrel containing 3 bushels. Required the number of barrels, and the number of bushels shipped. *Ans.* 304 barrels; and 912 bushels.

24. A merchant bought 275 yards of cloth, at 6 dollars per yard, of which he has sold 133 yards, at 10 dollars per yard. What would he gain on the whole, by selling the remainder at 9 dollars per yard? *Ans.* 958 dollars.

25. If 10 masons can build a wall, in 34 days; in how many days ought one mason to build the same wall?

It would take 1 *mason* 10 *times as long as it would* 10 *masons, to build the wall; that is,* 10 *times* 34 *days.* *Ans.* 340 days.

26. How long ought one man to subsist on a stock of provisions which would support 7 men for 29 days?

Ans. 203 days.

27. If 9 men could mow a certain meadow, in 19 days; in how many days ought one man to mow the meadow?

Ans. 171 days.

28. If 115 bushels of oats will feed one horse for 3 months; what quantity of oats would feed 12 horses, the same time?

Ans. 1380 bushels.

29. Allowing 20 pieces of artillery to demolish a fortress in 48 hours, in what time ought one piece to demolish the fortress?

Ans. 960 hours.

30. A lent B 8 yoke of working oxen, for 13 days. How long ought B to lend A 1 yoke, to requite the favor?

Ans. 104 days.

31. C exchanges silk, at 1 dollar per yard, with D, for broadcloth, at 7 dollars per yard. How many yards of silk should be given for 125 yards of broadcloth? *Ans.* 875 yards.

32. Two merchants barter as follows: A gives 75 yards of cloth, at 9 dollars per yard, to B, for 109 beaver hats, at 7 dollars apiece; the deficiency on either side being made good in money. Which of them receives money, and how much?

Ans. B receives 88 dollars.

33. A gave B 3 horses, and 30 head of cattle, for 50 barrels of flour and 200 barrels of corn. A sold the flour at 6 dollars, and the corn at 3 dollars a barrel; while B sold his horses at 120 dollars, and his cattle at 25 dollars, each? Which of the two gained by the trade, and how much?

Ans. B gained 210 dollars.

34. Find the Product 10×24738730. *Ans.* 247387300.
35. Find the Product 200×3076801. *Ans.* 615360200.
36. Find the Product 1200×706840. *Ans.* 848208000.
37. Find the Product 90×10707800. *Ans.* 963702000.
38. Find the Product 300×7060000. *Ans.* 2118000000.
39. Find the Product 1100×870003. *Ans.* 957003300.
40. Find the Product 12000×70000. *Ans.* 840000000.

RULE VI.

§ 45. *To Multiply by any Number exceeding* 12, *and containing two or more* SIGNIFICANT FIGURES.

1. Multiply by each *significant figure*, separately, of the multiplier,—placing the several rows of products one under another, with the first figure of each product *under the multiplying figure*,—and, in that order, add the several products together for the *entire product.*

2. *Ciphers in the right of either or both of the factors*, are omitted in multiplying; but as many 0s must be placed in the right of the product.

EXAMPLES.

1. To Multiply 8072 by 39; that is, to find 39 *times* 8072.

```
  8072
    39
 -----
 72648
24216
------
314808
```

Multiplying first by 9, we say 9 times 2 is 18, and set the first product figure 8 under the multiplying figure 9; then, 9 times 7 is 63, and 1 makes 64; 9 times 0 is 0, and 6 makes 6, &c.

Multiplying next by 3, we say 3 times 2 is 6, and set the 6 under the multiplying figure 3; 3 times 7 is 21; 3 times 0 is 0, and 2 is 2, &c.

Adding the two rows of product figures together, in the order in which they stand, we say 8 is 8; 6 and 4 are 10, &c.

The entire Product is 314808.

2. To Multiply 8420 by 30900; or to find 30900 *times* 8420;

```
     842|0
     309|00
   --------
    7578
  2526
---------
260178000
```

Omitting 0s in the right of both factors, we multiply 842 by 309, setting the first figures, 8 and 6, of the products under the multiplying figures, 9 and 3, respectively. To the product thus found, we annex three 0s, for the 0s omitted in multiplying.

☞ When the multiplying figure is *tens*, or *hundreds*, &c., the first product figure is set under *tens*, or *hundreds*, &c., respectively, to *increase the product* in the same degree in which the multiplying figure is increased in value, by distance from the unit's place (§ 16).

In the *first* example, 2 multiplied by 3 produces 6. But the 3 is 3 *tens*, or 10 *times* the simple 3; the 6 must therefore be 10 *times* the simple 6; and this *tenfold* value is assigned to 6 by setting it under the 3 *tens*, and *adding it in the ten's column.*

In the *second* example, we multiplied 842 by 309. In 309 the 3 is 3 *hundreds*, or 100 *times* the simple 3. The first product figure 6 must therefore be 100 *times* the simple 6; and this *hundredfold* value is assigned to 6 by setting it under the 3 *hundreds*, and *adding it in the hundreds' column.*

The first product figure being set in its proper place, the second, third, &c., fall in their proper places, in ascending orders of units towards the left.—The sum of the *partial* products is the entire product (§ 23).

As already shown, in connexion with Rule V, for each 0 omitted in the right of either factor, a 0 must be placed in the right of the product. ☜

EXERCISES.

41. Find the Product 24×360730. *Ans.* 8657520
42. Find the Product 307×80379. *Ans.* 24676353.
43. Find the Product 5372×7684. *Ans.* 41278448.
44. Find the Product 80760×870. *Ans.* 70261200.
45. Find the Product 13×730000. *Ans.* 9490000.
46. Find the Product 740×87305. *Ans.* 64605700.
47. Find the Product 9034×8076. *Ans.* 72958584.
48. Find the Product 30407×307. *Ans.* 9334949.
49. Find the Product 400070×990. *Ans.* 396069300.

50. The number of yards in a mile being 1760, how many yards are there in 15 miles?

The number of yards in 15 *miles is* 15 *times* 1760 *yards.*

Ans. 26400 yards.

51. There are 24 hours in one day. How many hours then are there in a year of 365 days?

The number of hours in a year is 365 *times* 24 *hours;* but 24 times 365 will produce the same number, and it is more convenient to make the *less* number the *multiplier.*

Ans. 8760 hours.

52. A hogshead of wine or brandy contains 63 gallons. How many gallons would there be in 250 hogsheads?

Ans. 15750 gallons.

53. What sum should be paid for a plantation containing 765 acres, at 43 dollars per acre? *Ans.* 32895 dollars.

54. If a man can walk 35 miles in a day, how far could he walk, at that rate, in a year, or 365 days? *Ans.* 12775 miles.

55. A has 340 acres of land worth 18 dollars an acre; and B has 239 acres worth 22 dollars an acre. How many acres have the two together, and what is the value of the whole.

Ans. 579 acres; and 11378 dollars.

56. A merchant bought 475 barrels of flour, at 15 dollars a barrel. He sold 280 barrels of it, at 16 dollars, and the rest at 14 dollars a barrel; what did he gain or lose?

Ans. gained 85 dollars.

57. One manufacturer exported 234 bales of cotton cloth,—each bale containing 2400 yards; another exported 370 bales, each containing 1050 yards. Which of them exported the greater quantity, and by how many yards?

Ans. The first, 173100 yards.

58. Farmer A had in wheat 205 acres, which produced 27 bushels per acre. Farmer B had 320 acres, which produced 19 bushels per acre. What quantity of wheat was raised by them both. *Ans.* 11615 bushels.

59. A speculator bought 150 head of cattle, and 47 mules. He made a profit of 13 dollars a head on the former, and 17 on the latter; what was gained by the speculation?

Ans. 2749 dollars.

60. Bought 360 acres of land, at 35 dollars per acre; and at another time double that quantity, at double the price per acre. What was the whole quantity of land purchased, and the sum paid for it? *Ans.* 1080 acres; and 63000 dollars.

61. Two persons start together from the same place, and travel in the same direction. One proceeds at the rate of 29 miles per day, and the other at the rate of 31 miles per day. What distance will be between them at the end of 25 days?

Ans. 50 miles.

62. A merchant bought 18 bales of linen, each containing 22 pieces, and each piece containing 40 yards. How many pieces and how many yards did he buy?

Ans. 396 pieces; and 15840 yards.

63. In a certain orchard there are 30 rows of apple trees, with 44 trees in each row. Allowing 2500 apples to each tree what number of apples would there be in the orchard?

Ans. 3300000 apples.

64. A farmer bought three tracts of land. The first and second contained each 280 acres, and the third as many as both the other two; how many acres did the farmer purchase, and what did the whole amount to, at 33 dollars per acre?

Ans. 1120 acres; and 36960 dollars.

65. Allowing a person's annual income to be 5000 dollars, and his daily expenses 3 dollars, what would be the amount of his annual saving,—there being 365 days in a year?

Ans. 3905 dollars.

66. If 327 head of cattle were purchased at 13 dollars a head, and 405 were purchased at 11 dollars a head, what would be the profit or loss on the whole at 12 dollars a head?

Ans. Profit, 78 dollars.

67. A planter sold 139 bales of cotton, at an average of 32 dollars per bale, and out of the proceeds bought 29 mules, at 49 dollars each, and 4 pair of oxen, at 52 dollars a pair; what sum had he left from the sale of his cotton?

Ans. 2819 dollars.

68. A sends 209 tons of coal to New York city; B sends as much as A, wanting 10 tons; and C sends as much as A and B together. What was each man's proceeds of sale, at 13 dollars per ton? *Ans.* A's 2717 dollars; B's 2587; C's 5304.

69. The President of the United States receives a salary of 25 thousand dollars a year. To what sum does his salary amount in 4 years, or one presidential term?

Ans. 100000 dollars.

70. The circumference of the Earth is about 25 thousand miles, and the distance to the Sun is 3 thousand 8 hundred times the Earth's circumference. What then is the distance to the Sun? *Ans.* 95000000 miles.

71. The velocity of light is 192 thousand 500 miles per second. Through what distance then does light move in one minute, which is 60 seconds? *Ans.* 11550000 miles.

72. The Earth turns around its axis once in every 24 hours, and moves 68 thousand miles an hour in its orbit around the Sun. How far then are we carried along the Earth's orbit during one revolution of the Earth on its axis?

Ans. 1632000 miles.

DIVISION.

§ 46. DIVISION consists in finding *how many times* a greater number *contains* a less, or *what part* a less number is of a greater.

The number to be divided is called the *dividend;* the dividing number the *divisor;* and the *number* or *part* found, the *quotient.*

One *half* is one of the *two equal parts,*—two *thirds* are two of the *three equal parts,*—and so on, into which any quantity may be divided.

What is meant by one *third?* By one *fourth?* By three *fourths?* By one *fifth?* By two *fifths?* By one *tenth?* By five *ninths?*

When we say 2 is contained in 6, 3 times, we divide 6 by 2. 6 is the *dividend,* 2 the *divisor,* and 3 the *quotient.*

Also, 2 is *one third* of 6, because if 6 were divided into *three equal parts,* each part would be 2.

How many times is 3 contained in 6? 3 is what part of 6?
How many times is 4 contained in 12? 4 is what part of 12?
How many times is 5 contained in 20? 5 is what part of 20?
If the dividend be 24, and the divisor 4, what will the *quotient* be? If the dividend be 35, and the divisor 5? If the dividend be 42, and the divisor 7? If the dividend be 56, and the divisor 8?

§ 47. The Quotient of a *less number* divided by a greater, is the *part that the less is* of the greater; and is denoted by the less over the greater, with a line between them.

Thus 1 divided by 2 is $\frac{1}{2}$ *one half,* because 1 is *one half* of 2.

How much is 1 divided by 3; that is, 1 is what part of 3?
How much is 1 divided by 4? 1 divided by 5? 1 divided by 6?
How much is 2 divided by 3; that is, 2 is what part of 3?
How much is 2 divided by 5? 2 divided by 7? 3 divided by 5?

Subtraction and Division.

§ 48. The subtraction of a less number from a greater, repeatedly, is equivalent to *dividing the greater* by the less, because it shows how many times the greater *contains the less.*

Thus 5 from 15 leaves 10, 5 from 10 leaves 5, and 5 from 5 leaves 0; so that 5 may be *subtracted* 3 times from 15, or is *contained* 3 times in 15.

How many times may 5 be subtracted from 20 ? How many times may 6 be subtracted from 24 ? 7 from 35 ? 8 from 48 ?

Multiplication and Division.

§ **49.** Multiplication and Division are the *reverse of each other.*

In Multiplication, two numbers or *factors* are given, to find their product; in Division, a product and one of its factors are given, to find the *other factor.*

The product being 15, and one factor 3, what is the other factor ?
The product being 30, and one factor 5, what is the other factor ?
The product being 36, and one factor 9, what is the other factor ?
The product being 63, and one factor 7, what is the other factor ?

Reciprocal of a Number.

§ **50.** The *reciprocal* of a number is a unit or 1 *divided by that number.*

Thus the reciprocal of 2 is $\frac{1}{2}$, and of 3 is $\frac{1}{3}$.

What is the reciprocal of 4 ? Of 5 ? Of 6 ? Of 10 ? Of 20 ?

The Quotient as a Part of the Dividend.

§ **51.** The Quotient is always such a part of the dividend as is expressed by the *reciprocal of the divisor.*

Thus 15 divided by 5 gives 3, and 3 is $\frac{1}{5}$ of 15.

Again, if we divide 2 by 3, the quotient will be $\frac{2}{3}$ (§ 47); and since *two thirds* of any quantity is *one third* of *two such quantities,* $\frac{2}{3}$ is equal to $\frac{1}{3}$ of 2; *such a part of the dividend* 2 as is expressed by the *reciprocal* of the divisor 3.

$\frac{3}{4}$ of 1 *cent* is what part of 3 *cents ?* $\frac{3}{4}$ of 1 is what part of 3 ?
$\frac{2}{5}$ of 1 pint is what part of 2 pints ? $\frac{2}{5}$ of 1 is what part of 2 ?
$\frac{5}{7}$ of 1 mile is what part of 5 miles ? $\frac{5}{7}$ of 1 is what part of 5 ?

$\frac{1}{6}$ of 5 cents is what part of 1 cent ? $\frac{1}{6}$ of 5 is what part of 1 ?
$\frac{1}{8}$ of 7 pints is what part of 1 pint ? $\frac{1}{8}$ of 7 is what part of 1 ?
$\frac{1}{9}$ of 8 miles is what part of 1 mile ? $\frac{1}{9}$ of 8 is what part of 1 ?

How would you find $\frac{1}{2}$ of any number ? $\frac{1}{3}$ of any number ? $\frac{1}{7}$ of any number ? $\frac{1}{10}$ of any number ? $\frac{1}{25}$ of any number ?

Sign of Division

§ **52.** The *sign* ÷, called *by,* placed between two numbers, signifies that the first of them is to be *divided by the second.*

Thus 36÷9, 36 *by* 9, signifies that 36 is to be divided by 9.

How many is 24÷6? 35÷7? 63÷9? 100÷10? 144÷12?
How much is 5÷13? 8÷23? 9÷34? 11÷65? 13÷95?

Division is also denoted by the dividend over the divisor with a line between them.

Thus $\frac{63}{9}$ denotes the same as 63÷9.

Quotient of Concrete Numbers.

§ 53. When the dividend and divisor are *similar* concrete numbers, the quotient is the number of times the dividend *contains the divisor*, or the *part the dividend is* of the divisor.

Thus 12 *cents*÷3 *cents* gives 4; and 12 cents÷13 cents gives $\frac{12}{13}$ (§ 47).

How many times 4 miles in 12 miles? 5 pounds in 30 pounds? 6 inches in 42 inches? 7 yards in 77 yards? 12 dollars in 96 dollars? What part is 3 days of 7 days? What part is 9 ounces of 20 ounces? What part is 20 feet of 49 feet?

§ 54. When the dividend and divisor are *dissimilar* concrete numbers, the quotient is such a *part of the dividend* as is expressed by the reciprocal of the divisor taken *abstractly*.

For example, if 5 *pencils* cost 20 *cents*, one pencil will cost 20 *cents*÷5; that is, $\frac{1}{5}$ of 20 cents, which is 4 cents.

If 3 *slates* cost 36 *cents*, what will one slate cost? If 4 *hats* cost 16 *dollars*, what will one hat cost? If in 9 *hours* a stage runs 54 *miles*, at what rate does it run per hour?

Remainder in Division.

§ 55. A *remainder*, in Division, is an *overplus* or *excess* of *the dividend* above so many times the divisor as it is contained in the dividend.

Thus 5 is contained in 17, 3 times, with 2 *over*, since 3 times 5 is 15; then 2 is the *remainder* of the dividend.

If the divisor be 6, and the dividend 27, what will the *quotient* and the *remainder* be? If the divisor be 8, and the dividend 45? If the divisor be 9, and the dividend 70?

§ 56. The *remainder* divided by the divisor, and so annexed to the quotient, *completes the quotient*.

Thus 17÷5 gives quotient 3, and remainder 2. This 2÷5 gives $\frac{2}{5}$ (§ 47); the *complete quotient* is then $3\frac{2}{5}$, *three* and *two-fifths*.

In like manner find the quotient of 9÷2. Of 13÷4. Of 21÷5. Of 27÷6. Of 30÷7. Of 41÷8. Of 100÷9.

Constant Quotient.

§ 57. The quotient of two numbers remains *the same*, when those numbers are *both multiplied*, or both divided, by the same number.

For example, 6÷2 gives 3, and 5 times 6÷5 times 2, that is, 30÷10, also gives 3. By reversing the process, we find 30÷10 equal to $\frac{1}{5}$ of 30÷$\frac{1}{5}$ of 10.

RULE VII.

§ 58. *To Divide by a number not exceeding* 12; *or by such Number with* 0s *annexed.*

1. Take figures enough in the *left of the dividend* to contain the divisor, and set down the number of times the divisor goes therein, noting the *excess*, if any.

2. Take the next figure of the dividend; with the *preceding excess*, if any, *prefixed*, and set the number of times the divisor is found therein on *the right* of the first quotient; if the divisor *will not go therein*, put a 0 in the quotient, and include the next figure in dividing; and so on.

3. *Ciphers in the right of the divisor* are omitted in dividing; but as many figures must be omitted in the right of the dividend, and *annexed to the remainder*. If there be no other remainder, these figures will form the remainder.

4. When the divisor is 10 or 100, &c., take for the *remainder* as many figures from the right of the dividend as there are 0s in the right of the divisor, and the *other figures* of the dividend for the *quotient*.

5. *Under the remainder*, if any, set the given divisor, and annex the part so found to the quotient.

EXAMPLES.

1. To divide 7805 by 3.

$$\begin{array}{r} 3\underline{)7805} \\ 2601\frac{2}{3} \end{array}$$

We say, 3 in 7, *twice*, and 1 *over;* prefixing the 1 to the next figure 8 of the dividend, we have 18; 3 in 18, 6 times; 3 in 0, 0 time; 3 in 5, *once*, and 2 over. This excess 2 is the remainder,—under which setting the divisor 3, we have $\frac{2}{3}$ to annex to the quotient (§ 56).

The quotient $2601\frac{2}{3}$ shows that the dividend 7805 contains the divisor 3, 2601 times, and $\frac{2}{3}$ of the divisor, besides.

The quotient is also $\frac{1}{3}$ of 7805 (§ 51.)

2. To divide 13127 by 120.

$$\begin{array}{r} 12|0)\underline{1312|7} \\ 109\tfrac{47}{120} \end{array}$$

Omitting the 0 in the right of the divisor, and the 7 in the right of the dividend, we say,

12 in 13 *once*, and 1 over; prefixing the 1 over to the next figure of the dividend, we say 12 in 11, 0 time; including the next figure 2 of the dividend, we say 12 in 112, 9 times, and 4 over.

Annexing the 7, omitted, to the 4, we have the remainder 47, under which setting the *given divisor* 120, we have $\frac{47}{120}$ to annex to the quotient.

3. To divide 14723 by 100, we take the two figures 23 from the right of the dividend, for the *remainder*, and the other figures 147 for the *quotient*.

Hence the *complete* quotient is $147\frac{23}{100}$.

☞ The figures of the dividend first taken in dividing, have a *local value* 10 times, or 100 times, &c., their *simple value*, according as *one*, or *two*, &c., figures follow them on the right (§ 16); and the first quotient figure must therefore have 10 *times*, or 100 *times*, &c., its simple value. The proper local value is assigned to the quotient figure, by the succeeding quotient figures,—these being always just as many as the succeeding figures of the dividend. In like manner each quotient figure receives its proper value.

The *excess* belonging to any particular place in the dividend, is so many *tens* in the next place on the right (§ 11); and is made *tens* to the next figure by prefixing it to that figure.

In the second example, we divided 12 in 1312, instead of 120 in 13127. But 120 is 10 times 12, and 13127 is 13120+7, or 10 times 1312,+7; and 12 in 1312 gives the same quotient as 10 times 12 in 10 times 1312 (§ 57).

In the same way it may be shown that any number of 0s may be omitted in the right of the divisor, if an equal number of figures be omitted in the right of the dividend. ☜

The Operation Proved.

§ 59. Division may be *verified* or *proved*, by multiplying the divisor and quotient together, and adding the remainder, if any, to the product; the result must be equal to *the dividend*.

EXERCISES.

1. How many barrels of apples, at 2 dollars per barrel, may be bought for 150 dollars?

The number of barrels that may be bought, is the number of times that 2 dollars *is contained in* 150 dollars (§ 53).

Ans. 75 barrels.

2. At 3 dollars per yard, how many yards of broadcloth may be purchased for 387 dollars? *Ans.* 129 yards.

3. How many cords of wood, at 4 dollars per cord, may be purchased for 600 dollars? *Ans.* 150 cords.

4. At 5 dollars apiece, how many superfine beaver hats may be purchased for 3700 dollars? *Ans.* 740 hats.

5. How many dozen of shoes, at 6 dollars per dozen, may be purchased for 750 dollars? *Ans.* 125 dozen.

6. There being 7 days in a week, it is required to find how many weeks there are in 728 days? *Ans.* 104 weeks.

7. If one box will hold 80 pair of shoes, how many of such boxes will be required to contain 1840 pair? *Ans.* 23 boxes.

8. At the rate of 900 dollars each, how many dwelling houses could be built for 11700 dollars? *Ans.* 13 houses.

9. At 11 dollars each per month, how many laborers could be hired a month for 2530 dollars? *Ans.* 230 laborers.

10. At 120 dollars apiece, how many fine horses could I purchase for the sum of 4200 dollars? *Ans.* 35 horses.

11. What quantity of flour could be bought for 3 dollars, when the price is 5 dollars per barrel?

One dollar would buy $\frac{1}{5}$ *of a barrel; hence* 3 *dollars would buy* $\frac{3}{5}$ *of a barrel.*

Or, 3 *dollars would buy the* same part *of a barrel that* 3 *dollars is of* 5 *dollars:* 3 *dollars is* $\frac{3}{5}$ *of* 5 *dollars* (§ 47).

Ans. $\frac{3}{5}$ of a barrel.

12. At the rate of 15 dollars per ton, what quantity of hay could be bought for 7 dollars? *Ans.* $\frac{7}{15}$ of a ton.

13. If land sell at the rate of 50 dollars per acre, what quantity of land could be purchased for 17 dollars?

Ans. $\frac{17}{50}$ of an acre.

14. A ferryboat is valued at 100 dollars. What share or interest in the boat could be purchased for 37 dollars?

Ans. $\frac{37}{100}$ of it.

15. A person who undertook a journey of 425 miles, having traveled 89 miles; what part of the journey has he accomplished? *Ans.* $\frac{89}{425}$ of it.

16. A manufactory is estimated to be worth 12000 dollars. What interest in it could be purchased for 2143 dollars?

Ans. $\frac{2143}{12000}$ of it.

17. What quantity of cloth, at 10 dollars a yard, ought I to have for 503 dollars ?

The divisor being 10 *we omit the* 3 *in the right of the dividend,* 503, *and thus find* 10 *in* 503, 50 times, *with remainder* 3.

The quotient is therefore $50\frac{3}{10}$.

The 50 *times* 10 *in* 503 *gives* 50 *yards, and the* 3 *dollars over will buy* $\frac{3}{10}$ *of a yard.* *Ans.* $50\frac{3}{10}$ yards.

18. At 3 dollars per cord, what quantity of wood might be bought for 175 dollars ? *Ans.* $58\frac{1}{3}$ cords.

19. At the rate of 7 miles an hour, how many hours would a stage coach be in running 975 miles ? *Ans.* $139\frac{2}{7}$ hours.

20. Allowing a workman to build 9 rods of fence in a day, how many days would he require to build 1603 rods ?
Ans. $178\frac{1}{9}$ days.

21. How many tons of hay, at 11 dollars per ton, may be purchased for 3071 dollars ? *Ans.* $279\frac{2}{11}$ tons.

22. How many acres of land, at 40 dollars per acre, may be bought for 7309 dollars ? *Ans.* $182\frac{29}{40}$ acres.

23. How many tons of steel, at 120 dollars per ton, may be bought for the sum of 1577 dollars ? *Ans.* $13\frac{17}{120}$ tons.

24. Allowing 30 days to make a month, how many months would there be in 200 days+177 days ? *Ans.* $12\frac{17}{30}$ months.

25. A has 2345 dollars, and B 3000 dollars. What quantity of land can the two together purchase, at the rate of 50 dollars per acre ? *Ans.* $106\frac{45}{50}$ acres.

26. A farmer sold beef for 130 dollars, and pork for 200 dollars. With the proceeds of these sales, he wishes to purchase corn at 3 dollars per barrel; what quantity can he buy ?
Ans. 110 barrels.

27. A merchant sold cloth for 423 dollars, cotton for 125 dollars, and silk for 300 dollars; and invested the proceeds in sugar at 12 dollars per barrel. How many barrels of sugar did he buy ? *Ans.* $70\frac{8}{12}$ barrels.

28. If 5 acres of ground sell for 163 dollars, what is the price per acre ?

One acre being $\frac{1}{5}$ *of* 5 *acres, is worth* $\frac{1}{5}$ *of* 163 dollars; *which is* 163 dollars÷5 (§ 51 and 54). *Ans.* $32\frac{3}{5}$ dollars.

29. If a man travels 200 miles in 6 days, at what rate does he travel per day ? *Ans.* $33\frac{2}{6}$ miles.

30. If 7 head of horses sell for 850 dollars, what will be the average sum received for each? *Ans.* $121\frac{3}{7}$ dollars.

31. Allowing 1703 acres of land to be divided into 8 farms of equal size, what will be the number of acres in each?
Ans. $212\frac{7}{8}$ acres.

32. If 100 yards of li en cost 57 dollars, what is the price per yard?

One yard costs $\frac{1}{100}$ of 57 *dollars*, or 57 *dollars*$\div$100; *which is*, $\frac{57}{100}$ of a dollar (§ 47 and 51). *Ans.* $\frac{57}{100}$ of a dollar.

33. If 50 bushels of corn sell for 13 dollars, what is the selling price per bushel? *Ans.* $\frac{13}{50}$ of a dollar.

34. If 3 plantations of equal value sell for 8491 dollars, what sum would be received for each? *Ans.* $2830\frac{1}{3}$ dollars.

35. If 8 yards of Irish linen amount to 7 dollars, at what price per yard does the linen sell? *Ans.* $\frac{7}{8}$ of a dollar.

36. If the construction of 4 bridges on a turnpike road, cost 14803 dollars, what is the average cost of each?

Ans. $3700\frac{3}{4}$ dollars.

37. Allowing 600 acres of ground to produce 24000 bushels of wheat, what would be the produce per acre?

Ans. 40 bushels.

38. Allowing 900 barrels of flour to sell for 9900 dollars, what would be the selling price per barrel? *Ans.* 11 dollars.

39. A capital stock of 225000 dollars is held in 1000 equal shares. What is the amount of each share?

Ans. 225 dollars.

40. A farmer has 70 acres of land worth 2450 dollars, and 110 acres worth 4500 dollars. What is the whole worth, and what is each tract worth per acre?

Ans. The whole, 6950 dollars;
The 1st, 35 dollars, and
The 2d, $40\frac{10}{11}$ dolls. per acre.

41. A drover bought 54 head of cattle at one time, and 66 head at another time,—the whole amounting to 1800 dollars. What was the average cost per head? *Ans.* 15 dollars.

42. A planter bought 7 mules at 35 dollars apiece, 4 at 40 dollars apiece, and 9 at 37 dollars apiece. What sum was paid for all, and what was the average sum paid for each?

Ans. 738 dollars for all; and $36\frac{18}{20}$ for each.

43. In how many days could 10 men accomplish the same amount of work that one man could do in 349 days?

10 *men could do the work in* $\frac{1}{10}$ of the time *in which* 1 *man would do it; that is, in* $\frac{1}{10}$ *of* 349 days. *Ans.* $34\frac{9}{10}$ days.

44. In how many days ought 7 masons to build a wall which one mason could build in 175 days? *Ans.* 25 days.

45. How long ought 20 men to subsist on a stock of provisions which would suffice one man 433 days? *Ans.* $21\frac{13}{20}$ days.

46. How long ought 12 horses to be fed on a quantity of oats which would be sufficient for 1 horse 185 days?

Ans. $15\frac{5}{12}$ days.

47. How long might 30 workmen be employed for a sum of money which would pay 1 workman for 401 days?

Ans. $13\frac{11}{30}$ days.

48. In how many days could 10 men accomplish the same amount of work that 13 men could do in 349 days?

1 *man would do the work in* 13 times 349 days, *and* 10 *men would do it in* $\frac{1}{10}$ of the time *in which* 1 *man would do it.*

Ans. $453\frac{7}{10}$ days.

49. If 34 men can raise the walls of a fortification in 27 days, in how many days could 20 men do the same?

Ans. $45\frac{18}{20}$ days.

50. How long should 12 teams be employed in doing an amount of hauling which 23 teams could accomplish in 65 days? *Ans.* $124\frac{7}{12}$ days.

51. Allowing that 75 laborers could pave a street in 123 days, in how many days could 120 laborers pave the street?

Ans. $76\frac{105}{120}$ days.

52. A merchant bought 20 yards of cloth for 143 dollars; and, at another time, 30 yards for 165 dollars. At what price per yard was each purchase made? *Ans.* {The 1st, $7\frac{3}{20}$, and The 2d, $5\frac{15}{30}$ dollars.

53. A farmer sold his farm containing 273 acres, at 35 dollars per acre, and immediately invested the proceeds in another farm at 50 dollars per acre. How many acres did he buy?

Ans. $191\frac{5}{50}$ acres.

54. A plantation containing 1200 acres, was exchanged for another containing 1000 acres, and worth 53 dollars per acre. At what price per acre was the first plantation rated?

Ans. $44\frac{200}{1200}$ dollars.

55. A gentleman having on hand 5000 dollars, took 2300 dollars to purchase bank stock, at 100 dollars a share, and divided the remainder equally among three benevolent institutions. How many shares of stock did he purchase, and what sum did each institution receive?

Ans. 23 shares; and each inst'n rec'd 900 dollars.

56. Find the Quotient of $437630 \div 9$. *Ans.* $48625\frac{5}{9}$.
57. Find the Quotient of $873007 \div 11$. *Ans.* $79364\frac{3}{11}$.
58. Find the Quotient of $703687 \div 20$. *Ans.* $35184\frac{7}{20}$.
59. Find the Quotient of $937863 \div 110$. *Ans.* $8526\frac{3}{110}$.
60. Find the Quotient of $768377 \div 120$. *Ans.* $6403\frac{17}{120}$.
61. Find the Quotient of $3800370 \div 300$. *Ans.* $12667\frac{270}{300}$.
62. Find the Quotient of $70307600 \div 700$. *Ans.* $100439\frac{300}{700}$.

RULE VIII.

§ 60. *To Divide by any Number exceeding* 12, *and containing two or more* SIGNIFICANT FIGURES.

1. Take figures enough in the left of the dividend to contain the divisor, and *on the right* set the number of times the divisor goes therein.

2. Multiply the divisor by the quotient figure, and subtract the product from *those figures of the dividend* which were taken in dividing.

3. Bring down the next figure of the dividend, annexing it to the *remainder*, if any. Divide into the number so obtained, and set the quotient figure on the right of the first one; if the divisor will not go in the number, set a 0 in the quotient, and bring down the next figure of the dividend.

4. Multiply the divisor by the last quotient figure—subtract the product from *the number last divided*,—bring down the next figure of the dividend, and so on.

5. *Ciphers in the right of the divisor*, and the final remainder, are to be treated as directed in RULE VII.

EXAMPLE.

To divide 210490 by 690.

```
69|0)21049|0(305 40/690.
     207
     ---
       349
       345
       ---
        40
```

Omitting the 0 in the right of the divisor, and one figure in the right of the dividend, we say

69 in 210, 3 times, and set the 3 on the right; multiplying the 69 by 3, we get 207; subtracting 207 from 210, the remainder is 3; annexing the 4 from the dividend to the remainder 3, we say 69 in 34, 0 time; annexing the 9 from the dividend, we say 69 in 349, 5 times; multiplying 69 by 5, we get 345; subtracting 345 from 349, the remainder is 4, to which annexing the 0 omitted in the dividend, the *remainder* becomes 40. Under the remainder, setting the *given divisor* 690, we have $\frac{40}{690}$ to annex to the quotient.

The quotient $305\frac{40}{690}$ shows that the dividend contains the divisor 305 times, and $\frac{40}{690}$ of the divisor, besides. It is also $\frac{40}{690}$ *of the dividend* (§ 51).

In dividing by this method, observe that the product of the divisor and any quotient figure, must be *less than the number* from which it is *to be subtracted;* and that the remainder must always be *less than the divisor.*

☞ This Rule differs from RULE VII, only in requiring the products and remainders to be written down. By Rule VII, the divisor being a small number, the *multiplication* and *subtraction* are carried on *mentally.* Both Rules depend on the same principles. ☜

Proof by Addition.

§ 61. The operation by the last Rule may be proved, most readily, by adding up the *remainder,* if any, and the several *products* of the divisor and quotient figures, in *the order in which they stand.* The Sum must be equal to the *dividend.*

For the preceding example the proof may be presented thus:

```
           207
            345
              40
          ------
The Sum   210490  is equal to the dividend.
```

EXERCISES.

63. Find the Quotient of 34793÷21. *Ans.* $1656\frac{17}{21}$.
64. Find the Quotient of 70370÷32. *Ans.* $2199\frac{2}{32}$.
65. Find the Quotient of 64783÷430. *Ans.* $150\frac{283}{430}$.
66. Find the Quotient of 730864÷56. *Ans.* $13051\frac{8}{56}$.
67. Find the Quotient of 207863÷340. *Ans.* $611\frac{123}{340}$.
68. Find the Quotient of 734764÷431. *Ans.* $1704\frac{340}{431}$.
69. Find the Quotient of 973100÷3220. *Ans.* $302\frac{660}{3220}$.
70. Find the Quotient of 8746346÷5473. *Ans.* $1598\frac{492}{5473}$.
71. Find the Quotient of 98300794÷3290. *Ans.* $29878\frac{2174}{3290}$.
72. Find the Quotient of 37034803÷40700. *Ans.* $909\frac{38503}{40700}$.
73. Find the Quotient of 13476390÷53001. *Ans.* $254\frac{14136}{53001}$.

74. At the rate of 32 miles per day, how many days would a person be employed in walking 3968 miles?

The number of days will be the number of times 32 in 3968.
Ans. 124 days.

75. If 45 acres of ground sell for 1039 dollars, what is the price per acre?

The price per acre is $\frac{1}{45}$ *of* 1039 *dollars.* *Ans.* $23\frac{4}{45}$ dollars

76. Allowing 365 days to make a year, how many years are there in 9130 days? *Ans.* $25\frac{5}{365}$ years.

77. In how many days could 63 men accomplish a piece of work which one man could do in 1000 days? *Ans.* $15\frac{55}{63}$ days.

78. Allowing a barrel to contain 196 pounds of flour, how many barrels could be filled with 4900 pounds? *Ans.* 25 barrels.

79. There being 1760 yards in a mile, find how many miles there are in 65129 yards. *Ans.* $37\frac{9}{1760}$ miles.

80. How many days would 21 horses subsist on an amount of food which would suffice one horse 300 days? *Ans.* $14\frac{6}{21}$ days.

81. Allowing a steamboat to run 275 miles in a day, in what time would she make a trip of 5349 miles? *Ans.* $19\frac{124}{275}$ days.

82. A has 340 head of cattle worth 9860 dollars, and B has 760 acres of land worth 32680 dollars. Required the value of A's cattle per head, and of B's land per acre. *Ans.* 29 dollars; and 43 dollars.

83. Having a tract of land containing 540 acres, I wish to divide it into fields containing 45 acres each. What number of fields will it make? *Ans.* 12 fields.

84. In how many days ought a company of 54 men to complete an excavation, if 75 men could do it in 150 days? *Ans.* $208\frac{18}{54}$ days.

85. A company of 100 men have provisions sufficient for 4 months. If 33 men depart from the company, how long will the same provisions suffice the remainder?

After 33 *men depart,* 67 *men will remain. The question, then, is, how long may* 67 *men subsist on provisions which would support* 100 *men for* 4 *months.* *Ans.* $5\frac{65}{67}$ months.

86. If 27 barrels of flour be worth 135 dollars, what are 350 barrels worth, at the same price per barrel?

One barrel is worth 135 dollars÷27; *and* 350 *barrels are worth* 350 times *as much as one barrel is worth.* *Ans.* 1750 dollars.

87. If 39 acres of ground produce 2184 bushels of corn, how many bushels will 280 acres produce at the same rate? *Ans.* 15680 bushels.

88. A merchant bought 250 yards of cloth for 1750 dollars, and sold 133 yards of it at the same price at which he bought it. What did the cloth sold amount to? *Ans.* 931 dollars.

89. What sum of money ought a workman to earn in 503 weeks, allowing that in 297 weeks he could earn 3861 dollars? *Ans.* 6539 dollars.

90. A farmer has 10 head of horses, and oats enough to feed them for 5 months. If he purchase 3 more horses, how long will his oats suffice to feed the whole number?

Ans. $3\frac{11}{13}$ months.

91. A gentleman having on hand 6975 dollars, bought 5 pair of oxen, at 45 dollars a pair, and, with the remainder of the sum, purchased 230 acres of land. What was the price of the land per acre? *Ans.* $29\frac{8}{23}$ dollars.

92. A person who undertook a journey of 1000 miles, traveled the first 7 days at the rate of 35 miles per day. How long will he be in accomplishing the remaining distance, at the rate of 33 miles per day? *Ans.* $22\frac{29}{33}$ days.

93. A planter has 2280 dollars to lay out for mules and oxen, and wishes to purchase the same number of each. If he pay 65 dollars a head for mules, and 30 for oxen, how many of each can he buy?

$65+30=95$. *Then* 95 *dollars will buy* 1 mule and 1 ox; *and the number of times* 95 *in* 2280 *will be the number required.*

Ans. 24 of each.

94. How many yards of cloth at 7 dollars a yard, 8 dollars a yard, and 9 dollars a yard,—the quantity of each kind to be the same,—can a merchant buy for 1800 dollars?

Ans. 75 yards of each kind.

95. A cistern, the capacity of which is 10000 gallons, is to be filled with water by 3 pipes discharging into it. The first pipe discharges 200 gallons per hour, the second and third each 150 gallons per hour. In what time will the cistern be filled by the three pipes running together? *Ans.* 20 hours.

96. Another cistern, the capacity of which is 15000 gallons, is supplied with water by two pipes, each discharging 325 gallons, per hour; but, by leakage, the cistern loses 100 gallons per hour. In what time would the two pipes, running together, fill the cistern? *Ans.* $27\frac{150}{550}$ hours.

97. Allowing the Moon to be 240 thousand miles from the Earth, and the Sun 95 millions of miles from the Earth; how many times the Moon's distance from us is that of the Sun?

Ans. $395\frac{200000}{240000}$ times.

98. The velocity of light being 192 thousand 500 miles per second, and the distance from the Sun to the Earth 95 millions of miles; how many seconds does light require to pass from the Sun to the Earth? *Ans.* $493\frac{97500}{192500}$ seconds.

EXERCISES ON CHAPTER II.

Sign of Aggregation.

§ **62.** A *parenthesis* ( ) is used to denote the *Aggregate* of the inclosed expression; and this aggregate connects with whatever *sign* immediately precedes or follows the parenthesis.

Thus $(13+7)\times5$ denotes the *sum* of 13 and 7 multiplied by 5; that is, 20×5.

1. Find the *value of the expression,*
   $(246+34+9+16+1)\times4.$ *Ans.* 1224.
2. Find the value of the expression,
   $(370+65+100+3-90)\times5.$ *Ans.* 2240.
3. Find the value of the expression,
   $(500+18+73+25+19)\times(7-2).$ *Ans.* 3175.
4. Find the value of the expression,
   $(7348+400-100)\times(437-129).$ *Ans.* 2355584.
5. Find the value of the expression,
   $(97746+305+20)\div(320+30).$ *Ans.* $280\frac{71}{350}$.
6. Find the value of the expression,
   $(4093757-307609)\times(5083-3).$ *Ans.* 19233631840
7. Find the value of the expression,
   $(73017600-189976)\div(763+7).$ *Ans.* $94581\frac{254}{770}$.
8. Find the value of the expression,
   $(83073769+23764)\times(7307-4).$ *Ans.* 606861283499.
9. Find the value of the expression,
   $(7360793-283746+3848-500)\div(75013-113).$
   Ans. $94\frac{39795}{74900}$.

10. The sum of two numbers being 35745, and one of the numbers 1740, what is the other number? *Ans.* 34005.

11. The difference of two numbers being 13000, and the less number 635, what is the greater number? *Ans.* 13635.

12. The difference of two numbers being 37073, and the greater number 739860, what is the less number?
Ans. 702787.

13. The product of two numbers being 96350, and one of the numbers 470, what is the other number? *Ans.* 205.

Two Numbers Found from their Sum and Difference.

§ 63. The Sum+the *difference* of two numbers is TWICE the *greater number;* and the Sum—the *difference* of two numbers, is TWICE the *less number.*

For example, take the numbers 10 and 6.

Their *sum* 16+their *difference* 4 is 20, which is *twice* 10. and their sum 16—their difference 4 is 12, which is *twice* 6.

14. The sum of two numbers is 80, and their difference is 20. What is the greater, and what the less number?

80+20 is 100, *twice* the *greater;* and 80—20 is 60, *twice* the *less; hence the greater number is* $\frac{1}{2}$ of 100, *and the less is* $\frac{1}{2}$ of 60.

Ans. 50 and 30.

15. The sum of two numbers is 1000, and their difference is 200. What are the two numbers? *Ans.* 600 and 400.

16. The sum of two numbers being 1840, and their difference 500,—what are the two numbers? *Ans.* 1170 and 670.

17. The product of two numbers being 11600 and the multiplicand 80, what is the multiplier? *Ans.* 145.

18. The product of two numbers being 46400, and the multiplier 240, what is the multiplicand? *Ans.* $193\frac{80}{240}$.

19. The dividend being 23200, and the quotient 160, what is the divisor?

The *dividend* is a *product*, and the *quo.* a *factor*. *Ans.* 145.

20. The sum of 1728 dollars having been divided equally among a number of men, each man received 24 dollars. What was the number of men? *Ans.* 72 men.

21. A and B together have 2300 dollars, and A has 500 dollars more than B. What sum has each?

Ans. A has 1400, and B 900 dollars.

22. An ironmonger bought 15 tons of iron at 39 dollars per ton, and 23 tons at 37 dollars per ton. What would he gain by selling the whole at 42 dollars per ton? *Ans.* 160 dollars.

23. C and D together have 4348 dollars, and C has 375 dollars less than D. What sum has each?

Ans. C has $1986\frac{1}{2}$, and D $2361\frac{1}{2}$ dollars.

24. A merchant collected from A 230 dollars, from B 303 dollars, and from C 95 dollars. If he lay out the whole sum collected, for cloth at 7 dollars a yard, how much can he buy?

Ans. $89\frac{5}{7}$ yards.

25. A farmer wishes to fill three kinds of sacks, containing 3 bushels, 4 bushels, and 5 bushels, and the same number of each kind, with 1728 bushels of corn. How many sacks can he fill? *Ans.* 144 of each kind.

26. A farmer sold wheat for 900 dollars, corn for 274 dollars, and other produce for 329 dollars. Out of these proceeds he bought three pair of oxen at 55 dollars a pair, and paid the remainder for 65 acres of land. What did the land cost him per acre? *Ans.* $20\frac{38}{65}$ dollars.

27. A bought a building lot in town for 75 dollars, which was at the rate of 200 dollars per acre, and B purchased 3 pasture lots, each containing 13 acres, for 950 dollars. What quantity of ground was in A's lot, and what did B pay per acre? *Ans.* $\frac{75}{200}$ of an acre; and B $24\frac{14}{39}$ dollars.

28. If a person's income be 5000 dollars a year, and his expenses be at the rate of 5 dollars per day, at what rate would he save money per day,—there being 365 days in a year? *Ans.* $8\frac{255}{365}$ dollars per day.

29. An army of 5000 men have provisions for 3 months. If 1625 men be discharged, how long will the same provisions suffice for the remainder? *Ans.* $4\frac{1500}{3375}$ months.

30. A carpenter can earn 45 dollars a month, but his necessary expenditures are at the rate of 24 dollars a month. He wishes to purchase a certain lot of ground, which contains 19 acres, and is held at 35 dollars per acre. In what time can he save enough to make the purchase? *Ans.* $31\frac{14}{21}$ months.

31. A sold to B 15 cords of wood at 3 dollars per cord, 53 barrels of corn at 2 dollars per barrel, and 2 beeves at 30 dollars each. In payment, A takes 160 dollars in cash, 3 sacks of coffee at 14 dollars a sack, and 20 gallons of molasses. What did A's sales amount to, and what did the molasses cost him per gallon?

Ans. 211 dollars; and $\frac{9}{20}$ of a dollar per gallon.

CHAPTER III.

COMPOSITE NUMBERS.—PRIME FACTORS.—COMMON MEASURE.—
COMMON MULTIPLE.

COMPOSITE NUMBERS.

§ **64.** A *composite* number is one which is the *product* of two factors, each greater than a *unit*.

Thus 4 is a composite number, being 2×2.

Is 6 a composite number? Is 7? Is 12? Is 19? Is 36? Is 45?

Decomposition of Numbers.

§ **65.** *Decomposing a number* consists in resolving the number into *its factors.*

Thus 6 is decomposed when resolved into the factors 3 and 2.

Into what *two factors* may 15 be resolved? 21? 33? 84? 99?
Into what *three* factors may 24 be resolved? 30? 70? 36? 100?

§ **66.** In Division, the *dividend is resolved into two factors*, one of which is the divisor, and the other the quotient.

Taking 4 as a factor of 20, what is the other factor? 7 being one factor of 56, what is the other factor? 9 being one factor of 108, what is the other factor? 12 being one factor of 144, what is the other factor?

Any number whatever may be resolved into *itself multiplied by a unit.*

Thus 5 is 5×1; 7 is 7×1, &c.

Sign of Equality.

§ **67.** The *sign* =, *equal to,* placed between two numbers, or numerical expressions, signifies that they are *equal to each other.*

Thus 12+8=4×5 signifies that the sum of 12 and 8 is *equal to* the product of 4 and 5; and is read 12 *plus* 8 *is equal to* 4 *into* 5.

Constant Product of Several Factors.

§ **68.** The Product of several factors remains *the same* in *whatever order* the factors are multiplied together.

Take, for example, the product $2\times3\times5$.

Since $2\times3=3\times2$, we have $2\times3\times5=3\times2\times5$; and since $2\times5=5\times2$, we have $3\times2\times5=3\times5\times2$; and so on, there being *six* different ways in which the factors may be multiplied together.

Division by the Canceling of Factors.

§ **69.** A Product is divided by *either of its factors* by *canceling* that factor; or by the product of *two or more of its factors* by canceling those factors, (§ 66); and

Equal factors may be canceled from a *dividend and its divisor*, without changing the value of the quotient, (§ 57).

The cancellation of a number is denoted by a *line drawn across it.* Thus $2\times\cancel{5}$, denotes that the 5 is canceled, which is equivalent to *dividing* 2×5, or 10 by 5.

$(30=2\times3\times5)\div(15=3\times5)$, by canceling *equal factors*, becomes $(2\times\cancel{3}\times\cancel{5})\div(\cancel{3}\times\cancel{5})=2$; that is, $30\div15=2$.

Cancellation is thus employed to simplify Division, when the dividend and divisor contain *equal factors*. Its application will be seen hereafter.

COMPOSITE MULTIPLIERS AND DIVISORS.

When a multiplier or divisor can be resolved into factors, each of which shall be a *number not exceeding* 12, or such number with 0s annexed, it will sometimes shorten the operation to multiply or divide by means of such factors.

RULE IX.

§ **70.** *To multiply by means of* FACTORS.

Resolve the multiplier into two or more factors; multiply by one of the factors, and the *product thence arising* by another factor; and so on, until all the factors are employed. The last product will be the one required.

EXAMPLE.

To multiply 345 by 18.

Resolving 18 into the factors 3 and 6, we have $345\times3=1035$; and $1035\times6=6210$

Then $345\times18=345\times3\times6=6210$ (§ 68).

EXERCISES.

In performing these exercises, use the *factors* of the multiplier.

1. Required the value of 147 shares of rail-road stock, at the rate of 96 dollars per share. *Ans.* 14112 dollars.

2. Allowing 63 gallons to fill a hogshead, how many gallons will be required to fill 183 hogsheads? *Ans.* 11529 gallons.

3. A planter sold 230 bales of cotton at an average of 32 dollars per bale. What sum did he receive for his cotton? *Ans.* 7360 dollars.

4. Allowing a ship to sail at the rate of 117 miles per day, how many miles would she sail in 108 days? *Ans.* 12636 miles.

5. If 56 masons could build a certain wall in 310 days, in how many days could one mason build the same wall? *Ans.* 17360 days.

6. If 132 clerks can accomplish a certain amount of writing in 51 days, in what time could one clerk accomplish 3 times as great an amount of writing? *Ans.* 20196 days.

7. A gentleman purchased 42 bales of cotton cloth,—each bale containing 31 pieces, and each piece containing 36 yards Required the number of yards that he purchased? *Ans.* 46872 yards.

8. A speculator bought a tract of land containing 1200 acres, at 72 dollars per acre; and afterwards sold *one fifth* of the tract at 96 dollars per acre. What did he gain on the part sold? *Ans.* 5760 dollars

RULE X.

§ **71.** *To divide by means of* FACTORS.

1. Resolve the divisor into two or more factors; divide by one of the factors, and the *quotient thence resulting* by another factor, and so on, until all the factors are employed. The last quotient will be the one required.

2. If a remainder occur in the first division, and in none succeeding it, it is the *true remainder.*

3. If a remainder occur in the second division, and in none succeeding it, multiply it by the first divisor, and to the product add the first remainder, if any, for the *true remainder.*

4. *If three or more factors be used,* multiply the last remainder by the preceding divisor, and to the product add the corresponding remainder, if any; multiply the sum by the next preceding divisor, adding as before; and so on, until the divisors are all included, for the *true remainder.*

EXAMPLE.

To divide 273 by 36.

$$\begin{array}{r|l} 4 & 273 \\ \hline 9 & 68 \text{ times, 1 over.} \\ \hline & 7 \text{ times, 5 over.} \end{array}$$

Quotient 7, true remainder 21; or quotient $7\frac{21}{36}$.

Resolving 36 into 4×9, we divide first by 4, and the quotient 68 thence resulting by 9, and obtain 7, the quotient required.

To find the true remainder, we multiply the second remainder 5, by the first divisor 4, and add the first remainder 1. Thus $5\times4=20$, and 1 makes 21.

The divisor 4 is only *one ninth* of the whole divisor 36; hence it is contained in the dividend 9 *times* as often as 36 is. The true quotient is then $\frac{1}{9}$ of that found for the divisor 4.

A remainder after the first division is so many *units of the dividend.* A remainder after the second division is so many *units of the first quotient;* and since the *first quotient* $\times$ the *first divisor* produces the *dividend,* a remainder of the first quotient $\times$ the first divisor produces the corresponding *remainder of the dividend.* This remainder added to the first one, gives the true remainder of the dividend.

EXERCISES.

In performing these exercises, use the *factors* of the divisor.

1. A hogshead of ale or beer contains 54 gallons; how many hogsheads then will be filled by 9479 gallons?
Ans. $175\frac{29}{54}$ hogsheads.

2. If 81 men take equal shares of 13846 dollars, how many dollars will be the share of each man?

How will you find the answer to this question? How do you find $\frac{1}{81}$ of any number? *Ans.* $170\frac{76}{81}$ dollars.

3. Allowing a person to travel at the rate of 45 miles per hour, how long will he be in going 586 miles?
Ans. $13\frac{1}{45}$ hours.

4. Supposing 49 fat cattle to sell for 1975 dollars, what would be the average price for each? *Ans.* $40\frac{15}{49}$ dollars.

5. If one man can reap a field of hemp in 19 days, in what time ought 14 men to reap the same field? *Ans.* $1\frac{5}{14}$.

6. In what time ought 72 men to accomplish the same amount of work that 9 men could do in 300 days?
Ans. $37\frac{36}{72}$ days.

7. If 77 cords of wood be purchased for 231 dollars, for what sum ought 521 cords to be bought at the same rate?
Ans. 1563 dollars.

8. Allowing 144 yards of cloth to sell for 864 dollars, what sum should be received for double the quantity of cloth, at double the price per yard? *Ans.* 3456 dollars.

9. A garrison of 140 men has provisions sufficient for 54 days. If 8 of the men depart, how long will the same provisions suffice the remainder of the garrison?

Ans. $57\frac{36}{132}$ days.

PRIME FACTORS.

§ 72. A *prime* number is one which cannot be resolved into two factors, each greater than a *unit;* thus 3 is a *prime* number.

Is 5 a *prime* or a *composite* number? Is 8? Is 11? Is 15? Is 23? Name all the *prime* numbers, in succession from 1 to 23.

§ 73. Every *composite* number may be resolved into *prime factors;* that is, into factors each of which shall be a prime number.

For example, 30 may be at once resolved into 3×10; then resolving 10 into 2×5, we have $30=3\times2\times5$; and 3, 2, and 5 are prime numbers.

What are the prime factors of 8? Of 20? Of 24? Of 36? Of 63? Of 16? Of 21? Of 27? Of 33? Of 100? Of 1000?

Table of Prime Numbers above 23.

This table, which might be extended without limit, may be useful to the pupil, by way of reference, in the application of subsequent Rules.

29	107	199	311	421	541	647	769	883	1019	1129	1279	1427	1543	1663	1801	1951
31	109	211	313	431	547	653	773	887	1021	1151	1283	1429	1549	1667	1811	1973
37	113	223	317	433	557	659	787	907	1031	1153	1289	1433	1553	1669	1823	1979
41	127	227	331	439	563	661	797	911	1033	1163	1291	1439	1559	1693	1831	1987
43	131	229	337	443	569	673	809	919	1039	1171	1297	1447	1567	1697	1847	1993
47	137	233	347	449	571	677	811	929	1049	1181	1301	1451	1571	1699	1861	1997
53	139	239	349	457	577	683	821	937	1051	1187	1303	1453	1579	1709	1867	1999
59	149	241	353	461	587	691	823	941	1061	1193	1307	1459	1583	1721	1871	2003
61	151	251	359	463	593	701	827	947	1063	1201	1319	1471	1597	1723	1873	2011
67	157	257	367	467	599	709	829	953	1069	1213	1321	1481	1601	1733	1877	2017
71	163	263	373	479	601	719	839	967	1087	1217	1327	1483	1607	1741	1879	2027
73	167	269	379	487	607	727	853	971	1091	1223	1361	1487	1609	1747	1889	2029
79	173	271	383	491	613	733	857	977	1093	1229	1367	1489	1613	1753	1901	2039
83	179	277	389	499	617	739	859	983	1097	1231	1373	1493	1619	1759	1907	2053
89	181	281	397	503	619	743	863	991	1103	1237	1381	1499	1621	1777	1913	2063
97	191	283	401	509	631	751	871	997	1109	1249	1399	1511	1627	1783	1931	2069
101	193	293	409	521	641	757	877	1009	1117	1259	1409	1523	1637	1787	1933	2081
103	197	307	419	523	643	761	881	1013	1123	1277	1423	1531	1657	1789	1949	2083

RULE XI.

§ **74.** *To resolve a* COMPOSITE *number into its prime factors.*

1. Divide the given number by any *prime number* that will divide it without a remainder; divide the quotient in like manner; and so on, until the quotient becomes a prime number.

2. The *several divisors* and the last *quotient* will be the prime factors required.

3. If the given number can only be divided by *itself*, or a *unit*, without a remainder, it is itself a prime number.

EXAMPLE.

To resolve 210 into its prime factors.

$$\begin{array}{r|l} 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

The prime divisor 2 resolves 210 into 2×105 (§ 66). The divisor 3 resolves 105 into 3×35; and the divisor 5 resolves 35 into 5×7.

Hence 210 is resolved into the prime factors, 2, 3, 5, and 7.

EXERCISES.

1. Resolve 735 into its *prime factors*. *Ans.* 5, 7, 3, and 7.
2. Resolve 330 into its prime factors. *Ans.* 2, 3, 5, and 11.
3. Resolve 510 into its prime factors. *Ans.* 2, 3, 5, and 17.
4. Resolve 390 into its prime factors. *Ans.* 5, 2, 3, and 13.
5. Resolve 550 into its prime factors. *Ans.* 5, 2, 5, and 11.
6. Resolve 930 into its prime factors. *Ans.* 2, 3, 5, and 31.
7. Resolve 1330 into its prime factors. *Ans.* 2, 5, 7, and 19.
8. Resolve 1610 into its prime factors. *Ans.* 2, 5, 7, and 23.
9. Resolve 4350 into its prime factors. *Ans.* 2, 3, 5, 5, and 29.
10. Resolve 6020 into its prime factors. *Ans.* 2, 2, 5, 7, and 43.

The application of the preceding Rule will be seen in finding Common Measures, and Common Multiples.

COMMON MEASURE.

§ 75. One number is called a *measure* of another, if it is contained in the other an exact number of times, *without a remainder*.

Thus 2 is a *measure* of 6, and 3 is also a measure of 6.

Name a measure of 15? Of 35? Of 72? Of 99? Of 132?

If a number can be *measured by* 2, it is called an *even number*; otherwise, it is an *odd number*.

Name all the *even* numbers to 50. The *odd* numbers to 55.

§ 76. A *common measure* of two or more numbers, is any number which will *measure each of them;* that is, divide each of them without a remainder.

Thus 3 is a *common* measure of 12 and 15.

Name a common measure of 18 and 27. Of 20 and 35. Of 32 and 48. Of 12, 18, and 30. Of 36, 54, and 72.

A common measure, it is evident, is any *factor* which is *common* to the given numbers; that is, any *factor found in each of them.*

Greatest Common Measure.

§ 77. The *greatest common measure* of two or more numbers, is the greatest number that will measure each of them; that is, divide each of them without a remainder.

Thus 9 is the greatest common measure of 18 and 27.

What is the greatest common measure of 16 and 24? Of 30 and 40? Of 36 and 48? Of 64 and 12? Of 8, 12, and 32?

What is the greatest common measure of 20 and 30? Of 25 and 35? Of 24 and 72? Of 15 and 40? Of 12, 36, and 60?

When two numbers have *no common measure* greater than a *unit*, they are said to be *prime to each other;* thus 16 and 21 are *prime to each other*.

A common measure is sometimes, though not so properly, called a common *divisor;* and the *greatest* common measure, the *greatest* common divisor.

RULE XII.

§ **78.** *To find the* COMMON MEASURES *of two or more numbers*

1. Resolve each number into its *prime factors*, and select all the factors which are *common to the several numbers*, that is, which are found in each number.

2. Any *one*, or the product of any *two* or *more*, of these common factors, will be *a common measure*—and the product of *all the common factors* will be the *greatest* common measure of the given numbers.

EXAMPLE.

To find the *common measures* of 30, 45, and 75.

Resolving each number into its prime factors, we find

$30=2\times3\times5$; $45=3\times3\times5$; and $75=3\times5\times5$ (§ 74).

The factors which are *common to the three numbers*, are 3 and 5; then 3 and 5 are each a common measure, and $3\times5,=15$, is the greatest common measure of 30, 45, and 75.

It is plain that both 3 and 5 will divide each of the given numbers, without a remainder, as will also $3\times5=15$; and this last is the greatest number that will so divide them (§ 69).

EXERCISES.

1. Find the greatest *common measure* of 252, 180, and 288. *Ans.* 36.
2. Find the greatest common measure of 120, 144, and 168. *Ans.* 24.
3. Find the greatest common measure of 240, 336, and 432. *Ans.* 48.
4. Find the greatest common measure of 392, 504, and 560. *Ans.* 56.
5. Find the greatest common measure of 504, 567, and 630. *Ans.* 63.
6. Find the greatest common measure of 336, 588, and 756. *Ans.* 84.
7. Find the greatest common measure of 288, 480, and 672. *Ans.* 96.
8. Find the greatest common measure of 460, 1035, and 1150. *Ans.* 115.
9. Find the greatest common measure of 620, 1116, and 1488. *Ans.* 124.
10. Find the several common measures of 42, 210, and 126, *Ans.* 2, 3, 7, 6, 14, 21, and 42.

Another Method of Finding the Greatest Common Measure.

§ **79.** The greatest common measure of the *divisor* and *dividend* is the same as that of the *divisor* and the *remainder*, if any, after division.

Or, more generally,

§ **80.** The greatest common measure of two or more numbers, is the same as that of the *least* of those numbers and the *remainder*, or remainders, if any, after dividing the least number into the other, or each of the others.

For example, take the numbers, 12, 28, and 42,

or 12, *twice* 12+4, and 3 *times* 12+6.

It is plain that every *measure* of 12 is also a measure of *twice* 12, and of 3 *times* 12; hence no number can measure 12, *twice* 12+4, and 3 *times* 12+6, unless it also measures 4 and 6; the greatest common measure, therefore, of 12, 4, and 6, will be the greatest common measure of 12, 28, and 42; and 4 and 6 are the remainders after dividing 12, into 28, and 42.

On this principle is founded

RULE XIII.

§ **81.** *To find the greatest common measure of two or more numbers.*

1. *For two given numbers,*—divide the less number into the greater, and the remainder into the divisor, and the last remainder into the last divisor, and so on, until there is no remainder. The last *divisor* will be the common measure required.

2. *For three or more numbers,*—divide the least number into each of the others, and take the remainders and divisor for a new set of numbers, with which proceed as before, and so on, until there is no remainder. The last divisor will be the common measure required.

EXAMPLE.

To find the greatest common measure of 135 and 720.

```
135)720(5
    675
    ---
     45)135(3
```

We divide 135 into 720, and the remainder 45 into the divisor 135, when we get no remainder. The last divisor 45 is the greatest common measure of 135 and 720.

☞ By the principle of § 79, the greatest common measure of 135 and 720 is the same as that of 135 and 45, which is evidently 45.

The same principle generalized for *two or more numbers*, **establishes**, in like manner, the 2d part of the Rule. ☜

EXERCISES.

1. Find the greatest *common* measure of 324 and 480. *Ans.* 12.
2. Find the greatest common measure of 972 and 1260. *Ans.* 36.
3. Find the greatest common measure of 744 and 1680. *Ans.* 24.
4. Find the greatest common measure of 480 and 960. *Ans.* 480.
5. Find the greatest common measure of 636 and 1080. *Ans.* 12.
6. Find the greatest common measure of 375 and 1100. *Ans.* 25.
7. Find the greatest common measure of 120 and 1440. *Ans.* 120.
8. Find the greatest common measure of 780 and 1560. *Ans.* 780.

APPLICATION OF COMMON MEASURE.

1. A farmer has 66 bushels of corn, and 90 of wheat, which he wishes to put into sacks of equal size, and without mixing the two kinds of grain. How many bushels must each sack contain?

The size of each sack will evidently be any common measure of 66 and 90. *Ans.* 2 bushels, 3 bushels, or 6 bushels.

2. A gentleman has a corner of ground, the sides of which measure 225 feet, 297 feet, and 369 feet. He wishes to enclose it with a fence having panels of uniform length; what must be the length of each panel? *Ans.* 9 feet.

3. An upholsterer has 125 yards of carpeting of one kind, 175 of another, and 225 of another. He wishes to divide the whole into pieces of equal length, and the longest that can be obtained; what must be the length of each piece? *Ans.* 25 yards.

4. Having 140 acres of land at one place, and 252 at another I wish to divide the whole into fields which shall be of equal size, and the largest that will meet such requisition. What must be the number of acres in each field? *Ans.* 28 acres.

5. Three regiments of soldiers containing, respectively, 1538 men, 2307 men, and 3845 men, are to be formed, separately, into battalions, the largest that will admit the same number of men in each. What will be the number in each battalion and the number of battalions in each regiment?

Ans. $\begin{cases}\text{769 men in each battalion;}\\ \text{2 battal's in the 1st reg't, 3 in the 2d, and 5 in the 3d.}\end{cases}$

COMMON MULTIPLE.

§ **82.** One number is called a *multiple* of another, if it can be *measured* by the other; that is, divided by it without a remainder.

Thus 6 is a *multiple* of 3; and 9 is also a multiple of 3.

Name a multiple of 5. Of 7. Of 11. Of 20. Of 50. Of 100.
Name a multiple of 8. Of 9. Of 12. Of 30. Of 60. Of 200.

§ **83.** A *common multiple* of two or more numbers, is any number which can be *measured by each of them;* that is, divided by each of them without a remainder.

Thus 30 is a *common multiple* of 10 and 6.

Name a common multiple of 4 and 7. Of 3, 4, and 8. Of 5 and 9. Of 4, 3, and 6. Of 10 and 4. Of 4, 9, and 12.

Name a common multiple of 5 and 8. Of 5, 10, and 6. Of 7 and 8. Of 4, 3, and 9. Of 11 and 3. Of 4, 8, and 10.

A common multiple of two or more numbers, it is evident, is any number of which *each of the given* numbers is a *factor*.

Least Common Multiple.

§ **84.** The *least common multiple* of two or more numbers, is the *least number* that can be measured by each of them; that is, divided by each of them without a remainder.

Thus 15 is the least common multiple of 3 and 5.

What is the least common multiple of 4 and 5? Of 3 and 6? Of 5, 2, and 3? Of 7 and 9? Of 6, 4, and 3? Of 8 and 12?

What is the least common multiple of 3 and 4? Of 4 and 8? Of 2, 3, and 4? Of 10 and 12? Of 1, 4, and 6? Of 10 and 20?

The least common multiple of two or more numbers is evidently the *least number* of which each of the given numbers is a *factor*.

The *product* of two or more numbers will always be *a common multiple* of the numbers, since it will have each of those numbers for a factor.

RULE XIV.

§ 85. *To find the* COMMON MULTIPLES *of two or more numbers.*

1. Resolve each number into its prime factors, and take the *smallest selection* of those factors that includes *the factors of each given number.*

2. The product of these selected factors will be the *least* common multiple; and this product multiplied by any number whatever, will be *a common* multiple, of the given numbers

EXAMPLE.

To find the least common multiple of 6, 12, and 15.

Resolving each number into its prime factors, we have

$$6=2\times3;\quad 12=2\times2\times3;\quad 15=3\times5.$$

If we take 2, 3, 2, 5, we have the smallest selection of those factors that includes the factors of *each given number.*

Then $2\times3\times2\times5=60$ is the *least* common multiple of 6 12, and 15.

And $60\times2=120$, or 60 multiplied by any other number, is a common multiple of the same numbers.

It is plain that $2\times3\times2\times5$ can be divided by each of the given numbers, without a remainder, since it includes the factors of each of them (§ 69); and it is the smallest number that can be so divided, since it is the smallest that includes the factors of each. Hence it is their least common multiple

It is also evident that if one number be a multiple of another, any *number of times the former*, will also be a multiple of the latter.

EXERCISES.

1. Find the least *common multiple* of 10, 12, and 16. *Ans.* 240.
2. Find the least common multiple of 14, 9, and 25. *Ans.* 3150.
3. Find the least common multiple of 18, 21, and 30. *Ans.* 630.
4. Find the least common multiple of 7, 16, and 15. *Ans.* 1680.
5. Find the least common multiple of 6, 33, and 21. *Ans.* 462.
6. Find the least common multiple of 11, 15, and 32. *Ans.* 5280.
7. Find the least common multiple of 27, 44, and 4. *Ans.* 1188.
8. Find the least common multiple of 9, 18, and 20. *Ans.* 180.
9. Find the least common multiple of 8, 14, and 35. *Ans.* 280.
10. Find the common multiples of 6, 14, 20, 8, 12, and 24.
Ans. 840, 1680, 2520, &c

Another Method of Finding the Least Common Multiple.

The preceding Rule is given to show the *composition* of the least common multiple. The following is the common Rule: and is preferable in a practical point of view.

RULE XV.

§ 86. *To find the least common multiple of two or more numbers.*

1. Set the numbers in a line, from left to right, and divide any *two or more* of them by any *prime* number, greater than a unit, that will divide them without a remainder, placing the quotient and the *undivided* numbers in a line below.

2. Divide any two or more of the numbers in the lower line, as before; and so on, until no two numbers in the lowest line, can be so divided. The product of the *divisors* and numbers in the *lowest line,* will be the least common multiple of the given numbers.

3. *If no two of the given numbers* can be divided as above, the *product of all* the given numbers will be their least common multiple.

EXAMPLE.

To find the least common *multiple* of 6, 12, and 15.

$$\begin{array}{r|rrrl} 2 & 6 & 12 & 15 & \\ \hline 3 & 3 & 6 & 15 & \\ \hline & 1 & 2 & 5 & 2\times3\times2\times5=60. \end{array}$$

We divide 6 and 12 by 2; and place 15 in a line with the quotients, since 15 will not divide by two without a remainder. We then divide 3, 6, and 15, by 3; when we find that no two of the numbers, 1, 2, 5, in the lowest line, can be divided by any number greater than a *unit,* without a remainder.

Taking now the divisors, 2 and 3, and the numbers 2 and 5 in the lowest line,—omitting the 1, as not affecting the product,—we have $2\times3\times2\times5=60$, for the least common multiple of 6, 12, and 15.

☞ This method is the same *in principle* with that of Rule XIV. The divisors and numbers in the lowest line, are necessarily prime factors of the given numbers; and are the *smallest selection* of such factors that includes the factors of *each given number.* ☜

EXERCISES.

1. Find the least *common multiple* of 4, 6, 8, and 10
Ans. 120.
2. Find the least common multiple of 9, 3, 12, and 15.
Ans. 180.
3. Find the least common multiple of 21, 7, 4, and 9.
Ans. 252.
4. Find the least common multiple of 6, 4, 12, and 20.
Ans. 60.
5. Find the least common multiple of 8, 7, 10, and 14.
Ans. 280.
6. Find the least common multiple of 15, 2, 7, and 13.
Ans. 2730.
7. Find the least common multiple of 24, 5, 6, and 10.
Ans. 120.
8. Find the least common multiple of 5, 10, 13, and 24.
Ans. 1560.
9. Find the least common multiple of 6, 7, 2, and 17.
Ans. 714.
10. Find the least common multiple of 11, 4, 5, and 19.
Ans. 4180.

APPLICATION OF COMMON MULTIPLE.

1. What is the smallest sum of money for which a person could purchase, either a number of mules at 32 dollars a head, or a number of cows at 14 dollars a head,—the same sum to be employed in either purchase?

It is evident that the number of dollars to be employed, is the least common multiple of 32 and 14. *Ans.* 224 dollars.

2. A can build 7 rods of fence in a day; B can build 9 rods; and C 12 rods, in a day. What amount of fencing would afford a number of whole days' work for any one of the three?
Ans. 252 rods, or 504 rods, or 756 rods, &c.

3. If one team can haul to market 10 barrels of flour; another, 12 barrels; and another, 15 barrels;—what number of barrels would make a number of full loads for any of the three teams? *Ans.* 60 barrels, or 120 barrels, or 180 barrels, &c.

4. How many bushels of wheat would fill a number of barrels, each containing 3 bushels; or a number of sacks, each containing 4 bushels; or a number of hogsheads, each containing 15 bushels;—the quantity to be the same in each case?
Ans. 60 bushels, or 120 bushels, or 180 bushels, &c.

5. What is the smallest sum for which I could purchase a number of mules, at 35 dollars a head; or a number of horses, at 45 dollars a head; and what number of each could I purchase for that sum? *Ans.* 315 dollars; 9 mules, or 7 horses.

EXERCISES ON CHAPTER III.

Find the value of each of the following expressions — multiplying and dividing by means of *factors*.

1. $(1000+250-30+375)\times(81-9)$. *Ans.* 114840.
2. $(3874-250+30-375)\times(100-4)$. *Ans.* 314784.
3. $(4800+675-84+860)\div(150-6)$. *Ans.* $43\frac{59}{144}$.
4. $(4800-675+84-860)\div(127+5)$. *Ans.* $25\frac{49}{132}$.
5. $(9999+999-75+375)\div(130-9)$. *Ans.* $93\frac{45}{121}$.

6. Resolve 436 into its prime factors. *Ans.* 2, 2, and 109.
7. Resolve 780 into its prime factors. *Ans.* 2, 2, 5, 3, and 13.
8. Resolve 972 into its prime factors. *Ans.* 2, 2, 3, 3, 3, 3, and 3.
9. Resolve 1275 into its prime factors. *Ans.* 3, 5, 5, and 17.
10. Resolve 2000 into its prime factors. *Ans.* 2, 2, 2, 2, 5, 5, and 5.

11. Find the greatest common measure of 124, 200, and 350 *Ans.* 2.
12. Find the greatest common measure of 325, 240, and 460 *Ans.* 5.
13. Find the greatest common measure of 270, 800, and 960 *Ans.* 10.

14. Find the least common multiple of 14, 25, 8, and 20. *Ans.* 1400.
15. Find the least common multiple of 8, 36, 9, and 17. *Ans.* 1224.
16. Find the common multiples of 15, 20, 32, and 75. *Ans.* 2400, 4800, 7200, &c.
17. Find the common measures of 300, 400, 500, and 600. *Ans.* 2, 4, 5, 10, 20, 25, 50, and 100.
18 Find the common multiples of 24, 36, 60, and 84. *Ans.* 2520, 5040, 7560, &c.

19. Find the greatest common measure, and also the least common multiple, of 74 and 126.

Ans. Greatest com. meas. 2; least com. mult. 4662.

20. The junior class in a school consists of 132 students, and the senior of 99. How might each class be divided, so that the whole school should be disposed in equal sections?

Ans. Into sections of 3, 11, or 33.

21. For what sum of money could a carpenter hire journeymen for one month, at 15 dollars, 21 dollars, or 24 dollars each, allowing the whole sum to be thus expended?

Ans. 840 dollars, or 1680 dollars, &c.

22. What is the smallest sum of money for which I could purchase a number of plows at 14 dollars each, or a number of carts at 30 dollars each, or a number of wagons at 90 dollars each? *Ans.* 630 dollars.

23. A wine merchant has 111 gallons of Madeira, 185 gallons of Port, and 259 gallons of Malaga, with which he wishes to fill a number of casks, all containing the same number of gallons, and without mixing the different kinds of wine. What must be the contents of each cask? *Ans.* 37 gallons.

24. A has 413 dollars, B 531 dollars, and C 590 dollars; and they agree to purchase horses, at the same price per head, provided each man can thus invest all his money. How many horses could each man purchase?

Ans. A could purchase 7, B 9, and C 10.

25. An island is 200 miles in circumference, and three persons, A, B, and C, start together, and travel the same way around it. A goes 20 miles per day, B 25, and C 40 miles per day. In what time would they all come together again at the same point from which they started?

First find the number of days it would require each person to go around the island. *Ans.* 40 days.

CHAPTER IV.

PRELIMINARY DEFINITIONS AND PRINCIPLES — REDUCTION OF FRACTIONS.

FRACTIONS.

§ **87.** A *Fraction* is an expression of *one* or *more* of the *equal parts* into which any quantity may be divided.

One *half* is *one* of the *two* equal parts,—One *third* is *one* of the *three* equal parts,—Two *thirds* are *two* of the *three* equal parts, and so on, *of any quantity.*

What is meant by *one fourth* of a quantity? By *three fourths* of a quantity? By *one fifth* of a quantity? By *four fifths?*

Any quantity consists of how many *halves* of that quantity? Of how many *thirds?* Of how many *fourths?* Of how many *tenths?*

Which is the greater part, *one half* or *one third* of a quantity? One fourth or one seventh? One ninth or one fifth? One tenth or one hundredth? One sixth or one sixtieth?

How many is *one half* of 2? One third of 3? Two thirds of 3? Three fourths of 4? One fifth of 5? Five sixths of 6? One seventh of 7? Three eighths of 8? Four ninths of 9?

If the 2 *halves* of any quantity were each divided into 2 equal parts, into how many equal parts would the *whole quantity be divided?* What would each of those parts be called? One *half* is how many *fourths?*

If the 3 *thirds* of any quantity were each divided into 2 equal parts, into how many equal parts would the whole quantity be divided? What would each of those parts be called? One *third* is how many *sixths?*

If the 4 *fourths* of any quantity were each divided into 3 equal parts, into how many equal parts would the whole quantity be divided? What would each of those parts be called? One *fourth* is how many *twelfths?*

Numerator and Denominator.

§ **88.** A fraction is *denoted by two numbers,* one above the other, with a line between them. The upper number is called the *numerator;* and the lower, the *denominator.*

The *numerator* shows the number of *equal parts in the fraction;* the *denominator* shows the number of such parts in the *whole quantity* divided.

Thus $\frac{3}{4}$, *three fourths;* 3 is the *numerator,* and 4 the *denominator.*

In $\frac{2}{3}$, which is the numerator, and what does it show? Which the denominator, and what does it show? In $\frac{4}{7}$? In $\frac{3}{8}$? In $\frac{4}{9}$?

The numerator and the denominator are together called the *terms* of the fraction.

Proper and Improper Fractions.

§ **89.** A *proper fraction* is one whose numerator is *less* than *its denominator;* and whose *value* is, consequently, *less* than a *unit* or *whole one.*

Thus $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$, are proper fractions.

§ **90.** An *improper fraction* is one whose numerator is *equal to,* or *greater than,* its denominator; and whose value is, accordingly, equal to, or greater than, a unit or whole one.

Thus $\frac{3}{3}$ is an improper fraction,—equal to a whole one; and $\frac{5}{3}$ is an improper fraction,—equal to $1\frac{2}{3}$, *one* and *two thirds.*

$\frac{6}{2}$ is equal to how many whole ones? $\frac{9}{4}$ is equal to how many whole ones? $\frac{10}{5}$? $\frac{13}{6}$? $\frac{21}{7}$? $\frac{35}{8}$? $\frac{45}{9}$? $\frac{60}{10}$ $\frac{100}{11}$ $\frac{130}{12}$?

Fractions Express Division.

§ **91.** Every *proper fraction* expresses *the part that its numerator is* of its denominator.

For example take the fraction $\frac{4}{9}$, *four ninths.*

Since 1 is *one ninth* of 9, 4 is *four ninths* of 9; that is, the fraction expresses *the part that its numerator* 4 is of its denominator 9.

2 is what part of 3? 4 is what part of 7? 5 is what part of 13? 9 is what part of 16? 11 is what part of 25? 17 is what part of 39?

§ **92.** Every fraction, whether proper or improper, is equal to its *numerator divided by its denominator.*

For example, the fraction $\frac{4}{9}$ is the quotient of $4 \div 9$ (§ 47); being the same as $\frac{1}{9}$ of 4 (§ 51).

And the fraction $\frac{9}{4}$ is $9 \div 4$, equal to $\frac{1}{4}$ of 9, equal to $2\frac{1}{4}$ (§ 56).

$\frac{2}{3}$ is equal to what part of 2? $\frac{5}{6}$ is equal to what part of 5?
$\frac{3}{13}$ is equal to what part of 3? $\frac{23}{25}$ is equal to what part of 23?
$\frac{7}{30}$ is equal to what part of 7? $\frac{31}{45}$ is equal to what part of 31?

$\frac{4}{3}$ is how many units or whole ones? $\frac{9}{3}$ is how many units or whole ones?
$\frac{27}{4}$ is how many units or whole ones? $\frac{41}{9}$ is how many units or whole ones?
$\frac{64}{8}$ is how many units or whole ones? $\frac{30}{10}$ is how many units or whole ones?

Constant Value of a Fraction.

§ **93.** The value of a fraction remains *the same* when its numerator and denominator are both multiplied, or both divided, by the *same number*.

Taking, for example, $\frac{3}{4}$, and multiplying both its terms by 2, we have $\frac{6}{8}$.

Now if any quantity were divided into 4 *fourths*, each *one* of these fourths, divided into *two equal parts*, would make 2 *eighths* of the quantity; then 3 *fourths* would make 6 *eighths;* that is, $\frac{3}{4}=\frac{3\times 2}{4\times 2}=\frac{6}{8}$.

Prove that $\frac{3}{5}$ is equal to $\frac{6}{10}$. Prove that $\frac{2}{6}$ is equal to $\frac{6}{18}$.

The truth of this principle is also evident from regarding a traction as a *quotient*, its numerator being a *dividend*, and its denominator the *divisor* (§ 92 and 57).

REDUCTION OF FRACTIONS.

§ **94.** *Reduction*, in general, consists in changing the *form* or *expression* of a quantity, without *altering its value.*

Thus $\frac{2}{5}$ may be changed to the form $\frac{4}{10}$, $\frac{6}{15}$, &c., without altering its value (§ 93).

FRACTIONS REDUCED TO THEIR LOWEST TERMS.

§ **95.** A fraction is *reduced to lower terms* when its numerator and denominator are diminished, without altering its value.

For example, $\frac{12}{16}$ reduced to *lower terms*, is $\frac{6}{8}$, found by dividing 12 and 16 by 2 (§ 93).

Reduce $\frac{6}{8}$ to lower terms. Reduce $\frac{16}{24}$ to lower terms.
Reduce $\frac{10}{20}$ to lower terms. Reduce $\frac{50}{100}$ to lower terms.

§ **96.** A fraction is *reduced to its lowest terms* when its numerator and denominator are made the *smallest* that will express the value of the given fraction.

Thus $\frac{12}{16}$ reduced to its *lowest terms* is $\frac{3}{4}$.

What is $\frac{5}{10}$ in its lowest terms? What is $\frac{6}{12}$ in its lowest terms?
What is $\frac{6}{9}$ in its lowest terms? What is $\frac{16}{24}$ in its lowest terms?

When a fraction is in its lowest terms, its numerator and denominator will be *prime to each other* (§ 77).

RULE XVI.

§ **97.** *To reduce a fraction to its* LOWEST TERMS.

1. Divide both terms of the fraction by any *common measure* greater than a unit, and the quotients by any common measure greater than a unit, and so on, until the quotients become *prime to each other*. The last quotients will be the lowest terms of the fraction. Or,

2. Divide both terms of the fraction by their *greatest common measure;* the quotients will be the lowest terms.

EXAMPLE.

To reduce $\frac{90}{120}$ to its lowest terms.

$$\begin{array}{r} 10\overline{)90} \\ 3\overline{)9} \\ 3 \end{array} \qquad \begin{array}{r} 10\overline{)120} \\ 3\overline{)12} \\ 4 \end{array} \qquad \frac{90}{120}=\frac{3}{4}.$$

Dividing 90 and 120 both by 10, and the quotients 9 and 12 both by 3, the quotients 3 and 4 are prime to each other, and we have the given fraction in its lowest terms equal to $\frac{3}{4}$ (§ 93).

Or, dividing 90 and 120 both by 30, which is their greatest common measure (§ 77), we at once have $\frac{90}{120}=\frac{3}{4}$.

The advantage of reducing a fraction to its lowest terms, is, that the *value* of the fraction is then more readily perceived.

Thus we more readily perceive the value of $\frac{3}{4}$ than of $\frac{90}{120}$.

EXERCISES.

1. Reduce $\frac{25}{75}$ and $\frac{216}{360}$ to their lowest terms. *Ans.* $\frac{1}{3}$ and $\frac{3}{5}$.
2. Reduce $\frac{30}{90}$ and $\frac{125}{150}$ to their lowest terms. *Ans.* $\frac{1}{3}$ and $\frac{5}{6}$.
3. Reduce $\frac{45}{50}$ and $\frac{279}{403}$ to their lowest terms. *Ans.* $\frac{9}{10}$ and $\frac{9}{13}$.
4. Reduce $\frac{55}{75}$ and $\frac{99}{225}$ to their lowest terms. *Ans.* $\frac{11}{15}$ and $\frac{11}{25}$.
5. Reduce $\frac{39}{84}$ and $\frac{180}{468}$ to their lowest terms. *Ans.* $\frac{13}{28}$ and $\frac{5}{13}$.

6. At 25 dollars per acre, what quantity of land could be purchased for 15 dollars? *Ans.* $\frac{3}{5}$ of an acre.

7. At 18 dollars per ton for hay, what quantity of hay could be bought for 10 dollars? *Ans.* $\frac{5}{9}$ of a ton.

8. Allowing 365 days to a year, what part of a year is included in 146 days? *Ans.* $\frac{2}{5}$ of a year.

9. There being 1760 yards in a mile what part of a mile is included in 1320 yards? *Ans.* $\frac{3}{4}$ of a mile.

10. If 133 dollars were equally divided among 152 men, what would be the portion of each man (§ 92)? *Ans.* $\frac{7}{8}$ of a dollar.

11. If a person saves out of his income 290 dollars in a year, at what rate does he save per day? *Ans.* $\frac{58}{73}$ of a dollar.

12. What quantity of ground must a man plow per day, in order to accomplish 63 acres in 72 days? *Ans.* $\frac{7}{8}$ of an acre.

13. A footman performed a journey of 500 miles in 15 days. At what rate did he walk per day? *Ans.* $33\frac{1}{3}$ miles.

14. A farmer bought a tract of land containing 400 acres, for 18100 dollars. What was the price per acre?
Ans. $45\frac{1}{4}$ dollars.

15. A speculator purchased 1000 head of cattle at 16 dollars each, and sold the whole of them for 21875 dollars. How much did he gain per head? *Ans.* $5\frac{7}{8}$ dollars

FRACTIONS REDUCED TO A COMMON DENOMINATOR.

§ **98.** Two or more fractions are said to have a *common denominator*, when they have the *same number* for a denominator

Thus $\frac{2}{7}$, $\frac{4}{7}$, and $\frac{6}{7}$, have a common denominator.

Give another example of fractions having a common denominator.

§ **99.** Two or more fractions may often be reduced, *mentally*, to a common denominator, by multiplying, or dividing, *both terms* of one or more of them, so as to make the denominator *the same for each.*

For example, $\frac{1}{2}$ and $\frac{2}{3}$ are reduced to a common denominator, by multiplying both terms of $\frac{1}{2}$ by 3, and of $\frac{2}{3}$ by 2.

We thus find $\frac{1}{2}=\frac{1\times3}{2\times3}=\frac{3}{6}$; and $\frac{2}{3}=\frac{2\times2}{3\times2}=\frac{4}{6}$ (§ 93). The *equivalent* of $\frac{1}{2}$ is $\frac{3}{6}$; and the *equivalent* of $\frac{2}{3}$ is $\frac{4}{6}$.

Give the *equivalents* of $\frac{2}{3}$ and $\frac{3}{4}$ in fractions having a *common denominator.* Of $\frac{1}{2}$ and $\frac{3}{5}$. Of $\frac{3}{4}$ and $\frac{2}{5}$. Of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. Of $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{3}{10}$.

Least Common Denominator.

§ **100.** Two or more fractions are reduced to their *least common denominator*, when the common denominator found is the smallest by which the *equivalent* of each fraction can be expressed.

For example, $\frac{3}{4}$ and $\frac{5}{6}$, reduced to their least common denominator, are equal, respectively, to $\frac{9}{12}$ and $\frac{10}{12}$.

Give the equivalents of $\frac{3}{8}$ and $\frac{5}{6}$, by their least common denominator. Of $\frac{5}{8}$ and $\frac{3}{12}$. Of $\frac{1}{15}$ and $\frac{2}{15}$. Of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$. Of $\frac{1}{3}$, $\frac{5}{6}$ and $\frac{7}{12}$.

RULE XVII.

§ **101.** *To reduce two or more fractions to a* COMMON DENOMINATOR.

1. Multiply each numerator by all the denominators except its own, for the *new numerators;* and multiply all the denominators together for the *common denominator.*

2. *If the least common denominator be required,*—take the *least* COMMON MULTIPLE of the given denominators, for the common denominator. Divide this multiple by the denominator of each given fraction, and multiply the quotient by the numerator, for the *new numerators.*

EXAMPLES.

1. To reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, to a common denominator.

For the new numerators, we have

$$2\times6\times8=96;\quad 5\times3\times8=120;\quad 7\times6\times3=126;$$

and for the common denominator, $3\times6\times8=144$.

The equivalents of the given fractions, are, then,

$\frac{96}{144}$, $\frac{120}{144}$, and $\frac{126}{144}$, respectively.

2. To reduce the same fractions $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, to their *least common denominator.*

The least *common multiple* of the denominators 3, 6, and 8, is 24 (§ 84). Then 24 is the common denominator required.

Dividing 24 by each given denominator, and multiplying the *quotients* by the given numerators, respectively, we have for the new numerators,

$$(24\div3)\times2=16;\ (24\div6)\times5=20;\ (24\div8)\times7=21.$$

The given fractions are, then, respectively equal to,

$\frac{16}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.

This method will not find the least common denominator, in all cases, unless each of the given fractions is in its *lowest terms.*

For instance, if, in the above example, we take $\frac{14}{16}$ instead of its equal, $\frac{7}{8}$, the least common multiple will be 48, which is not the least common denominator by which the equivalent of each fraction can be expressed.

☞ In finding a common denominator, as above, both terms of each given fraction are multiplied by the *same number*.

Thus $\frac{2}{3}$, in the first example, has both its terms multiplied by 6×8,—producing the new terms $\frac{96}{144}$.

In the second example, the numerator 2 is multiplied by $8, = 24 \div 3$, and this is the same number by which the denominator 3 is multiplied, to produce the common denominator 24.

Hence the values of the given fractions remain the same in reducing them to a common denominator (§ 93). ☜

EXERCISES.

1. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{8}$ to a common denominator.
   Ans. $\frac{80}{120}$, $\frac{96}{120}$, and $\frac{45}{120}$.
2. Reduce $\frac{5}{11}$, $\frac{9}{7}$, and $\frac{3}{2}$ to a common denominator.
   Ans. $\frac{70}{154}$, $\frac{198}{154}$, and $\frac{231}{154}$.
3. Reduce $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{8}{13}$ to a common denominator.
   Ans. $\frac{351}{585}$, $\frac{260}{585}$, and $\frac{360}{585}$.
4. Reduce $\frac{5}{8}$, $\frac{6}{7}$, and $\frac{3}{12}$ to a common denominator.
   Ans. $\frac{420}{672}$, $\frac{576}{672}$, and $\frac{168}{672}$.
5. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to the least common denominator.
   Ans. $\frac{18}{24}$, $\frac{20}{24}$, and $\frac{21}{24}$.
6. Reduce $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{5}{6}$ to the least common denominator.
   Ans. $\frac{6}{12}$, $\frac{3}{12}$, and $\frac{10}{12}$.
7. Reduce $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{7}{12}$ to the least common denominator.
   Ans. $\frac{24}{36}$, $\frac{16}{36}$, and $\frac{21}{36}$.
8. Reduce $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{9}{10}$ to the least common denominator.
   Ans. $\frac{24}{30}$, $\frac{25}{30}$, and $\frac{27}{30}$.
9. Reduce $\frac{4}{6}$, $\frac{2}{8}$, and $\frac{8}{12}$ to the least common denominator.
   Ans. $\frac{16}{24}$, $\frac{6}{24}$, and $\frac{16}{24}$.
10. Reduce $\frac{2}{3}$, $\frac{6}{10}$, and $\frac{15}{20}$ to the least common denominator.
    Ans. $\frac{40}{60}$, $\frac{36}{60}$, and $\frac{45}{60}$.
11. Reduce $\frac{3}{7}$, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{1}{9}$ to a common denominator.
    Ans. $\frac{1080}{2520}$, $\frac{2016}{2520}$, $\frac{1575}{2520}$, $\frac{280}{2520}$.
12. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{9}$, and $\frac{1}{10}$ to a common denominator.
    Ans. $\frac{360}{720}$, $\frac{540}{720}$, $\frac{160}{720}$, $\frac{72}{720}$.
13. Reduce $\frac{2}{5}$, $\frac{1}{8}$, $\frac{3}{7}$, and $\frac{4}{5}$ to a common denominator.
    Ans. $\frac{560}{1400}$, $\frac{175}{1400}$, $\frac{600}{1400}$, $\frac{1120}{1400}$.
14. Reduce $\frac{1}{9}$, $\frac{2}{11}$, $\frac{3}{8}$, and $\frac{4}{7}$ to a common denominator.
    Ans. $\frac{616}{5544}$, $\frac{1008}{5544}$, $\frac{2079}{5544}$, $\frac{3168}{5544}$.
15. Reduce $\frac{6}{11}$, $\frac{1}{3}$, $\frac{7}{13}$, and $\frac{1}{5}$ to a common denominator.
    Ans. $\frac{1170}{2145}$, $\frac{715}{2145}$, $\frac{1155}{2145}$, $\frac{429}{2145}$.

One advantage of expressing Fractions by a *common denominator*, is, that we are thus enabled, most readily, to compare the *values* of fractions in certain cases.

Comparison of Fractions.

§ **102.** Fractions having different numerators and denominators, are most readily *compared with each other*, by reducing them to a common denominator, and then comparing the new *numerators*.

For example, we should not very readily perceive which is the greater fraction, $\frac{5}{7}$ or $\frac{8}{11}$. By reducing them to a common denominator, we have $\frac{55}{77}$ and $\frac{56}{77}$; from which we see at once that $\frac{8}{11}$ is $\frac{1}{77}$ the greater.

11. One person expends 5 dollars for coal, at 7 dollars per ton; and another, 6 dollars, at 9 dollars per ton. What quantity does each of them purchase, and which of them the greater quantity?

Ans. { The 1st $\frac{5}{7}$, the 2d $\frac{2}{3}$ of a ton; The 1st the greater quantity. }

12. A, B, and C, purchase iron;—A, at 37 dollars per ton: B, at 42; and C, at 50 dollars per ton. A lays out 20 dollars; B, 25; and C 30 dollars: what quantity of iron does each of them purchase,—which of them the largest, and which the smallest quantity?

Ans. { A, $\frac{20}{37}$, B, $\frac{25}{42}$, and C, $\frac{3}{5}$ of a ton; C the largest, and A the smallest quantity. }

INTEGERS AND MIXED NUMBERS REDUCED TO IMPROPER FRACTIONS.

§ **103.** An *integer*, or *integral* number, is any whole number, in opposition to a *fraction*.

Thus 1, 5, 10, &c., are *integers*.

§ **104.** A mixed number is an integer, or whole number, with a fraction annexed to it.

Thus $5\frac{3}{4}$, $7\frac{1}{2}$, are *mixed* numbers.

What kind of quantity is $\frac{3}{7}$? $\frac{15}{4}$? 25? $37\frac{1}{2}$?

§ **105.** An integer is reduced to the form of an *improper fraction*, by taking a *unit* for a *denominator*.

Thus 5 is $\frac{5}{1}$, 5 *ones;* 10 is $\frac{10}{1}$, 10 *ones;* &c.

Or, an integer may be reduced to an improper fraction having any *proposed denominator*.

Thus 5 is equal to $\frac{10}{2}$, or $\frac{15}{3}$, or $\frac{20}{4}$; and so on.

7 is equal to how many *halves?* How many *4ths?* *8ths?*
9 is equal to how many *halves?* How many *6ths?* *14ths?*
12 is equal to how many *halves?* How many *10ths?* *18ths?*

§ **106.** A *mixed* number may be readily reduced to an improper fraction whose denominator is *that of the fraction annexed.*

For example, 5 being equal to $\frac{15}{3}$, $5\frac{2}{3}=\frac{17}{3}$.

$2\frac{1}{2}$ is how many *halves?* $3\frac{1}{4}$ is how many $4ths$? $4\frac{2}{3}$ is how many $3rds$?
$5\frac{3}{5}$ is how many $5ths$? $7\frac{2}{9}$ is how many $9ths$? $9\frac{5}{8}$ is how many $8ths$?
11 is how many $9ths$? $12\frac{7}{10}$ is how many $10ths$?

RULE XVIII.

§ **107.** *To reduce an integer, or a mixed number, to an improper fraction.*

1. To REDUCE AN INTEGER. Under the given integer, regarded as a *numerator*, place 1 for the denominator. Or multiply the integer by any proposed denominator; the product will be the numerator.

2. To REDUCE A MIXED NUMBER. Multiply the integer contained in it, by the denominator annexed; and to the product *add the numerator*, for a numerator to be placed over said denominator.

EXAMPLES.

1. To reduce the integer 25 to *7ths*.

$$25\times7=175;\ \text{hence}\ 25=\frac{175}{7}.$$

2. To reduce the mixed number $25\frac{3}{7}$ to an improper fraction

$$25\times7=175;\ \text{and}\ 175+3=178;\ \text{hence}\ 25\frac{3}{7}=\frac{178}{7}$$

EXERCISES.

1. Reduce 13, 35, and 74, each to *3rds*. *Ans.* $\frac{39}{3}$, $\frac{105}{3}$, and $\frac{222}{3}$.
2. Reduce 29, 83, and 90, each to *4ths*. *Ans.* $\frac{116}{4}$, $\frac{332}{4}$, and $\frac{360}{4}$.
3. Reduce 71, 55, and 89, each to *7ths*. *Ans.* $\frac{497}{7}$, $\frac{385}{7}$, and $\frac{623}{7}$.
4. Reduce 19, 91, and 100, each to *10ths*. *Ans.* $\frac{190}{10}$, $\frac{910}{10}$, and $\frac{1000}{10}$.
5. Reduce $25\frac{1}{2}$ and $34\frac{1}{3}$ to improper fractions. *Ans.* $\frac{51}{2}$ and $\frac{103}{3}$.
6. Reduce $83\frac{1}{5}$ and $94\frac{1}{4}$ to improper fractions. *Ans.* $\frac{416}{5}$ and $\frac{377}{4}$.
7. Reduce $74\frac{1}{6}$ and $47\frac{1}{12}$ to improper fractions. *Ans.* $\frac{445}{6}$ and $\frac{565}{12}$.
8. Reduce $39\frac{1}{10}$ and $16\frac{3}{17}$ to improper fractions. *Ans.* $\frac{391}{10}$ and $\frac{275}{17}$
9. Reduce $12\frac{2}{3}$ and $14\frac{1}{2}$ first to improper fractions, and then to a common denominator. *Ans.* $\frac{38}{3}$ and $\frac{29}{2}$; $\frac{76}{6}$ and $\frac{87}{6}$.
10. Reduce $25\frac{1}{2}$, $39\frac{1}{4}$, and $234\frac{3}{8}$ first to improper fractions, and then to their least common denominator.

Ans. $\frac{51}{2}$, $\frac{157}{4}$, and $\frac{1875}{8}$; $\frac{204}{8}$, $\frac{314}{8}$, and $\frac{1875}{8}$.

11. How many bushels of apples, at $\frac{1}{4}$ of a dollar per bushel, may be purchased for 13 dollars?

The number of bushels *that may be purchased, is equal to the number of* 4ths *of a dollar in* 13 dollars. *Ans.* 52 bushels.

12. What number of horses could be fed on 150 bushels of oats, giving $\frac{1}{5}$ of a bushel to each? *Ans.* 750 horses.

13. At the rate of $\frac{1}{2}$ of a dollar per day, how many laborers could be hired, one day, for $4\frac{1}{2}$ dollars? *Ans.* 9 laborers.

14. How many lots of ground, each to contain $\frac{1}{4}$ of an acre, could be made out of a field containing $129\frac{3}{4}$ acres? *Ans.* 519 lots.

15. Required how long a company of workmen would be employed in graduating $33\frac{2}{5}$ miles of road, at the rate of $\frac{1}{5}$ of a mile per month. *Ans.* 167 months.

16. Allowing a mechanic to earn 35 dollars in 2 months, or $\frac{1}{6}$ *of a year,* what sum ought he to earn in 3 years?

In 3 *years there are* 18 sixths *of a year. If he earn* 35 *dollars in* 1 sixth *of a year, he will earn* 18 times 35 *dollars in* 18 sixths *of a year, that is, in* 3 *years.* *Ans.* 630 dollars.

17. If a steamer run at the rate of 5 miles in $\frac{1}{4}$ of an hour, how far will she run in 13 hours? *Ans.* 260 miles.

18. If $\frac{1}{2}$ of a ton of hay brings 9 dollars, what should be paid for $34\frac{1}{2}$ tons of hay, at the same rate? *Ans.* 621 dollars.

19. Allowing $\frac{1}{4}$ of an acre of ground to produce 11 bushels of wheat, how many bushels would be raised from two fields, containing, respectively, 39 acres, and $41\frac{3}{4}$ acres? *Ans.* 3553 bushels.

IMPROPER FRACTIONS REDUCED TO INTEGERS OR MIXED NUMBERS.

Since 2 *halves,* or 3 *thirds,* or 4 *fourths,* and so on, make a *unit* or whole one, any improper fraction may readily be reduced to an integer, or a mixed number.

For example $\frac{12}{3}$ is equal to 4, and $\frac{14}{3}$ is equal to $4\frac{2}{3}$.

$\frac{13}{4}$ is how many units or whole ones? $\frac{15}{4}$ is how many units or whole ones? $\frac{25}{5}$ is how many units or whole ones? $\frac{45}{6}$ is how many units or whole ones? $\frac{72}{9}$ is how many units or whole ones? $\frac{13}{3}$ is how many units or whole ones? $\frac{21}{7}$ is how many units or whole ones? $\frac{17}{8}$ is how many units or whole ones? $\frac{47}{11}$ is how many units or whole ones? $\frac{100}{12}$ is how many units or whole ones?

The following Rule is the reverse of the preceding one.

RULE XIX.

§ **108.** *To reduce an improper fraction to an integer, or a mixed number.*

1. Divide the numerator by the denominator, placing the divisor, or denominator, under the remainder, if any, and annexing the fraction so formed to the quotient.

2. The fraction annexed to the quotient may often be reduced to lower terms.

EXAMPLE.

To reduce $\frac{140}{25}$ to an integer, or a mixed number.

$140 \div 25$, gives *quotient* 5, and *remainder* 15,

Hence $\frac{140}{25}=5\frac{15}{25}=5\frac{3}{5}$.

The fraction formed of the divisor and remainder, will be in its *lowest terms*, or not, according as the improper *fraction reduced,* is, or is not, in its lowest terms.

For, if the dividend and divisor have any *common measure*, the divisor and remainder will have the same common measure (§ 79).

EXERCISES.

1. Reduce $\frac{75}{25}$ to an integer, or a mixed number. *Ans.* 3.
2. Reduce $\frac{130}{10}$ to an integer, or a mixed number. *Ans.* 13.
3. Reduce $\frac{372}{13}$ to an integer, or a mixed number. *Ans.* $28\frac{8}{13}$.
4. Reduce $\frac{480}{48}$ to an integer, or a mixed number. *Ans.* 10.
5. Reduce $\frac{1384}{15}$ to an integer, or a mixed number. *Ans.* $92\frac{4}{15}$.
6. Reduce $\frac{2783}{125}$ to an integer, or a mixed number. *Ans.* $22\frac{33}{125}$.

7. How many dollars must be paid for 120 pounds of cheese, at $\frac{1}{8}$ of a dollar per pound?

The 120 pounds *will cost* $\frac{120}{8}$ *of a dollar.* *Ans.* 15 dollars.

8. How many dollars must be paid for 875 pounds of rice at $\frac{1}{16}$ of a dollar per pound? *Ans.* $54\frac{11}{16}$ dollars.

9. If $\frac{1}{15}$ of a barrel of flour will serve a family for a week how many barrels would serve them 52 weeks?

Ans. $3\frac{7}{15}$ barrels.

10. If a person walk at the rate of $\frac{1}{20}$ of a mile per minute how many miles would he walk in 245 minutes?

Ans. $12\frac{1}{4}$ miles

11. How many dollars should be paid for 7 yards of linen, at $\frac{3}{4}$ of a dollar per yard?

The 7 yards *will cost* 7 times $\frac{3}{4}$ *of a dollar, which will be* $\frac{21}{4}$ *of a dollar.* *Ans.* $5\frac{1}{4}$ dollars.

12. How many dollars should be paid for 25 bushels of wheat, at $\frac{7}{8}$ of a dollar per bushel? *Ans.* $21\frac{7}{8}$ dollars.

13. How many dollars should be paid for 19 pounds of tobacco, at $\frac{9}{16}$ of a dollar per pound? *Ans.* $10\frac{11}{16}$ dollars.

14. If a laborer's wages be $\frac{4}{5}$ of a dollar per day, how many dollars will he earn in 237 days? *Ans.* $189\frac{3}{5}$ dollars.

15. If a farmer raise $\frac{9}{10}$ of a ton of hay per acre, how many tons of hay would he raise on 45 acres? *Ans.* $40\frac{1}{2}$ tons.

16. How many dollars should be paid for 9 barrels of flour, at $5\frac{1}{2}$ dollars per barrel?

$5\frac{1}{2}=\frac{11}{2}$; then 9 times 11 *halves* is $\frac{99}{2}$. *Ans.* $49\frac{1}{2}$ dollars.

17. How many dollars should be paid for 3 hundred weight of beef, at $4\frac{1}{4}$ dollars per hundred weight? *Ans.* $12\frac{3}{4}$ dollars.

18. If a stage coach run at the rate of $7\frac{1}{2}$ miles per hour, how many miles would it run in 13 hours? *Ans.* $97\frac{1}{2}$ miles.

19. A farmer had in oats 64 acres of ground, which produced $30\frac{3}{4}$ bushels per acre. What was the entire produce? *Ans.* 1968 bushels.

20. A grazier sold 75 head of fat cattle, at the rate of $35\frac{7}{8}$ dollars a head. What did he get for the whole number? *Ans.* $2690\frac{5}{8}$ dollars.

21. A merchant sold 37 yards of superfine cloth, at $11\frac{1}{4}$ dollars per yard. What did the whole amount to? *Ans.* $416\frac{1}{4}$ dollars.

22. A miller sold 100 barrels of flour, at $3\frac{3}{16}$ dollars per barrel. What did the whole amount to? *Ans.* $318\frac{3}{4}$ dollars.

23. If a ship sail at the rate of $125\frac{3}{4}$ miles per day, how far will she sail in 73 days? *Ans.* $9179\frac{3}{4}$ miles.

24. If one acre of ground produce $25\frac{1}{3}$ bushels of wheat, how many bushels would 150 acres produce at the same rate? *Ans.* 3800 bushels.

25. Allowing a person on a journey to travel at the rate of $40\frac{1}{4}$ miles per day, how far would he go in 31 days? *Ans.* $1247\frac{3}{4}$ miles.

26. Allowing 24 men to accomplish a certain work in $5\frac{1}{2}$ days, in how many days ought one man to accomplish the same work? *Ans.* 132 days.

27. Allowing a certain quantity of provisions to suffice 35 men for $13\frac{3}{4}$ days, how long ought the same quantity to suffice one man? *Ans.* $481\frac{1}{4}$ days.

EXERCISES ON CHAPTER IV.

1. Reduce $\frac{142}{378}$ and $\frac{230}{755}$ to their lowest terms.
Ans. $\frac{71}{189}$ and $\frac{46}{151}$.

2. Reduce $\frac{345}{800}$ and $\frac{436}{940}$ to their lowest terms.
Ans. $\frac{69}{160}$ and $\frac{109}{235}$.

3. Reduce $\frac{475}{670}$ and $\frac{738}{804}$ to their lowest terms.
Ans. $\frac{95}{134}$ and $\frac{123}{134}$.

4. Reduce $\frac{284}{300}$ and $\frac{900}{845}$ to their lowest terms.
Ans. $\frac{71}{75}$ and $\frac{180}{169}$.

5. Reduce $\frac{432}{868}$ and $\frac{820}{1000}$ to their lowest terms.
Ans. $\frac{108}{217}$ and $\frac{41}{50}$.

6. Reduce $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{13}{15}$ to a common denominator.
Ans. $\frac{240}{360}$, $\frac{315}{360}$, and $\frac{312}{360}$.

7. Reduce $\frac{9}{11}$, $\frac{3}{14}$, and $\frac{17}{20}$ to a common denominator.
Ans. $\frac{2520}{3080}$, $\frac{660}{3080}$, and $\frac{2618}{3080}$.

8. Reduce $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{11}{12}$ to a common denominator.
Ans. $\frac{288}{360}$, $\frac{300}{360}$, and $\frac{330}{360}$.

9. Reduce $\frac{3}{10}$, $\frac{5}{13}$, and $\frac{7}{15}$ to a common denominator.
Ans. $\frac{585}{1950}$, $\frac{750}{1950}$, and $\frac{910}{1950}$.

10. Reduce $\frac{7}{11}$, $\frac{8}{17}$, and $\frac{9}{20}$ to a common denominator.
Ans. $\frac{2380}{3740}$, $\frac{1760}{3740}$, and $\frac{1683}{3740}$.

11. Reduce $\frac{3}{16}$, $\frac{13}{20}$, and $\frac{25}{60}$ to the least common denominator.
Ans. $\frac{45}{240}$, $\frac{156}{240}$, and $\frac{100}{240}$.

12. Reduce $\frac{4}{16}$, $\frac{5}{20}$, and $\frac{21}{45}$ to the least common denominator.
Ans. $\frac{180}{720}$, $\frac{180}{720}$, and $\frac{336}{720}$.

13. Reduce $\frac{3}{20}$, $\frac{4}{16}$, and $\frac{11}{32}$ to the least common denominator.
Ans. $\frac{24}{160}$, $\frac{40}{160}$, and $\frac{55}{160}$.

14. Reduce $\frac{1}{30}$, $\frac{7}{40}$, and $\frac{19}{100}$ to the least common denominator.
Ans. $\frac{20}{600}$, $\frac{105}{600}$, and $\frac{114}{600}$.

15. Reduce 9, 13, $28\frac{4}{5}$, and $31\frac{3}{5}$, respectively to 5ths.
Ans. $\frac{45}{5}$, $\frac{65}{5}$, $\frac{144}{5}$, and $\frac{158}{5}$.

16. Reduce 10, 23, $40\frac{3}{8}$, and $73\frac{5}{8}$, respectively to 8ths.
Ans. $\frac{80}{8}$, $\frac{184}{8}$, $\frac{323}{8}$, and $\frac{589}{8}$.

17. Reduce $3\frac{1}{4}$, $5\frac{2}{3}$, $10\frac{2}{5}$, and $13\frac{1}{6}$, to improper fractions.
Ans. $\frac{13}{4}$, $\frac{17}{3}$, $\frac{52}{5}$, and $\frac{79}{6}$.

18. Reduce $7\frac{3}{12}$, $9\frac{4}{15}$, $75\frac{7}{8}$, and $90\frac{1}{9}$ to improper fractions.
Ans. $\frac{87}{12}$, $\frac{139}{15}$, $\frac{607}{8}$ and $\frac{811}{9}$.

19. Reduce $\frac{275}{5}$, $\frac{483}{10}$, $\frac{1750}{12}$, and $\frac{2365}{15}$ to integral or mixed numbers. *Ans.* 55, $48\frac{3}{10}$, $145\frac{5}{6}$, and $157\frac{2}{3}$.

20. Reduce $\frac{340}{13}$, $\frac{400}{12}$, $\frac{1860}{120}$, and $\frac{2930}{300}$ to integral or mixed numbers. *Ans.* $26\frac{2}{13}$, $33\frac{1}{3}$, $15\frac{1}{2}$, and $9\frac{23}{30}$.

21. A laid out 25 dollars for iron, at 40 dollars per ton, and B laid out 35 dollars for iron, at 45 dollars per ton. What quantity of the article did each of them purchase?

Ans. A $\frac{5}{8}$ and B $\frac{7}{9}$ of a ton.

22. A bought 9 yards of linsey for 12 dollars, and B bought 21 yards for 24 dollars. At what rate did each of them pay per yard? *Ans.* A $1\frac{1}{3}$ and B $1\frac{1}{7}$ dollar per yard.

23. If $\frac{1}{4}$ of an acre of land sell for 11 dollars, what should be received, at the same rate, for two farms, one containing 175, and the other $217\frac{3}{4}$ acres? *Ans.* 17281 dollars.

24. A farmer sold to A 37 bushels of wheat, to B 100 bushels, and to C 123 bushels, at $\frac{5}{8}$ of a dollar per bushel. What did the whole quantity sold amount to? *Ans.* $162\frac{1}{2}$ dollars.

25. A merchant bought 45 yards of Irish linen for 30 dollars, and sold the same for 35 dollars. At what price per yard was the linen bought and sold?

Ans. Bought at $\frac{2}{3}$ and sold at $\frac{7}{9}$ of a dollar.

26. If $\frac{1}{8}$ of a yard of cloth cost 2 dollars, what should be paid, at the same rate, for two pieces, each containing 13 yards, and a third one containing $9\frac{7}{8}$ yards? *Ans.* 574 dollars.

27. Bought of one person 45 sheep; of another 170; of another 63; of another 21; and of another 71; at an average of $\frac{3}{4}$ of a dollar a head. What did the whole amount to?

Ans. $277\frac{1}{2}$ dollars.

28. A sold to B 35 pounds of loaf sugar, at $\frac{3}{16}$ of a dollar a pound; to C 24 pounds of cheese, at $\frac{1}{12}$ of a dollar a pound; and to D 41 pounds of coffee, at $\frac{4}{25}$ of a dollar a pound. What did each of the articles amount to?

Ans. $6\frac{9}{16}$, 2, and $6\frac{14}{25}$ dollars.

29. Bought a lot of ground for $25, at the rate of $75 per acre. What was the quantity purchased? *Ans.* $\frac{1}{3}$ of an acre.

30. Two travelers having a journey of 1000 miles to perform, proceed as follows, namely, the first goes 25 days at the rate of 30 miles per day, and the second 20 days at the rate of 40 miles per day. What part of the journey has each accomplished? *Ans.* The first $\frac{3}{4}$, and the second $\frac{4}{5}$ of the journey.

31. A expended 75 dollars for steel at 87 dollars per ton, and B expended 87 dollars for steel at 93 dollars per ton. What quantity was purchased by each? and which of the two purchased the greater quantity?

Ans. A $\frac{75}{87}$, B $\frac{87}{93}$ of a ton; and B the greater quantity.

32. A merchant sold 4 yards of superfine cloth at $12\frac{1}{2}$ dollars per yard, 16 yards of silk at $2\frac{1}{4}$ dollars per yard, 13 yards of linen at $\frac{7}{8}$ of a dollar per yard, and 9 yards of calico at $\frac{1}{4}$ of a dollar per yard. What did each of the articles amount to?

Ans. 50, 36, $11\frac{3}{8}$, and $2\frac{1}{4}$ dollars.

CHAPTER V.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF FRACTIONS.

ADDITION OF FRACTIONS.

§ **109.** ADDITION OF FRACTIONS consists in finding the sum of two or more fractions.

The sum of two or more fractions is found by means of a *common denominator*.

Thus the sum of $\frac{2}{7}$ and $\frac{3}{7}$, is $\frac{5}{7}$; just as the sum of 2 *cents* and 3 *cents* is 5 *cents*.

What is the sum of $\frac{4}{8}$ and $\frac{3}{8}$? Of $\frac{5}{9}$ and $\frac{2}{9}$? Of $\frac{4}{10}$ and $\frac{5}{10}$? Of $\frac{3}{11}$, $\frac{2}{11}$, and $\frac{4}{11}$? Of $\frac{5}{12}$, $\frac{2}{12}$, and $\frac{3}{12}$? Of $\frac{7}{20}$, $\frac{9}{20}$, and $\frac{3}{20}$?

How many *units* or whole ones are there in the sum of $\frac{3}{5}$ and $\frac{2}{5}$? In the sum of $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$? In the sum of $\frac{6}{7}$, $\frac{5}{7}$, and $\frac{3}{7}$? In the sum of $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$? In the sum of $\frac{4}{10}$, $\frac{8}{10}$, and $\frac{9}{10}$?

§ **110.** Two or more fractions may often be reduced, mentally, to a common denominator, (§ 99), and then added together.

Thus to add together $\frac{2}{3}$ and $\frac{3}{4}$, we say $\frac{2}{3}$ is equal to $\frac{8}{12}$, and $\frac{3}{4}$ is equal to $\frac{9}{12}$; then $\frac{8}{12}+\frac{9}{12}=\frac{17}{12}=1\frac{5}{12}$.

What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? Of $\frac{1}{2}$ and $\frac{3}{4}$? Of $\frac{3}{5}$ and $\frac{5}{10}$? Of $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$? Of $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{4}{9}$? Of $\frac{3}{8}$, $\frac{3}{4}$, and $\frac{9}{16}$?

RULE XX.

§ **111.** *To add two or more fractions together.*

1. If the fractions have not a *common denominator*, reduce them to a common denominator.

2. Add all the numerators together, and place the sum, as a numerator, over their common denominator.

3. *Mixed numbers* may be added under the form of *improper fractions*; or, the fractions contained in them may be added, separately, and their sum added to that of the integers.

EXAMPLES.

1. To add together, $\frac{3}{5}$, $\frac{4}{7}$, and $\frac{5}{8}$.

Reducing these fractions to a common denominator,

$$\text{we have } \tfrac{3}{5} = \tfrac{168}{280}$$
$$\tfrac{4}{7} = \tfrac{160}{280}$$
$$\text{and } \tfrac{5}{8} = \tfrac{175}{280}$$

Adding the new numerators together, we have

$$168 + 160 + 175 = 503.$$

Placing the sum 503 over the common denominator 280 we have $\frac{3}{5} + \frac{4}{7} + \frac{5}{8} = \frac{503}{280}$, which is equal to $1\frac{223}{280}$.

2. To add together 56, $15\frac{2}{3}$, $8\frac{3}{4}$, and $7\frac{5}{12}$.

Reducing the mixed numbers to improper fractions,

$$\text{we have } 15\tfrac{2}{3} = \tfrac{47}{3},$$
$$8\tfrac{3}{4} = \tfrac{35}{4},$$
$$\text{and } 7\tfrac{5}{12} = \tfrac{89}{12},$$

Reducing these fractions to a common denominator,

we have, $\frac{188}{12}$, $\frac{105}{12}$, and $\frac{89}{12}$.

Then $15\frac{2}{3} + 8\frac{3}{4} + 7\frac{5}{12} = \frac{188}{12} + \frac{105}{12} + \frac{89}{12} = \frac{382}{12} = 31\frac{5}{6}$; and the integer 56 added to $31\frac{5}{6}$ makes the entire sum $87\frac{5}{6}$.

Otherwise. Adding together the fractions contained in the given mixed numbers, we shall find $\frac{2}{3} + \frac{3}{4} + \frac{5}{12} = 1\frac{10}{12} = 1\frac{5}{6}$.

Adding together all the integers, $56 + 15 + 8 + 7 = 86$; then the entire sum is $86 + 1\frac{5}{6} = 87\frac{5}{6}$, as before.

Note. In all subsequent exercises, improper fractions in the answers are to be reduced to integers or mixed numbers; and proper fractions, to their lowest terms.

EXERCISES.

1. Add together $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$, and $\frac{4}{9}$. *Ans.* $2\frac{25}{36}$.
2. Add together $\frac{5}{8}$, $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{2}{12}$. *Ans.* $2\frac{11}{120}$.
3. Add together $\frac{3}{7}$, $\frac{4}{5}$, $\frac{9}{20}$, and $\frac{5}{18}$. *Ans.* $1\frac{241}{252}$.
4. Add together 25, 34, $\frac{1}{3}$, and $\frac{7}{24}$ *Ans.* $59\frac{5}{8}$.
5. Add together 13, $\frac{5}{13}$, 73, and $\frac{13}{5}$. *Ans.* $88\frac{64}{65}$.
6. Find the sum of $3\frac{1}{4}$, $5\frac{2}{3}$, $10\frac{1}{2}$, and $35\frac{3}{8}$. *Ans.* $54\frac{19}{24}$.
7. Find the sum of $7\frac{3}{4}$, $9\frac{1}{5}$, $25\frac{5}{8}$, and $19\frac{7}{12}$. *Ans.* $62\frac{19}{120}$
8. Find the sum of $18\frac{1}{2}$, $\frac{9}{16}$, $75\frac{7}{8}$, and $50\frac{1}{5}$. *Ans.* $145\frac{11}{80}$.
9. Find the sum of 100, $15\frac{5}{6}$, $3\frac{2}{3}$, and $37\frac{1}{2}$. *Ans.* 157.
10. Find the sum of $10\frac{3}{4}$, 49, $\frac{5}{24}$, and $83\frac{3}{4}$. *Ans.* $143\frac{17}{24}$.

11. What sum should be paid for a vest at $4\frac{3}{4}$ dollars, and a hat at $5\frac{7}{8}$ dollars? *Ans.* $10\frac{5}{8}$ dollars.

12. What sum should be paid for a cord of wood at $3\frac{1}{2}$ dollars, a barrel of flour at $5\frac{3}{4}$ dollars, and a shote at $2\frac{1}{3}$ dollars? *Ans.* $11\frac{7}{12}$ dollars.

13. Bought a quantity of corn for $15\frac{7}{8}$ dollars, a ton of hay for 13 dollars, and a lot of pork for $19\frac{3}{4}$ dollars. What did the whole amount to? *Ans.* $48\frac{5}{8}$ dollars.

14. Sold wheat for 275 dollars, oats for $37\frac{3}{8}$ dollars, and rye for $27\frac{3}{5}$ dollars. What did the whole amount to? *Ans.* $339\frac{39}{40}$ dollars.

15. A merchant's bill was as follows: for calico $\frac{3}{4}$ of a dollar, linen $3\frac{3}{5}$ dollars, silk $13\frac{3}{4}$ dollars, and for groceries $21\frac{1}{8}$ dollars. What was the amount of the bill? *Ans.* $39\frac{9}{40}$ dollars.

16. A farmer paid three laborers for a month's work as follows: to the first, $15\frac{1}{2}$ bushels of corn; to the second, $19\frac{1}{4}$ bushels; to the third, $23\frac{7}{8}$ bushels. How much corn did he pay them all? *Ans.* $58\frac{5}{8}$ bushels.

17. A manufacturer sold four pieces of cloth. The first piece contained $39\frac{2}{3}$ yards, the second $41\frac{5}{8}$ yards, the other two each $93\frac{1}{4}$ yards. How many yards did he sell? *Ans.* $267\frac{19}{24}$ yards.

18. A person on a journey traveled the first day 31 miles; the second and third, each $29\frac{3}{7}$ miles; the fourth and fifth, each $27\frac{5}{8}$ miles. How far did he go in the five days? *Ans.* $145\frac{3}{28}$ miles.

19. A stage coach ran for two hours at the rate of $8\frac{7}{10}$ miles per hour, and for two hours more at the rate of $7\frac{1}{9}$ miles per hour; how far was that in the whole time? *Ans.* $31\frac{28}{45}$ miles.

20. Sold to A, 25 barrels of apples, for $56\frac{1}{4}$ dollars; to B, $30\frac{1}{2}$ barrels, for 75 dollars; and to C, $10\frac{3}{4}$ barrels for $21\frac{7}{8}$ dollars. Required the quantity sold, and the sum received. *Ans.* $66\frac{1}{4}$ barrels: $153\frac{1}{8}$ dollars.

21. Bought of a grocer a sack of coffee, for $13\frac{5}{8}$ dollars; a barrel of sugar, for $18\frac{4}{10}$ dollars; and a keg of rice, for $5\frac{1}{16}$ dollars. What sum should be paid for the whole? *Ans.* $37\frac{7}{80}$ dollars.

22. Laid out for goods, at one time, $\frac{5}{7}$ of a dollar; at another time, $3\frac{2}{5}$ dollars; at another, $21\frac{1}{3}$ dollars; and at another, $9\frac{3}{4}$ dollars. What was the whole sum disbursed? *Ans.* $35\frac{83}{420}$ dollars.

23. A merchant sold to one person, 4 yards of cloth for 24 dollars; to another, $9\frac{3}{8}$ yards for $43\frac{3}{16}$ dollars; and to another, $13\frac{1}{8}$ yards for $40\frac{1}{5}$ dollars. Required the quantity of cloth sold, and the sum received. *Ans.* $26\frac{1}{2}$ yards: $107\frac{31}{80}$ dollars.

24. Bought in market a pound of butter for $18\frac{3}{4}$ cents, a

dozen eggs for $12\frac{1}{2}$ cents, a quarter of veal for $56\frac{1}{4}$ cents, and a quart of peas for $6\frac{1}{4}$ cents. What did the whole amount to?

Ans. $93\frac{3}{4}$ cents.

25. Bought of a merchant a bunch of tape for $6\frac{1}{4}$ cents, a yard of cotton for 15 cents, a paper of pins for $31\frac{1}{4}$ cents, a yard of cambric for 25 cents, and a pair of gloves for $43\frac{3}{4}$ cents. What was the amount of the bill? *Ans.* $121\frac{1}{4}$ cents.

26. Bought of a farmer a quarter of beef for $8\frac{1}{2}$ dollars, a cord of wood for $2\frac{3}{4}$ dollars, a ton of hay for 13 dollars, a quantity of corn for $18\frac{1}{4}$ dollars, and a lot of bacon for $15\frac{7}{8}$ dollars. What did the whole amount to? *Ans.* $58\frac{3}{8}$ dollars.

27. On a journey I traveled the first day $41\frac{1}{2}$ miles, the second $40\frac{3}{4}$ miles, the third and fourth each 45 miles, the fifth and sixth each $39\frac{1}{5}$ miles. What distance did I accomplish in the six days? *Ans.* $250\frac{13}{20}$ miles.

28. Going out to collect money, I received from A $37\frac{1}{2}$ dollars, from B 20 dollars more than from A, from C $5\frac{3}{4}$ dollars more than from B, and from D as much as from the other three together. What was the whole sum collected?

Ans. $316\frac{1}{2}$ dollars.

29. A farmer bought at one time $97\frac{1}{4}$ acres of land, for 1000 dollars; at another, $127\frac{2}{5}$ acres, for $1375\frac{1}{2}$ dollars; at another, $500\frac{3}{8}$ acres, for 6831 dollars; and at another, $333\frac{1}{3}$ acres, for $4013\frac{3}{16}$ dollars. What was the whole quantity of land that he purchased, and the sum that he paid for it?

Ans. $1058\frac{43}{120}$ acres; $13219\frac{11}{16}$ dollars.

30. A contributed toward a charitable purpose, $23\frac{1}{2}$ dollars, B contributed twice as much as A, C as much as B, D as much as A and B together, and E as much as all the rest. What was the whole contribution? *Ans.* 376 dollars.

31. An agriculturist sold to A, $135\frac{3}{4}$ bushels of corn, for $65\frac{1}{2}$ dollars; and $20\frac{1}{2}$ bushels of oats, for $5\frac{1}{8}$ dollars. He also sold to B, $17\frac{3}{8}$ bushels of oats, for $4\frac{3}{4}$ dollars; and 79 bushels of corn for 39 dollars. What quantity of each did he sell, and what sum did he receive for the whole?

Ans. $214\frac{3}{4}$ bushels of corn, and $37\frac{7}{8}$ of oats: sum received $114\frac{3}{8}$ dollars.

SUBTRACTION OF FRACTIONS.

§ 112. SUBTRACTION OF FRACTIONS consists in finding the *difference* between two fractions; that is, the *remainder* when the less is taken from the greater.

The difference between two fractions is found by means of *a common denominator*.

Thus the difference between $\frac{4}{9}$ and $\frac{7}{9}$ is $\frac{3}{9}$; just as the difference between 4 *dollars* and 7 *dollars* is 3 *dollars*.

What is the difference between $\frac{3}{10}$ and $\frac{8}{10}$? Between $\frac{5}{12}$ and $\frac{11}{12}$? Between $\frac{9}{13}$ and $\frac{3}{13}$? Between $\frac{11}{15}$ and $\frac{4}{15}$? Between $\frac{3}{17}$ and $\frac{13}{17}$?

§ 113. Two fractions may often be reduced, mentally, to a common denominator, (§ 99), and then subtracted, the one from the other.

Thus to subtract $\frac{2}{5}$ from $\frac{4}{7}$, we say $\frac{2}{5}$ is equal to $\frac{14}{35}$, and $\frac{4}{7}$ is equal to $\frac{20}{35}$; then $\frac{20}{35}-\frac{14}{35}=\frac{6}{35}$.

What is the difference between $\frac{2}{8}$ and $\frac{3}{4}$? Between $\frac{2}{5}$ and $\frac{2}{3}$? Between $\frac{5}{6}$ and $\frac{7}{12}$? Between $\frac{1}{2}$ and $\frac{1}{3}$? Between $\frac{4}{5}$ and $\frac{8}{15}$? Between $\frac{3}{4}$ and $\frac{5}{8}$? Between $\frac{5}{7}$ and $\frac{11}{14}$? Between $\frac{9}{10}$ and $\frac{9}{20}$?

§ 114. A *proper fraction* may be subtracted from an *integer*, by first subtracting the fraction from a *unit*, and then subtracting a unit from the integer.

Thus to subtract $\frac{3}{5}$ from 7, we say $\frac{3}{5}$ from 1 or $\frac{5}{5}$ leaves $\frac{2}{5}$, and 1 from 7 leaves 6; then $7-\frac{3}{5}=6\frac{2}{5}$.

Subtract $\frac{2}{3}$ from 13. $\frac{3}{4}$ from 17. $\frac{4}{5}$ from 20. $\frac{1}{6}$ from 18. $\frac{3}{7}$ from 20.

RULE XXI.

§ 115. *To subtract one fraction from another.*

1. If the fractions have not a *common denominator*, reduce them to a common denominator.

2. Subtract the less numerator from the greater, and place the remainder, as a numerator, over the common denominator.

3. *Mixed numbers* may be used in subtraction under the form of *improper fractions:* or, the fractions in them may be taken first in subtracting, and then the integers.

EXAMPLES.

1. To find the difference between $\frac{7}{12}$ and $\frac{8}{15}$.

Reducing these fractions to a common denominator,

$$\text{we have } \tfrac{7}{12}=\tfrac{105}{180},$$
$$\text{and } \tfrac{8}{15}=\tfrac{96}{180},$$

Subtracting the less numerator 96 from the greater 105, and placing the remainder 9 over the common denominator 180, we find $\frac{7}{12}-\frac{8}{15}=\frac{9}{180}=\frac{1}{20}$.

2. To subtract $15\frac{2}{3}$ from $37\frac{4}{5}$.

Reducing these mixed numbers to improper fractions,

$$\text{we have } 15\tfrac{2}{3}=\tfrac{47}{3},$$
$$\text{and } 37\tfrac{4}{5}=\tfrac{189}{5}.$$

Reducing these improper fractions to a common denominator, we have $\frac{47}{3}=\frac{235}{15}$, and $\frac{189}{5}=\frac{567}{15}$;

then $37\frac{4}{5}-15\frac{2}{3}=\frac{567}{15}-\frac{235}{15}=\frac{332}{15}=22\frac{2}{15}$,

Otherwise. Taking the fractions $\frac{2}{3}$ and $\frac{4}{5}$, contained in the two mixed numbers, and reducing them to a common denominator, we shall find $15\frac{2}{3}=15\frac{10}{15}$,

and $37\frac{4}{5}=37\frac{12}{15}$.

Then subtracting $\frac{10}{15}$ from $\frac{12}{15}$, and 15 from 37

we have $37\frac{12}{15}-15\frac{10}{15}=22\frac{2}{15}$ as before.

3. To subtract $26\frac{3}{4}$ from $139\frac{2}{3}$.

Reducing the fractions contained in these two mixed numbers, to a common denominator,

$$\begin{array}{rr} \text{we shall find } 139\tfrac{2}{3}= & 139\tfrac{8}{12} \\ \text{and } 26\tfrac{3}{4}= & \underline{26\tfrac{9}{12}} \\ & 102\tfrac{11}{12} \end{array}$$

As $\frac{9}{12}$ cannot be subtracted from $\frac{8}{12}$, we add a *unit*, that is, $\frac{12}{12}$ to $\frac{8}{12}$, making $\frac{20}{12}$, and say $\frac{9}{12}$ from $\frac{20}{12}$ leaves $\frac{11}{12}$.

We then add 1 to the 6, and say 7 from 9 leaves 2, &c. (§ 34.)

4. To subtract $1843\frac{13}{15}$ from 2745.

$$\begin{array}{r} 2745(\tfrac{15}{15}) \\ \underline{1843\ \tfrac{13}{15}} \\ 901\ \tfrac{2}{15} \end{array}$$

Here we annex, mentally, $\frac{15}{15}$, equal to a *unit*, to the upper number, and say $\frac{13}{15}$ from $\frac{15}{15}$ leaves $\frac{2}{15}$. Then adding 1 to the 3, we say 4 from 5 leaves 1, &c. (§ 34).

EXERCISES.

1. Subtract $\frac{4}{9}$ from $\frac{7}{10}$, and $10\frac{3}{4}$ from $27\frac{6}{7}$. *Ans.* $\frac{23}{90}$ and $17\frac{3}{28}$.
2. Subtract $\frac{9}{10}$ from $\frac{12}{13}$, and $8\frac{1}{3}$ from $94\frac{4}{5}$. *Ans.* $\frac{3}{130}$ and $86\frac{7}{15}$.
3. Subtract $\frac{7}{12}$ from $\frac{11}{14}$, and $13\frac{7}{8}$ from $87\frac{2}{3}$. *Ans.* $\frac{17}{84}$ and $73\frac{19}{24}$.
4. Subtract $\frac{17}{20}$ from $\frac{44}{25}$, and $24\frac{4}{5}$ from $99\frac{1}{2}$. *Ans.* $\frac{91}{100}$ and $74\frac{7}{10}$.
5. Subtract $\frac{8}{30}$ from $\frac{39}{45}$, and $9\frac{5}{9}$ from $74\frac{3}{7}$. *Ans.* $\frac{3}{5}$ and $64\frac{55}{63}$.

6. Find the difference between 349 and $574\frac{3}{4}$. *Ans.* $225\frac{3}{4}$.
7. Find the difference between 730 and $625\frac{5}{8}$. *Ans.* $104\frac{3}{8}$.
8. Find the difference between 287 and $720\frac{3}{7}$. *Ans.* $433\frac{3}{7}$.
9. Find the difference between 934 and $373\frac{5}{9}$. *Ans.* $560\frac{4}{9}$.
10. Find the difference between 870 and $780\frac{7}{10}$. *Ans.* $89\frac{3}{10}$.

11. If flour were bought at $4\frac{1}{16}$ dollars per barrel, and sold at $5\frac{3}{8}$ dollars per barrel, what would be the gain per barrel? *Ans.* $1\frac{5}{16}$ dollars.

12. From a barrel of wine which contained $31\frac{1}{2}$ gallons, $13\frac{3}{4}$ gallons were drawn. What quantity remained in the barrel? *Ans.* $17\frac{3}{4}$ gallons.

13. A person who had to make a journey of 500 miles, has traveled $275\frac{5}{8}$ miles on his way. How far has he yet to go? *Ans.* $224\frac{3}{8}$ miles.

14. A farmer having 1000 acres of land, sells to one of his neighbors $479\frac{7}{16}$ acres. How many has he remaining? *Ans.* $520\frac{9}{16}$ acres.

15. A manufacturer who had on hand $700\frac{5}{8}$ yards of cloth has sold 534 yards of it. What quantity remains on hand? *Ans.* $166\frac{5}{8}$ yards.

16. A merchant bought a quantity of bacon for $15\frac{3}{8}$ dollars, and a quantity of pork for $23\frac{1}{2}$ dollars. He sold the whole for 48 dollars; what did he gain by the sale? *Ans.* $9\frac{1}{8}$ dollars.

17. Bought at one time $147\frac{2}{3}$ bushels of coal, and at another time $320\frac{1}{5}$ bushels. Having consumed 200 bushels, I desire to know what quantity is still on hand? *Ans.* $267\frac{13}{15}$ bushels.

18. A bought of B 75 yards of cloth; of which he sold to C $18\frac{3}{4}$ yards, and to D $20\frac{5}{8}$ yards. How many yards has he left? *Ans.* $35\frac{5}{8}$ yards.

19. A gentleman having 3000 dollars to divide among his three sons, gives $753\frac{1}{2}$ dollars to the first, 1284 dollars to the second, and the remainder to the third. What sum does the third receive? *Ans.* $962\frac{1}{2}$ dollars.

20. A merchant bought two pieces of cotton each containing $34\frac{3}{8}$ yards, of which he has sold $18\frac{3}{4}$ yards. How many yards has he left? *Ans.* 50 yards.

21. If I should collect from A 200 dollars, from B and C each $175\frac{1}{2}$ dollars, and then pay to D $56\frac{7}{8}$ dollars, and to E the remainder of the sum collected, how many dollars would E receive? *Ans.* $494\frac{1}{8}$ dollars.

22. Bought of A 40 cords of wood for $81\frac{3}{4}$ dollars, of which I sold to B 20 cords for $45\frac{1}{4}$ dollars. If I sell the rest of the wood to C for $43\frac{3}{4}$ dollars, what sum will I gain?

Ans. $7\frac{1}{4}$ dollars.

23. If a quantity of cloth be purchased for $321\frac{1}{4}$ dollars, a quantity of silk for 137 dollars, and a quantity of linen for $93\frac{7}{8}$ dollars, what will be the gain or loss if the whole be sold for 600 dollars? *Ans.* Gain $47\frac{7}{8}$ dollars.

24. A person having 100 dollars on hand, laid out $17\frac{1}{2}$ dollars for provisions, and paid taxes amounting to $21\frac{3}{4}$ dollars; what sum had he remaining? *Ans.* $60\frac{3}{4}$ dollars.

25. From the sum of 1500 dollars which I deposited in bank, having drawn, at different times, 200 dollars, $137\frac{3}{4}$ dollars, $313\frac{1}{2}$ dollars, and $79\frac{3}{8}$ dollars; what sum have I yet in bank?

Ans. $769\frac{3}{8}$ dollars.

26. Bought a quantity of iron for 95 dollars, and of coal for $81\frac{7}{8}$ dollars. The iron was sold for $115\frac{3}{4}$ dollars, and the coal for 100 dollars; what profit was made on both commodities?

Ans. $38\frac{7}{8}$ dollars.

27. Bought 350 acres of land for $4327\frac{1}{4}$ dollars. Having sold $137\frac{5}{8}$ acres for $1387\frac{1}{8}$ dollars, I desire to know how many acres remain, and for what sum the remainder should be sold to make a profit of 500 dollars on the whole?

Ans. $212\frac{3}{8}$ acres: $3440\frac{1}{8}$ dollars.

28. A merchant bought one piece of cloth containing $53\frac{1}{2}$ yards, another containing $39\frac{3}{8}$ yards, and another containing 40 yards. Having sold 13 yards from the first piece, $24\frac{3}{4}$ from the second, and $19\frac{1}{8}$ from the third, the merchant wishes to know the whole number of yards he has remaining.

Ans. 76 yards.

29. A speculator bought 1000 acres of land for $1587\frac{5}{8}$ dollars, and 500 acres for $737\frac{7}{8}$ dollars. Having sold $945\frac{1}{2}$ acres for 2000 dollars, he wishes to know what quantity of land he has remaining, and for what sum he could afford to sell the remainder, so as to lose nothing on the whole.

Ans. $554\frac{1}{2}$ acres; $325\frac{18}{40}$ dollars.

MULTIPLICATION OF FRACTIONS.

§ **116.** *Multiplying by a fraction* consists in finding *such a part of the multiplicand* as is expressed by the multiplier.

For example, 12 *multiplied by* $\frac{5}{6}$ is 5 *times* $\frac{1}{6}$ of 12, which is $\frac{5}{6}$ of 12, equal to 10.

How many is $\frac{1}{2}$ of 6, or 6 *multiplied by* $\frac{1}{2}$? 6 multiplied by $\frac{1}{3}$?
How many is $\frac{1}{4}$ of 12, or 12 multiplied by $\frac{1}{4}$? $\frac{3}{4}$ of 12, or $12\times\frac{3}{4}$?

$\frac{1}{5}$ of 20, or $20\times\frac{1}{5}$? $\frac{4}{5}$ of 20, or $20\times\frac{4}{5}$? $\frac{4}{7}$ of 35, or $35\times\frac{4}{7}$?
$\frac{1}{6}$ of 30, or $30\times\frac{1}{6}$? $\frac{5}{6}$ of 30, or $30\times\frac{5}{6}$? $\frac{3}{8}$ of 40, or $40\times\frac{3}{8}$?

How many is 12 multiplied by $3\frac{1}{2}$; that is, 3 times 12, together with $\frac{1}{2}$ of 12? 8 multiplied by $4\frac{1}{4}$? $10\times6\frac{2}{5}$? $12\times5\frac{3}{4}$?

How many is 20 multiplied by $4\frac{1}{5}$: that is, 4 times 20, together with $\frac{1}{5}$ of 20? 24 multiplied by $3\frac{1}{6}$? $8\times9\frac{3}{4}$? $12\times10\frac{5}{6}$?

Compound Fractions.

§ **117.** A fraction *multiplied by another fraction*, or divided by an *integer*, may be expressed as a *compound* fraction, that is, *a fraction of a fraction.*

Thus $\frac{5}{6}\times\frac{2}{3}$ is $\frac{2}{3}$ of $\frac{5}{6}$, (§ 116;) and $\frac{5}{6}\div4$ is $\frac{1}{4}$ of $\frac{5}{6}$, (§ 51).

The expressions $\frac{2}{3}$ of $\frac{5}{6}$, and $\frac{1}{4}$ of $\frac{5}{6}$ are called *compound fractions*, in contradistinction to *simple fractions*, which consist of a single numerator and denominator. Hence,

§ **118.** Multiplying *two or more fractions* together is equivalent to reducing a *compound* to a *simple* fraction.

RULE XXII.

§ **119.** *To multiply two or more fractions together.*

1. Multiply the numerators together for a numerator, and the denominators together for a denominator.

2. *An integer and a fraction are multiplied together*, by multiplying the numerator, or dividing the denominator, of the fraction, by the integer.

3. A *mixed number* may be used in multiplication under the form of an *improper fraction;* or the integer and fraction in it may be taken separately in multiplying,—observing to add together the separate products.

EXAMPLES.

1. To multiply $\frac{5}{7}$ by $\frac{3}{4}$; that is, to find $\frac{3}{4}$ of $\frac{5}{7}$.

$$\frac{5}{7}\times\frac{3}{4}=\frac{5\times3}{7\times4}=\frac{15}{28}.$$

2. To multiply $\frac{5}{24}$ by 6; that is to find 6 times $\frac{5}{24}$.

Multiplying the *numerator* by the *integer*,

we have $\frac{5}{24}\times6=\frac{5\times6}{24}=\frac{30}{24}=1\frac{6}{24}=1\frac{1}{4}$.

Or, *dividing the denominator* by the integer,

we have $\frac{5}{24}\times6=\frac{5}{24\div6}=\frac{5}{4}=1\frac{1}{4}$; as before.

3. To multiply $5\frac{2}{3}$ by $2\frac{1}{2}$; that is, to find *twice* $5\frac{2}{3}$, together with $\frac{1}{2}$ of $5\frac{2}{3}$.

Reducing the two mixed numbers to *improper fractions*,

we have $5\frac{2}{3}=\frac{17}{3}$, and $2\frac{1}{2}=\frac{5}{2}$.

Then $5\frac{2}{3}\times2\frac{1}{2}=\frac{17}{3}\times\frac{5}{2}=\frac{85}{6}=14\frac{1}{6}$.

Otherwise. Taking the integer and fraction in each mixed number, separately, we shall find $5\frac{2}{3}\times2=11\frac{1}{3}$.

and $\frac{1}{2}$ of $5\frac{2}{3}=2\frac{5}{6}$. Then $11\frac{1}{3}+2\frac{5}{6}=14\frac{1}{6}$. (§ 23).

In multiplying, we say *twice* $\frac{2}{3}$ is $\frac{4}{3}$, equal to $1\frac{1}{3}$. Setting down $\frac{1}{3}$, and *carrying* 1,—*twice* 5 is 10, and 1 is 11.

Next, $\frac{1}{2}$ of 5 is 2, with 1 over; this 1, equal to $\frac{3}{3}$, added to the $\frac{2}{3}$ makes $\frac{5}{3}$; then $\frac{1}{2}$ of $\frac{5}{3}$ is $\frac{5}{3}\times\frac{1}{2}=\frac{5}{6}$.

☞ Recurring to the first example, we remark, first, that $\frac{1}{4}$ of $\frac{1}{7}$ is $\frac{1}{28}$. For if any quantity were divided into 7 *sevenths*, and each one of these *sevenths* were divided into 4 *equal parts*, these last parts would be 28*ths* of the quantity, since 7 times 4=28.

That is, 1 *fourth* of 1 *seventh* is $\frac{1}{28}$. 1 *fourth* of 5 *sevenths* is, therefore, $\frac{5}{28}$; and 3 *fourths* of 5 *sevenths* is 3 times as much as 1 *fourth* of it, that is, 3 times $\frac{5}{28}$, which is $\frac{15}{28}$.

This demonstration discovers two principles, as involved in the Rule for multiplying a fraction by a fraction.

First. Multiplying the *denominator* of a fraction, finds ***such a part*** of the fraction as is expressed by the *reciprocal of the multiplier*; and, then, secondly, multiplying the *numerator* finds as many of such parts as are expressed by the multiplier.

Thus, multiplying the denominator of $\frac{5}{7}$ by 4, finds $\frac{5}{28}=\frac{1}{4}$ of $\frac{5}{7}$;

then, multiplying the numerator of $\frac{5}{28}$ by 3, finds $\frac{15}{28}=\frac{3}{4}$ of $\frac{5}{7}$. ☜

From the preceding demonstration it follows that,

§ 120. In fractional, as in integral, multiplication, the pro duct remains *the same*, when the multiplicand and multiplier are taken, the one for the other. Thus $\frac{4}{7}$ of $\frac{3}{4}=\frac{3}{4}$ of $\frac{4}{7}$.

EXERCISES.

1. Multiply $\frac{5}{8}$ by $\frac{2}{3}$, and $\frac{7}{20}$ by 5. *Ans.* $\frac{5}{12}$ and $1\frac{3}{4}$.
2. Multiply $\frac{4}{9}$ by $\frac{3}{5}$, and $\frac{11}{30}$ by 10. *Ans.* $\frac{4}{15}$ and $3\frac{2}{3}$.
3. Multiply $\frac{6}{13}$ by $\frac{1}{2}$, and 13 by $\frac{1}{3}$. *Ans.* $\frac{3}{13}$ and $4\frac{1}{3}$.
4. Multiply $\frac{8}{11}$ by $\frac{3}{2}$, and 15 by $\frac{4}{5}$. *Ans.* $1\frac{1}{11}$ and 12.
5. Multiply $\frac{7}{12}$ by $\frac{5}{6}$, and 39 by $\frac{3}{7}$. *Ans.* $\frac{35}{72}$ and $16\frac{5}{7}$.

S. Multiply $8\frac{3}{4}$ by $\frac{1}{2}$, and $9\frac{2}{3}$ by 3. *Ans.* $4\frac{3}{8}$ and 29.
7. Multiply $10\frac{1}{2}$ by 5, and $12\frac{1}{4}$ by $\frac{2}{3}$. *Ans.* $52\frac{1}{2}$ and $8\frac{1}{6}$.
8. Multiply 15 by $3\frac{1}{4}$, and 20 by $4\frac{1}{2}$. *Ans.* $48\frac{3}{4}$ and 90.
9. Multiply $9\frac{3}{8}$ by $5\frac{3}{4}$, and $10\frac{2}{5}$ by $1\frac{7}{8}$. *Ans.* $53\frac{29}{32}$ and $19\frac{1}{2}$.
10. Multiply $25\frac{5}{6}$ by $4\frac{1}{4}$, and 30 by $9\frac{4}{5}$. *Ans.* $109\frac{19}{24}$ and 294.

11. What should be paid for $\frac{3}{4}$ of a yard of linen, at the rate of $\frac{7}{8}$ of a dollar per yard?

It is plain that $\frac{3}{4}$ of a yard will cost $\frac{3}{4}$ of $\frac{7}{8}$ of a dollar. $\frac{3}{4}$ of $\frac{7}{8}$, the same as $\frac{7}{8}\times\frac{3}{4}$, is a compound fraction, *to be reduced to a simple fraction.* (§ 118). *Ans.* $\frac{21}{32}$ of a dollar.

12. What should be paid for $\frac{2}{3}$ of a barrel of apples, if the whole barrel be worth $\frac{15}{16}$ of a dollar? *Ans.* $\frac{5}{8}$ of a dollar.

13. A person owning $\frac{7}{12}$ of a tract of land, sells $\frac{3}{5}$ of his share to A; what part of the whole tract does A purchase? *Ans.* $\frac{7}{20}$ of the whole.

14. What should be paid for $\frac{1}{2}$ of $\frac{7}{8}$ of a pound of tea, at the rate of $\frac{13}{16}$ of a dollar per pound? *Ans.* $\frac{91}{256}$ of a dollar.

15. A gentleman owning $\frac{1}{4}$ of a ship, sells $\frac{2}{7}$ of his share to A, and the rest of it to B. What part of the whole ship does he sell to each? *Ans.* To A $\frac{1}{14}$, and to B $\frac{5}{28}$ of the whole.

16. Bought $\frac{3}{5}$ of an acre of ground at the rate of $18\frac{3}{4}$ dollars per acre; required the sum to be paid for it. *Ans.* $11\frac{1}{4}$ dollars.

17. Sold $25\frac{1}{2}$ bushels of clover seed at $7\frac{1}{4}$ dollars per bushel, and 3 bushels at 7 dollars per bushel; what did the whole amount to? *Ans.* $205\frac{7}{8}$ dollars.

18. Find the entire cost of $\frac{3}{4}$ of a pound of pepper at $\frac{1}{4}$ of a dollar a pound, $\frac{3}{4}$ of a hundred weight of flour at $2\frac{1}{2}$ dollars a hundred, and $2\frac{1}{8}$ yards of cloth at 7 dollars a yard. *Ans.* $16\frac{15}{16}$ dollars.

In multiplying two or more fractions together, *equal factors may be cancelled in the resulting numerator and denominator*, without affecting the value of the product. (§ 92, § 69).

$$\tfrac{3}{4} \text{ of } \tfrac{4}{5} \text{ of } \tfrac{5}{7} = \frac{3\times\cancel{4}\times\cancel{5}}{\cancel{4}\times\cancel{5}\times7} = \tfrac{3}{7}.$$

19. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{13}$ to a simple fraction. *Ans.* $\frac{5}{26}$.

20. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{5}$ of $7\frac{1}{2}$ to a simple expression. *Ans.* $1\frac{1}{2}$.

A factor of a *composite number* is canceled by substituting *the other factor* of that number for the number itself. (§ 66).

$$\tfrac{7}{9}\times\tfrac{5}{14} = \frac{1\times5}{9\times2}, \text{ by dividing 7 and 14 by 7.}$$

21. Multiply $\frac{3}{13}$ of $\frac{3}{16}$ of $\frac{5}{12}$ by $\frac{12}{35}$ of $\frac{7}{24}$. *Ans.* $\frac{1}{208}$.

22. Multiply $\frac{10}{11}$ of $\frac{3}{20}$ of $\frac{1}{6}$ by $\frac{5}{7}$ of 21. *Ans.* $\frac{15}{44}$.

23. Multiply $\frac{6}{17}$ of $\frac{1}{18}$ of $8\frac{1}{2}$ by $\frac{9}{11}$ of $10\frac{2}{3}$. *Ans.* $1\frac{15}{11}$.

24. Having purchased 350 bushels of wheat at $\frac{7}{8}$ of a dollar a bushel, and sold $\frac{2}{3}$ of the quantity at 1 dollar a bushel, what would I gain or lose on the whole by selling the remainder at $\frac{3}{4}$ dollar a bushel? *Ans.* Gain $14\frac{7}{12}$ dollars.

25. In how many days ought one man to accomplish an undertaking which 17 men could perform in $13\frac{3}{8}$ days?
Ans. $227\frac{3}{8}$ days.

26. A farmer bought $\frac{4}{5}$ of a tract of land which contained $735\frac{1}{2}$ acres, and sold to his neighbor $\frac{1}{2}$ of his purchase. What part of the whole tract, and how many acres, did he sell?
Ans. $\frac{2}{5}$ of the tract; $294\frac{1}{5}$ acres.

27. Bought of A $3\frac{1}{2}$ cords of wood, at $2\frac{3}{4}$ dollars per cord; of B $7\frac{3}{8}$ cords, at $1\frac{5}{8}$ dollars per cord; and of C 10 cords, at $2\frac{1}{16}$ dollars per cord. What was the whole quantity of wood purchased, and the whole sum paid for it?
Ans. $20\frac{7}{8}$ cords; $42\frac{15}{64}$ dollars.

28. If 3 masons can build a wall of a certain length, height, and thickness, in $13\frac{1}{2}$ days, in what time ought one mason to build a wall of the same height and thickness, but $2\frac{1}{4}$ times as long? *Ans.* $91\frac{1}{8}$ days.

29. A speculator bought of A 189 acres of land at 10 dollars an acre, and of B $250\frac{1}{4}$ acres at 13 dollars an acre. He sold $\frac{2}{3}$ of the first tract at $18\frac{1}{2}$ dollars, and $\frac{3}{5}$ of the second at 19 dollars an acre; what would he make, on the whole, by selling the remainder of both tracts at 20 dollars an acre?
Ans. $3352\frac{13}{20}$ dollars

DIVISION OF FRACTIONS.

§ **121.** *Dividing by a fraction*, as well as by an integer, consists in finding *how many times* the dividend *contains the divisor*, or *what part* the dividend is of the divisor.

Thus $\frac{6}{7}$ divided by $\frac{2}{7}$ gives the quotient 3, because $\frac{6}{7}$ contains $\frac{2}{7}$ 3 *times*; and $\frac{1}{7}$ divided by $\frac{2}{7}$ gives the quotient $\frac{1}{2}$, because $\frac{1}{7}$ is $\frac{1}{2}$ of $\frac{2}{7}$.

How many times does $\frac{8}{9}$ contain $\frac{2}{9}$, or what is the quotient of $\frac{8}{9}$ divided by $\frac{2}{9}$? Of $\frac{6}{10}$ divided by $\frac{2}{10}$? Of $\frac{30}{41}$ divided by $\frac{6}{41}$?

What part is $\frac{1}{4}$ of $\frac{3}{4}$, or what is the quotient of $\frac{1}{4}$ divided by $\frac{3}{4}$? Of $\frac{1}{5}$ divided by $\frac{4}{5}$? Of $\frac{2}{6}$ divided by $\frac{3}{6}$? Of $\frac{3}{7}\div\frac{5}{7}$? Of $\frac{4}{3}\div\frac{10}{3}$?

§ **122.** In fractional, as in integral, division, the dividend is a *product* given, and the divisor *one of its factors* given, to find the other factor.

Thus $\frac{8}{15}\div\frac{2}{3}$ is equal to $\frac{4}{5}$, because $\frac{8}{15}$ is the product of $\frac{2}{3}\times\frac{4}{5}$, that is, $\frac{8}{15}$ is $\frac{4}{5}$ of $\frac{2}{3}$; the quotient $\frac{4}{5}$ expressing the *part that the dividend* $\frac{8}{15}$ is of the divisor $\frac{2}{3}$.

What is the quotient of $\frac{9}{24}$ divided by $\frac{3}{4}$; that is, $\frac{3}{4}$ being one factor of $\frac{9}{24}$, what is the other factor? Of $\frac{18}{30}\div\frac{3}{10}$? Of $\frac{20}{36}\div\frac{5}{6}$? Of $\frac{32}{45}\div\frac{8}{9}$?

Reciprocal of a Fraction.

§ **123.** The *reciprocal* of a fraction is the fraction *inverted*; and is equal to a *unit* divided by the fraction. (§ 50).

Thus the reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$; equal to 1 or $\frac{7}{7}$ divided by $\frac{2}{7}$.

What is the reciprocal of $\frac{3}{4}$? Of $\frac{2}{5}$? Of $\frac{8}{13}$? Of $\frac{23}{39}$?

The reciprocal of a *mixed number* is that of its equivalent improper fraction. Thus the reciprocal of $5\frac{2}{3}=\frac{17}{3}$ is $\frac{3}{17}$.

What is the reciprocal of $2\frac{1}{2}$? Of $7\frac{3}{4}$? Of $10\frac{1}{4}$? Of $9\frac{3}{10}$?

Complex or Mixed Fractions.

§ **124.** When the dividend or divisor is a *fraction* or a *mixed* number, the dividend placed over the divisor, with a line between, forms a *complex* or *mixed fraction*.

Thus, $\frac{3}{4}\div5$, $2\div3\frac{1}{2}$, may be expressed, respectively, by the *complex fractions* $\frac{\frac{3}{4}}{5}$, and $\frac{2}{3\frac{1}{2}}$.

These complex fractions may be read—*numerator* $\frac{3}{4}$, *denominator* 5; numerator 2, denominator $3\frac{1}{2}$.

A complex, as well as a simple, fraction is equal to its numerator divided by its denominator. (§ 92).

RULE XXIII.

§ **125.** *To divide a fraction by a fraction.*

1. Divide the numerator of the dividend by that of the divisor, and the denominator by the denominator; or

2. Multiply the dividend by the *reciprocal of the divisor.*

3. *A fraction is divided by an integer*, by dividing the numerator by the integer, or multiplying the denominator by the integer.

4. *An integer is divided by a fraction*, by multiplying the integer by the reciprocal of the fraction.

5. A *mixed number* may be used in division under the form of an improper fraction. A mixed number may also be divided by an *integer*, by dividing the integer and fraction in the mixed number, separately.

EXAMPLES.

1. To divide $\frac{18}{20}$ by $\frac{2}{5}$.

Dividing numerator by numerator, and denominator by denominator,

we have $\frac{18}{20} \div \frac{2}{5} = \frac{18 \div 2}{20 \div 5} = \frac{9}{4} = 2\frac{1}{4}$.

2. To divide $\frac{9}{35}$ by $\frac{2}{3}$.

Here we cannot divide numerator by numerator, and denominator by denominator, without remainders. Multiplying, therefore, the dividend by the *reciprocal of the divisor,*

we have $\frac{9}{35} \div \frac{2}{3} = \frac{9}{35} \times \frac{3}{2} = \frac{27}{70}$.

3. To divide $\frac{10}{13}$ by 5.

Dividing the numerator of the fraction by the integer,

we have $\frac{10}{13} \div 5 = \frac{10 \div 5}{13} = \frac{2}{13}$.

Or, *multiplying* the *denominator* by the integer,

we have $\frac{10}{13} \div 5 = \frac{10}{13 \times 5} = \frac{10}{65} = \frac{2}{13}$, as before.

4. To divide 123 by $\frac{4}{5}$.

Multiplying the integer by the reciprocal of the fraction,

we have $123 \div \frac{4}{5} = 123 \times \frac{5}{4} = \frac{615}{4} = 153\frac{3}{4}$.

5. To divide $163\frac{2}{3}$ by 5.

Reducing the mixed number to an improper fraction,

we have $163\frac{2}{3} \div 5 = \frac{491}{3} \div 5 = \frac{491}{15} = 32\frac{11}{15}$.

Otherwise. Dividing the integer and fraction in the mixed number, separately,

$$\begin{array}{r} 5)\underline{163\ \frac{2}{3}} \\ 32\ \frac{11}{15} \end{array}$$

We say 5 in 16, 3 times and 1 over; 5 in 13, *twice* and 3 over: this 3 and the $\frac{2}{3}$ make $3\frac{2}{3}$, equal to $\frac{11}{3}$; then $\frac{11}{3} \div 5$, that is, $\frac{1}{5}$ of $\frac{11}{3}$, is $\frac{11}{15}$.

☞ Dividing *a fraction by a fraction* is the *reverse* of *multiplying* a fraction by a fraction. (§122).

When the terms of the dividend cannot be divided by those of the divisor without a remainder, suppose each term of the former $\times$ by *both terms* of the latter; this will not *alter the value* of the dividend. (§93). Then the *resulting numerator* $\div$ the numerator of the divisor, and the *denominator* by the denominator, gives, for the quotient, the *dividend* $\times$ the *reciprocal of the divisor.*

Thus, in the second example,

$$\frac{9}{35} = \frac{9 \times 2 \times 3}{35 \times 2 \times 3}; \text{ and } \frac{9 \times 2 \times 3}{35 \times 2 \times 3} \div \frac{2}{3} = \frac{9 \times 3}{35 \times 2} = \frac{9}{35} \times \frac{3}{2} = \text{the } \textit{quotient.} ☜$$

From the Rule, thus demonstrated, it follows that,

§ **126.** Dividing by a Fraction is equivalent to *multiplying the dividend* by the *reciprocal of the divisor.*

In thus multiplying, *equal factors may be canceled* in the resulting numerator and denominator, (§ 92, § 69).

Thus $\frac{8}{15} \div \frac{4}{5} = \frac{8}{15} \times \frac{5}{4} = \frac{2}{3} \times \frac{1}{1}$, or simply $\frac{2}{3}$, by only dividing 8 and 4 by 4, and 15 and 5 by 5.

EXERCISES.

1. Divide $\frac{18}{25}$ by $\frac{3}{5}$, and $\frac{21}{35}$ by 7. *Ans.* $1\frac{1}{5}$ and $\frac{3}{35}$.
2. Divide $\frac{13}{50}$ by $\frac{4}{9}$, and $\frac{8}{15}$ by 9. *Ans.* $\frac{117}{200}$ and $\frac{8}{135}$.
3. Divide $\frac{9}{14}$ by $\frac{10}{11}$, and 29 by $\frac{5}{7}$. *Ans.* $\frac{99}{140}$ and $40\frac{3}{5}$.
4. Divide $\frac{12}{17}$ by $\frac{4}{5}$, and $\frac{9}{75}$ by 13. *Ans.* $\frac{15}{17}$ and $\frac{3}{325}$.
5. Divide $\frac{35}{72}$ by $\frac{7}{9}$, and 95 by $\frac{7}{10}$. *Ans.* $\frac{5}{8}$ and $135\frac{5}{7}$.

6. Divide $8\frac{3}{4}$ by $5\frac{2}{3}$.

Reducing the mixed numbers to improper fractions,

we have $8\frac{3}{4} \div 5\frac{2}{3} = \frac{35}{4} \div \frac{17}{3} = \frac{35}{4} \times \frac{3}{17} = \frac{105}{68} = 1\frac{37}{68}$.

7. Divide $\frac{54}{75}$ by 9, and $10\frac{2}{3}$ by $3\frac{1}{4}$. *Ans.* $\frac{2}{25}$ and $3\frac{11}{39}$.
8. Divide 95 by $\frac{7}{12}$, and $125\frac{1}{2}$ by 7. *Ans.* $162\frac{6}{7}$ and $17\frac{13}{14}$.
9. Divide $\frac{27}{95}$ by 12, and 38 by $9\frac{4}{5}$. *Ans.* $\frac{9}{380}$ and $3\frac{43}{49}$.
10. Divide 100 by $\frac{4}{5}$, and $15\frac{2}{3}$ by $6\frac{1}{2}$. *Ans.* 125 and $2\frac{16}{39}$.
11. Divide $\frac{36}{53}$ by 3, and $340\frac{3}{8}$ by 8. *Ans.* $\frac{12}{53}$ and $42\frac{35}{64}$.
12. Divide 236 by $\frac{5}{8}$, and 425 by $7\frac{3}{7}$. *Ans.* $377\frac{3}{5}$ and $57\frac{11}{52}$.
13. Divide $\frac{4}{85}$ by $\frac{7}{10}$, and $18\frac{3}{4}$ by $10\frac{2}{5}$. *Ans.* $\frac{8}{119}$ and $1\frac{167}{208}$.
14. Divide 370 by $\frac{3}{11}$, and $213\frac{1}{4}$ by 11. *Ans.* $1356\frac{2}{3}$ and $19\frac{17}{44}$.
15. Divide $\frac{21}{100}$ by 21, and 11 by $213\frac{1}{4}$. *Ans.* $\frac{1}{100}$ and $\frac{44}{853}$.

16. How many yards of calico, at $\frac{1}{4}$ of a dollar per yard, may be purchased for $\frac{7}{8}$ of a dollar?

The number of yards will be the number of times $\frac{1}{4}$ of a dollar *is contained in* $\frac{7}{8}$ of a dollar, *that is,* $\frac{7}{8} \div \frac{1}{4}$. *Ans.* $3\frac{1}{2}$ yards.

17. How many weeks will a family be in consuming $19\frac{4}{5}$ barrels of flour, at the rate of $\frac{3}{4}$ of a barrel per week? *Ans.* $26\frac{2}{5}$ weeks.

18. How many days would a person be in traveling $175\frac{1}{2}$ miles, at the rate of $31\frac{3}{4}$ miles per day? *Ans.* $5\frac{67}{127}$ days.

19. A gentleman purchased a farm for $4379\frac{1}{16}$ dollars, paying $16\frac{7}{8}$ dollars per acre. Required the number of acres purchased. *Ans.* $259\frac{1}{2}$ acres.

20. A merchant laid out for broadcloth 5727 dollars, paying $5\frac{3}{4}$ dollars a yard. Required the number of yards purchased. *Ans.* 996 yards.

21. How many barrels of wine are there in 2753 gallons, allowing $31\frac{1}{2}$ gallons to make one barrel? *Ans.* $87\frac{25}{63}$ barrels.

22. How many years would there be in $5783\frac{1}{2}$ days, allowing $365\frac{1}{4}$ days to make one year? *Ans.* $15\frac{1219}{1461}$ years.

23. What quantity of salt may be purchased for $\frac{1}{4}$ of a dollar, at $\frac{5}{8}$ of a dollar per bushel?

$\frac{1}{4}$ *of a dollar will buy the same part of a bushel that* $\frac{1}{4}$ *of a dollar is of* $\frac{5}{8}$ *of a dollar; that is,* $\frac{1}{4} \div \frac{5}{8}$. *Ans.* $\frac{2}{5}$ of a bushel.

24. What quantity of iron may be purchased for $22\frac{1}{2}$ dollars, at the rate of 45 dollars per ton? *Ans.* $\frac{1}{2}$ of a ton.

25. What quantity of land may be purchased for $15\frac{3}{4}$ dollars, at the rate of 29 dollars per acre? *Ans.* $\frac{63}{116}$ of an acre.

26. If a person could accomplish a certain work in $25\frac{1}{2}$ days what part of the work could he perform in $9\frac{1}{4}$ days? *Ans.* $\frac{37}{102}$ of the work.

27. A laborer agreed to work 30 days for a certain sum of money; having worked but $17\frac{1}{2}$ days, what part of the stipulated sum ought he to receive? *Ans.* $\frac{7}{12}$ of it.

28. A mason having undertaken to build a wall of a specified length, height, and thickness in 100 days, it is required to determine what part of it he ought to finish in $39\frac{1}{2}$ days.
Ans. $\frac{79}{200}$ of it.

29. If $\frac{2}{3}$ of a yard of cloth cost 5 dollars, what is the price of a yard of the cloth at the same rate?

2 *thirds of a yard being worth* 5 *dollars,* 1 *third of a yard is worth* $\frac{1}{2}$ *of* 5 *dollars; hence* a *whole yard,* or 3 *thirds* of a yard, *is worth* $\frac{3}{2}$ *of* 5 *dollars; that is,* $5\times\frac{3}{2}$, or $5\div\frac{2}{3}$.

We may also reason thus: *the price of a yard will be such that* $\frac{2}{3}$ *of the price will be* 5 *dollars;* or the *price* $\times\ \frac{2}{3}=5$ dollars.

Hence 5 is a *dividend* or product *given,* $\frac{2}{3}$ a *divisor* or one factor *given, to find the quotient, which will be the other factor.*
Ans. $7\frac{1}{2}$ dollars.

30. If $\frac{3}{4}$ of a ton of hay sell for 10 dollars, what should be paid for a ton of hay at the same rate? *Ans.* $13\frac{1}{3}$ dollars.

31. If $\frac{5}{8}$ of an acre of ground bring $21\frac{1}{4}$ dollars, what should be given for an acre at that rate? *Ans.* 34 dollars.

32. Allowing $\frac{4}{5}$ of a ton of coal to amount to $5\frac{3}{8}$ dollars, what is the value of a ton at that rate? *Ans.* $6\frac{23}{32}$ dollars.

33. Allowing $\frac{7}{10}$ of an acre of ground to produce $19\frac{1}{2}$ bushels of wheat, what would one acre produce at the same rate?
Ans. $27\frac{6}{7}$ bushels.

34. What should be paid for 4 yards of cloth, when $\frac{3}{4}$ of a yard costs $6\frac{1}{2}$ dollars?

First find what should be paid for one yard.
Ans. $34\frac{2}{3}$ dollars.

35. What should be paid for 7 gallons of wine, when $\frac{1}{2}$ of a gallon costs 1 dollar? *Ans.* 14 dollars.

36. What should be paid for 10 acres of ground, when $\frac{3}{4}$ of an acre sells for 28 dollars? *Ans.* $373\frac{1}{3}$ dollars.

37. What should be paid for 12 yards of silk, when $\frac{7}{8}$ of a yard sells for $1\frac{1}{4}$ dollars? *Ans.* $17\frac{1}{7}$ dollars.

38. What should be paid for $9\frac{1}{2}$ barrels of flour, when $\frac{1}{2}$ of a barrel sells for $3\frac{3}{4}$ dollars? *Ans.* $71\frac{1}{4}$ dollars.

39. Allowing $\frac{3}{8}$ of an acre of land to be worth $15\frac{7}{8}$ dollars, what should be given for $120\frac{1}{2}$ acres at the same rate?
Ans. $5101\frac{1}{6}$ dollars.

40. Allowing $\frac{9}{10}$ of a ton of steel to amount to $100\frac{1}{2}$ dollars, what would $3\frac{4}{5}$ tons amount to at the same rate?
Ans. $424\frac{1}{3}$ dollars.

42. If $5\frac{1}{2}$ barrels of flour sell for 24 dollars, what is the price of one barrel at the same rate?

$5\frac{1}{2}=\frac{11}{2}$; 11 *halves* cost 24 dollars; 1 *half* will cost $\frac{1}{11}$ of 24 dollars; *one barrel*, or 2 *halves*, will cost $\frac{2}{11}$ of 24 dollars; that is, $24\times\frac{2}{11}$, *or the part of* 24 *expressed by the* reciprocal *of* $5\frac{1}{2}$.

Ans. $4\frac{4}{11}$ dollars.

43. If a man travels $27\frac{3}{5}$ miles in $\frac{3}{4}$ of a day, at what rate does he travel per day? *Ans.* $36\frac{4}{5}$ miles.

44. If a ship sails $575\frac{1}{2}$ miles in $6\frac{3}{4}$ days, what is the average distance that she sails in one day? *Ans.* $85\frac{7}{27}$ miles.

45. What is the price of silk per yard, when $3\frac{1}{2}$ yards cost $4\frac{3}{8}$ dollars? What would 7 yards cost?

Ans. $1\frac{1}{4}$ dollar; and $8\frac{3}{4}$ dollars.

46. What is the price of coal per ton, when $\frac{3}{5}$ of a ton costs $9\frac{3}{4}$ dollars? What would $10\frac{1}{4}$ tons cost?

Ans. $16\frac{1}{4}$ dollars; and $166\frac{9}{16}$ dollars.

47. What is the price of land per acre, when 13 acres cost $71\frac{1}{2}$ dollars? What would $\frac{9}{10}$ of an acre cost?

Ans. $5\frac{1}{2}$ dollars; and $4\frac{19}{20}$ dollars.

48. If 25 cords of wood sell for $68\frac{3}{4}$ dollars, what is the price per cord? What should be paid for $\frac{3}{4}$ of a cord?

Ans. $2\frac{3}{4}$ dollars; $2\frac{1}{16}$ dollars.

49. If a man walk $62\frac{1}{3}$ miles in $18\frac{1}{2}$ hours, at what rate does he walk per hour? How far could he go in 20 hours?

Ans. $3\frac{41}{111}$ miles; and $67\frac{43}{111}$ miles.

50. If a rail road car run 230 miles in $10\frac{1}{4}$ hours, what is its average run per hour? What distance ought it to run in 23 hours? *Ans.* $22\frac{18}{41}$ miles; and $516\frac{4}{41}$ miles.

51. If $2\frac{1}{2}$ hundred weight of sugar cost $17\frac{1}{2}$ dollars, what will 15 hundred weight cost at the same rate?

First find the cost of one hundred weight. *Ans.* 105 dollars.

52. If $9\frac{3}{4}$ tons of hay amount to 78 dollars, what would $7\frac{1}{2}$ tons amount to at the same rate? *Ans.* 60 dollars.

53. A farmer bought $3\frac{1}{2}$ tons of plaster for 14 dollars, and sold to his neighbor $1\frac{1}{4}$ tons of it, at the same price per ton. What did the part sold amount to? *Ans.* 5 dollars.

54. A sold to B 30 acres of land for $622\frac{1}{2}$ dollars; and B sold to C $12\frac{3}{4}$ acres of the same land at the same price per acre What did C pay for the land he bought?

Ans. $264\frac{9}{16}$ dollars.

55. D bought of E $35\frac{1}{2}$ bushels of clover seed, for 142 dollars; and afterwards sold to F $\frac{1}{4}$ of his purchase, at a profit of $1\frac{1}{4}$ dollar per bushel. What did the part sold to F amount to? *Ans.* $46\frac{19}{32}$ dollars.

ADDITION, ETC., OF FRACTIONS.

EXERCISES ON CHAPTER V.

§ **127.** *Compound* and *complex fractions* are prepared for addition, subtraction, &c. by reducing them to *simple fractions*; such reduction being nothing more than the *multiplication* and *division* of fractions. (§ 118, and § 124).

1. Find the sum of $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{3}$ of $\frac{4}{5}$, and $\frac{1}{6}$ of $\frac{3}{10}$ of $\frac{1}{20}$. *Ans.* $\frac{1367}{1500}$.

2. Find the difference between $\frac{1}{4}$ of $\frac{5}{8}$, and $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{7}{12}$. *Ans.* $\frac{145}{864}$.

3. Find the product of $\frac{3}{5}$ of $\frac{5}{8}$ multiplied by $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{4}{9}$. *Ans.* $\frac{1}{30}$.

4. Find the quotient of $\frac{4}{5}$ of $\frac{3}{10}$ divided by $\frac{1}{2}$ of $\frac{1}{3}$ of $15\frac{2}{3}$. *Ans.* $\frac{108}{1175}$.

5. Multiply the sum of $\dfrac{2\frac{1}{2}}{5}$ and $\dfrac{7}{3\frac{1}{2}}$ by the sum of $2\frac{1}{4}$ and $\frac{3}{4}$. *Ans.* $7\frac{1}{2}$.

6. Divide the sum of $\dfrac{1\frac{1}{3}}{4}$ and $\dfrac{9}{12\frac{1}{2}}$ by the sum of $3\frac{1}{8}$ and $\frac{5}{16}$. *Ans.* $\frac{1264}{4125}$.

7. Find the value of the expression
$$(\tfrac{2}{5}+\tfrac{3}{4}+5\tfrac{1}{2}-\tfrac{1}{2}\text{ of }\tfrac{3}{4})\times 4. \qquad \textit{Ans. } 25\tfrac{1}{10}.$$

8. Find the value of the expression
$$(10\tfrac{2}{5}-5\tfrac{3}{4}+2\tfrac{1}{3}-\tfrac{2}{3}\text{ of }\tfrac{5}{6})\times\tfrac{4}{5}. \qquad \textit{Ans. } 5\tfrac{32}{225}.$$

9. Find the value of the expression
$$(19\tfrac{4}{5}+\tfrac{7}{10}-15+\tfrac{1}{3}\text{ of }2\tfrac{1}{4})\times\frac{3}{5\frac{1}{2}} \qquad \textit{Ans. } 3\tfrac{9}{22}.$$

10. Find the value of the expression
$$(254+\tfrac{2}{7}+\tfrac{3}{8}-\tfrac{4}{5}\text{ of }15)\div\frac{2\frac{1}{2}}{5}. \qquad \textit{Ans. } 485\tfrac{9}{28}.$$

11. Find the value of the expression
$$(\frac{5}{3\frac{1}{2}}+\tfrac{1}{2}\text{ of }\tfrac{2}{3}\text{ of }\tfrac{2}{5}\text{ of }20)\div\frac{3\frac{1}{4}}{6\frac{1}{2}}. \qquad \textit{Ans. } 8\tfrac{4}{21}.$$

12. A and B together have $387\frac{5}{8}$ dollars, and B has $31\frac{1}{4}$ dollars more than A: what sum has each of them? (§ 63.)

Ans. A $178\frac{3}{16}$, and B $209\frac{7}{16}$ dollars.

13. Bought at one time 10 bushels of wheat, at another $12\frac{3}{4}$ bushels, and at another $25\frac{2}{3}$ bushels; at $1\frac{1}{4}$ dollar per bushel: what did the whole amount to? *Ans.* $60\frac{25}{48}$ dollars.

14. Having on hand $19\frac{4}{5}$ tons of iron, if I sell $10\frac{1}{4}$ tons of it at 45 dollars, and the remainder at 43 dollars per ton, what will the whole amount to? *Ans.* $871\frac{9}{10}$ dollars.

15. Paid 70 dollars for wood, at 2 dollars per cord; and afterwards sold $\frac{1}{4}$ of the quantity, at $3\frac{1}{4}$ dollars per cord. What did the wood sold amount to? *Ans.* $28\frac{7}{16}$ dollars.

16. Laid out $59\frac{5}{16}$ dollars for silk at $\frac{7}{8}$ of a dollar per yard, and sold $\frac{1}{2}$ of the quantity purchased, at a profit of $\frac{3}{8}$ of a dollar per yard. What did the part sold amount to?

Ans. $42\frac{3}{16}$ dollars.

17. If $\frac{3}{4}$ of a yard of broadcloth cost $4\frac{1}{2}$ dollars, what should be paid for $\frac{5}{8}$ of a yard at the same rate? *Ans.* $3\frac{3}{4}$ dollars.

18. If $3\frac{1}{4}$ hundred weight of hemp sell for $16\frac{1}{4}$ dollars, what sum should be paid for $10\frac{2}{3}$ hundred weight?

Ans. $53\frac{1}{3}$ dollars.

19. A bought of B $13\frac{1}{2}$ tons of hay at 9 dollars per ton, and of C $15\frac{2}{5}$ tons at $10\frac{1}{2}$ dollars per ton. A then sold to D 9 tons at 12 dollars, and the rest of his purchase to E at 13 dollars, per ton: what did he gain on the hay? *Ans.* $83\frac{1}{2}$ dollars.

20. If 5 barrels of flour will supply a company for 20 days, how many days would $7\frac{1}{2}$ barrels supply the company?

Ans. 30 days.

21. In how many days ought one man to accomplish a piece of work which 5 men could do in $33\frac{1}{2}$ days? In how many days ought 14 men to accomplish the same work?

Ans. $167\frac{1}{2}$ days; and $11\frac{27}{28}$ days.

22. How long ought 13 persons to subsist on a stock of provisions which would be sufficient for 10 persons $29\frac{1}{2}$ days?

Ans. $22\frac{9}{13}$ days.

23. A person bought 19 barrels of apples, at $2\frac{1}{4}$ dollars per barrel. Having sold $12\frac{1}{2}$ barrels of them at $2\frac{1}{2}$ dollars a barrel, at what price per barrel must he sell the remainder, to make a profit of $5\frac{3}{4}$ dollars on the whole? *Ans.* $2\frac{17}{26}$ dollars.

24. Bought at one time 320 acres of land, at $25\frac{1}{2}$ dollars an acre; and at another time 275 acres, at $31\frac{1}{4}$ dollars an acre. If $\frac{3}{7}$ of the whole quantity were sold at 20 dollars, and the remainder at 30 dollars an acre, what would be the gain or loss? *Ans.* Loss $1453\frac{3}{4}$ dollars.

25. If 3 men can plow $15\frac{1}{2}$ acres of ground in 4 days, how much ought 1 man to plow in one day? How much ought 5 men to plow in $7\frac{2}{5}$ days? *Ans.* $1\frac{7}{24}$ acres; and $47\frac{19}{24}$ acres.

26. A purchased of B 40 yards of cloth for 260 dollars. He then sold to C $\frac{3}{5}$ of his purchase at a profit of $\frac{3}{8}$ dollar per yard, and the remainder to D at a loss of $\frac{1}{8}$ dollar per yard. What did A gain or lose by these several transactions?

Ans. Gained 7 dollars.

CHAPTER VI.

DECIMAL FRACTIONS.—DECIMAL OR FEDERAL MONEY.

DECIMAL FRACTIONS.

§ **128.** A *Decimal Fraction* is one or more 10*ths*, or 100*ths*, or 1000ths, &c., of a quantity, expressed by its *numerator* only with a point prefixed;—its denominator being understood to be 1 with as many 0s annexed as there are *figures in the numerator*.

Thus .3 is $\frac{3}{10}$; the denominator being understood to be 1 with *one cipher* annexed, since there is one figure in the numerator.

But .03 is $\frac{3}{100}$; the denominator being understood to be 1 with *two* 0s annexed, because there are two figures in the numerator .03.

What is expressed by .4, 4 with a point prefixed? By .05, 5 with a 0 and point prefixed? By .006? By .0005? By .00001?

One *tenth* is how many *hundredths?* One hundredth is how many *thousandths?* One thousandth is how many *ten-thousandths?*

The simple term *decimal* is sometimes used to designate a decimal fraction.

§ **129.** A *vulgar fraction*, as distinguished from a *decimal*, is any fraction expressed by a numerator and denominator; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{10}$.

The term *fraction*, used alone, commonly denotes a *vulgar fraction*.

Notation of Decimals.

§ **130.** The first figure on the right of the *decimal point*, denotes *tenths;* the second, *hundredths;* the third, *thousandths;* and so on, t ten-thousandths, hundred-thousandths, millionths, ten-millionths, &c.

Thus in the decimal .1 2 3 4 5 6,

the 1 is 1 *tenth*, the 2 is 2 *hundredths*, the 3 is 3 thousandths, the 4 is 4 ten-thousandths, &c. But

§ **131.** The *first two* figures on the right of the decimal point will together denote *hundredths;* the *first three* will together denote *thousandths;* the first four, ten-thousandths, &c.

Thus in the decimal .12, the 1 being $\frac{1}{10}=\frac{10}{100}$, and the 2 being $\frac{2}{100}$, both together make 12 *hundredths.*

Again, in .123, the 12 is $\frac{120}{1000}$, the 3 is $\frac{3}{1000}$, and the whole 123 *thousandths.*

This is in accordance with the fact that the denominator understood to a decimal, is 1 with as many 0s annexed as there are figures in the numerator, (§ 128).

Complex or Mixed Decimals.

§ **132.** A *complex* or *mixed* decimal is a decimal fraction with a *vulgar fraction* annexed to it.

Thus .5½ is 5½ *tenths;* that is, 5 *tenths* and ½ of 1 *tenth;*$=\frac{5\frac{1}{2}}{10}$;

.25¾ is 25¾ *hundredths;* the same as $\frac{25\frac{3}{4}}{100}$.

The vulgar fraction annexed to a decimal, denotes its proper part of 1 *tenth,* or 1 *hundredth,* &c., according as it is annexed to *tenths,* or *hundredths,* &c.; and must not be reckoned as in a separate place of decimals.

Scale of Decimals.

§ **133.** In decimals, as in integers, *ten* of any *lower order* make *one* of the next higher order; or *one* of a higher order makes *ten* of the next lower order.

Thus beginning at *thousandths,* for example, 10 thousandths make 1 *hundredth;* 10 hundredths make 1 *tenth;* 10 *tenths* make 1 *unit.*

That is, the system of numbering by *tens,* is carried from units *up* through tens, hundreds, &c., and from units *down* through *tenths, hundredths,* &c.

RULE XXIV.

§ **134.** *To read a Decimal Fraction.*

Call the successive figures *tenths, hundredths, thousandths, ten thousandths,* &c., from the decimal point *toward the right;* then, disregarding 0s next the point, read the number as if it were an integer, and add the *decimal name* of the last figure.

EXAMPLES.

1. To read a decimal .03457.

Calling the figures *tenths, hundredths,* &c. toward the right we find the last figure 7 to be 7 *hundred thousandths.*

Hence the decimal is 3457 *hundred-thousandths.* (§ 131.)

2. To read the mixed number 305.004.

The 4 is 4 thousandths; the whole is read, *three hundred* and *five*, and *four thousandths*.

Note.—An ambiguity may sometimes arise in the enunciation of decimals and mixed decimal numbers.

For example, .305 is three hundred and five *thousandths;* and 300.005 is three hundred, and five *thousandths.*

But when read in this manner, the two are not sufficiently distinguished from each other. The ambiguity may be prevented by using the word *decimal* before the fraction; thus, .3 5, *decimal* 305 *thousandths;* 300.005, 300 and decimal 5 thousandths.

EXERCISES.

Read each of the following decimals and mixed numbers.

1.	.34.	16.	.37403.
2.	.037.	17.	54386.7.
3.	4.309.	18.	509.703.
4.	38.63.	19.	72.3436.
5.	58.704.	20.	.38007.
6.	.3368.	21.	943.479.
7.	43.009.	22.	83.4863.
8.	6.0007.	23.	73.8400.
9.	8739.3.	24.	8.3000.
10.	373.854.	25.	1.26835.
11.	4.00006.	26.	8.73473.
12.	3068.37.	27.	7638.67.
13.	27.1007.	28.	83.748.
14.	39.3834.	29.	7367.85
15.	341.0101.	30.	96.73867

Ciphers Annexed or Prefixed to Decimals.

§ **135.** Ciphers *annexed* to a decimal fraction do not *alter the value* of the decimal.

Thus .3=.30=.300, &c., since $\frac{3}{10}=\frac{30}{100}=\frac{300}{1000}$, &c. (§ 93)

§ **136.** Each 0 between the decimal point and the first significant decimal figure, *diminishes the decimal to* $\frac{1}{10}$ of its value without the 0.

Thus .3 is 3 *tenths*, .03 is 3 *hundredths*, and .003 is 3 *thousandths.*

$\frac{3}{100}$ is $\frac{1}{10}$ of $\frac{3}{10}$, and $\frac{3}{1000}$ is $\frac{1}{10}$ of $\frac{3}{100}$; (§ 119).

How may the *integer* 1 be made to denote 1 *tenth?* 1 *hundredth?* 1 *thousandth?* 1 ten-thousandth? 1 hundred-thousandth?

How may the integer 5 be made to denote 5 *hundredths?* 5 thousandths? 5 hundred-thousandths? 5 millionths?

RULE XXV.

§ 137. *To denote tenths, hundredths, &c., by decimal fractions.*

Prefix the decimal point to the numerator, interposing so many 0s, when necessary, next the point, that—the successive figures being called *tenths, hundredths, thousandths,* &c., toward the right,—the last figure shall have the same *decimal name* with the parts to be expressed.

EXAMPLE.

1. To denote 54 *ten-thousandths* by a decimal fraction.

.0054

Prefixing the decimal point to the numerator 54, and interposing two 0s next the point, we find that, when the several figures are called *tenths, hundredths,* &c., toward the right, the 4 is *ten-thousandths,*—which is the name of the parts to be expressed.

EXERCISES.

In the notation of both integers and decimals.

Write in figures the following numbers—observing that, in the verbal expression, the integral and fractional parts are separated by a *comma.*

1. Fifteen hundredths.
2. Nineteen thousandths.
3. Six ten-thousandths.
4. Twenty-four thousandths.
5. Five hundred thousandths.
6. Thirty-nine millionths.
7. One hundred thousandths.
8. Ten ten-millionths.
9. Forty-nine hundredths.
10. Seventeen ten-thous'dths.
11. Fifty-two thousandths.
12. Seventy-one hundredths.
13. Eight hundred thous'dths.
14. Ninety-one millionths.
15. One hundred thousandths.
16. Four thousand and nine, and five thousandths.
17. Twenty thousand and seven, and nineteen ten-thousandths.
18. Fifteen millions, and three hundred and two thous'dths.
19. Five hundred and four thousand, and nine ten-thous'dths.
20. Five millions two hundred and one thousand, and three tenths.
21. Seven hundred millions, and three hundred and nine thousandths.
22. Eighteen millions three hund. and seventy-six thousand and thirty, and twelve hundredths.

FEDERAL MONEY.

§ **138.** FEDERAL MONEY, or money of the United States, is expressed in units according to the *decimal* scale of numeration, that is, numeration by *tens*.

The units of Federal money are,

Eagles, Dollars, Dimes, Cents, and Mills.

10 mills	*m*	make	1 cent, *ct.*
10 cents		"	1 dime, *d.*
10 dimes, or 100 cts.		"	1 dollar, $.
10 dollars		"	1 eagle, *E.*

The character $ is prefixed to dollars; thus $5 is 5 dollars.

One *cent* is what part of a dollar? 3 cents are what part of a $?
One *mill* is what part of a cent? 7 mills are what part of a cent?
One mill is what part of a dollar? 9 mills are what part of a $?

§ **139.** The only denominations of Federal Money in common use, are *dollars* and *cents*;—eagles being expressed in dollars; dimes, in cents; and smaller values in fractions of a cent.

Thus, instead of 5 *eagles*, 4 *dollars*, 3 *dimes*, 2 *cents*, and 5 *mills*; we would say 54 *dollars*, 32½ *cents*.

DECIMAL NOTATION OF FEDERAL MONEY

Cents being expressed only in numbers less than 100, and mills in numbers less than 10, we have the following

RULE XXVI.

§ **140.** *For the decimal expression of Federal Money.*

Regarding dollars as *integers*, make cents and mills *decimals* of a dollar, by prefixing to them the decimal point,—observing to interpose a 0 next the point, when the number of cents is less than 10,—and two 0s next the point when only mills, or a fraction of a cent, are given.

EXAMPLES.

31 cents is $.31, 31 *hundredths* of a dollar;
6¼ cents is $.06¼, 6¼ *hundredths* of a dollar; (§132.)
5 mills is $.005, 5 *thousandths* of a dollar;
¾ of a cent is $.00¾, ¾ of 1 *hundredth* of a dollar;
7 *cts.* and 5 *m.* is $.075, 75 *thousandths* of a dollar.

It may be useful to observe here, conversely, that,

§ **141.** In a decimal fraction *of a dollar*, the first figure after the point denotes tenths of a dollar, or *tens* of cents; the second denotes *cents;* the first two together denote *cents;* the third, *mills;* the fourth. *tenths* of a mill, &c.

Thus \$.5 is 5 *tenths* of a dollar, equal to 50 cents;
\$.453 is 45 cents and 3 mills;
\$.0625 is 6 cents, 2 mills, and 5 *tenths* of a mill.

DECIMALS REDUCED TO VULGAR FRACTIONS.

RULE XXVII.

§ **142.** *To reduce a decimal to a vulgar fraction.*

1. Remove the decimal point from the numerator, and underneath set the proper denominator. (§ 128).

2. The fraction thus formed may often be reduced to lower terms.

EXAMPLE.

To reduce .125 to a vulgar fraction.

$$.125=\frac{125}{1000}=\frac{1}{8}.$$

EXERCISES.

Reduce each of the following decimals of a dollar to a vulgar fraction of a dollar.

1. \$.5.	*Ans.* \$$\frac{1}{2}$.	7. \$.9375.	*Ans.* \$ $\frac{15}{16}$.
2. \$.25.	*Ans.* \$$\frac{1}{4}$.	8. \$.5625.	*Ans.* \$ $\frac{9}{16}$.
3. \$.75.	*Ans.* \$$\frac{3}{4}$.	9. \$.1875.	*Ans.* \$ $\frac{3}{16}$.
4. \$.375.	*Ans.* \$$\frac{3}{8}$.	10. \$.0625.	*Ans.* \$ $\frac{1}{16}$.
5. \$.625.	*Ans.* \$$\frac{5}{8}$.	11. \$.0025.	*Ans.* \$$\frac{1}{400}$.
6. \$.875.	*Ans.* \$$\frac{7}{8}$.	12. \$.0075.	*Ans.* \$$\frac{3}{400}$.

13. Express the value of \$13.75 by an integer and a vulgar fraction. *Ans.* \$ 13$\frac{3}{4}$

14. Express the value of \$20.125 by an integer and a vulgar fraction. *Ans.* \$ 20$\frac{1}{8}$

15. Express the value of \$35.625 by an integer and a vulgar fraction. *Ans.* \$ 35$\frac{5}{8}$

16. Express the value of \$49.0625 by an integer and a vulgar fraction. *Ans.* \$49$\frac{1}{16}$

17. Express the value of \$57.9375 by an integer and a vulgar fraction. *Ans.* \$57$\frac{15}{16}$

VULGAR FRACTIONS REDUCED TO DECIMALS.

RULE XXVIII.

§ **143.** *To reduce a vulgar fraction to a decimal.*

1. Divide the denominator into the numerator, with as many 0s annexed to the latter as may be necessary to find an *exact quotient*, or the number of decimal figures required.

2. Point off in the right of the quotient as many decimal figures as there were 0s annexed to the numerator; observing to prefix 0s to the quotient, when necessary to make up the number.

EXAMPLE.

To reduce $\frac{3}{125}$ to a decimal fraction.

125)3000(24. Then $\frac{3}{125}$=.024.

Annexing three 0s to the numerator, and dividing 3000 by the denominator, we find the quotient 24.

Prefixing a 0 to the quotient, to make up three decimal figures, for the three 0s annexed to the numerator, we find $\frac{3}{125}$=.024.

☞ The fraction is equal to its numerator 3 divided by its denominator 125 (§ 92.)

Each 0 annexed to the numerator, *multiplies* the fraction by 10 (§ 119—2); but each decimal figure made in the quotient, *divides* the quotient by 10, since each quotient figure thus becomes $\frac{1}{10}$ of its former value.

Thus multiplying and dividing by the same number, we preserve a *constant value* to the fraction. (§ 93).☜

EXERCISES.

Reduce each of the following fractions of a dollar to a decimal of a dollar.

1. \$$\frac{1}{2}$.	*Ans.* \$.5.	6. \$$\frac{7}{8}$. . .	*Ans.* \$.875.
2. \$$\frac{1}{4}$.	*Ans.* \$.25.	7. \$$\frac{1}{16}$. . .	*Ans.* \$.0625.
3. \$$\frac{3}{4}$.	*Ans.* \$.75.	8. \$$\frac{3}{16}$. . .	*Ans.* \$.1875.
4. \$$\frac{3}{8}$.	*Ans.* \$.375.	9. \$$\frac{5}{16}$. . .	*Ans.* \$.3125
5. \$$\frac{5}{8}$.	*Ans.* \$.625.	10. \$$\frac{9}{16}$. . .	*Ans.* \$.5625.

11. Reduce \$17$\frac{1}{8}$ to a decimal expression. *Ans.* \$17.125.
12. Reduce \$13$\frac{7}{16}$ to a decimal expression. *Ans.* \$13.4375
13. Reduce \$25$\frac{11}{16}$ to a decimal expression. *Ans.* \$25.6875
14. Reduce \$31$\frac{13}{16}$ to a decimal expression. *Ans.* \$31.8125.
15. Reduce \$40$\frac{15}{16}$ to a decimal expression. *Ans.* \$40.9375.

A Complex Reduced to a Simple Decimal.

§ **144.** In a *complex* decimal, instead of the vulgar fraction annexed, we may put its equivalent decimal, without the point prefixed to it.

Thus in $.6\frac{1}{25}$, the $\frac{1}{25}=.04$; then $.6\frac{1}{25}=.604$.

16. Reduce $.25\frac{1}{2}$ to a simple decimal. . . . *Ans.* .255.
17. Reduce $.31\frac{1}{4}$ to a simple decimal. . . . *Ans.* .3125.
18. Reduce $.18\frac{3}{4}$ to a simple decimal. . . . *Ans.* .1875.
19. Reduce $.23\frac{1}{8}$ to a simple decimal. . . . *Ans.* .23125.
20. Reduce $.90\frac{1}{16}$ to a simple decimal. . . *Ans.* .900625.

Approximate Decimals.

§ **145.** A decimal which expresses a *near*, but not the *exact*, value of a vulgar fraction, or other quantity, is an *approximate* decimal.

In reducing $\frac{1}{3}$, for example, to a decimal,—if we annex one 0 to the 1 and divide by the 3, we find $\frac{1}{3}=.3\frac{1}{3}$;

by annexing two 0s to the 1, we find $\frac{1}{3}=.33\frac{1}{3}$;

by annexing three 0s to the 1, we find $\frac{1}{3}=.333\frac{1}{3}$; and so on.

In the first of these mixed decimals, the $\frac{1}{3}$ annexed is $\frac{1}{3}$ of 1 *tenth*, equal to $\frac{1}{30}$; in the second, it is $\frac{1}{3}$ of 1 *hundredth*, equal to $\frac{1}{300}$; and in the third, it is $\frac{1}{3}$ of 1 *thousandth*, equal to $\frac{1}{3000}$.

By omitting these small values, $\frac{1}{30}$, $\frac{1}{300}$, $\frac{1}{3000}$, we have .3 for a near or *approximate* value of $\frac{1}{3}$, .33 for a *nearer* value, and .333 for a *still nearer* value of $\frac{1}{3}$.

The sign + is commonly affixed to an approximate decimal; thus $\frac{1}{3}=.33+$, 33 hundredths, *nearly*.

Instead of the sign +, we shall employ a *comma* ', after the manner of an *apostrophe*, to denote an *approximate decimal.*

Thus $\frac{1}{3}=.33'$, 33 hundredths, *nearly*.

The number of figures to which an *approximate decimal* need be carried, in any particular case, will depend on the value of the whole quantity of which the decimal expresses a part.

In a decimal of a dollar, for example, two figures will give the number of *cents*, which is near enough for ordinary purposes. When greater accuracy is required, the third figure may be found, which will give the number of *mills*.

21. Reduce $\$\frac{1}{3}$ to an approximate decimal. . . *Ans.* .333'.
22. Reduce $\$\frac{2}{7}$ to an approximate decimal. . . *Ans.* .285'.
23. Reduce $\$\frac{4}{9}$ to an approximate decimal. . . *Ans.* .444'.

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF DECIMALS, AND FEDERAL MONEY.

§ **146.** The objects of Addition, Subtraction, Multiplication and Division, are the same for every kind of quantity; and, having been defined for integers and vulgar fractions, the definitions need not be repeated.

§ **147.** Addition, Subtraction, &c., of Federal Money, are brought under the Rules to be given for the same operations, respectively, on decimal fractions, by regarding dollars as *integers,* and making cents and mills *decimals* of a dollar.

§ **148.** The principles of notation being the same for decimals and integers, (§ 133 and 137) the methods of adding, subtracting, &c., will be the same for decimals and integers

ADDITION OF DECIMALS.

RULE XXIX.

§ **149.** *For the addition of decimals.*

Set *tenths* under *tenths, hundredths* under *hundredths,* &c., and add as in integers; observing to make in the right of the *sum* as many decimal figures as will be equal to the greatest number of decimal figures in any one of the given numbers.

EXAMPLE.

To find the sum of .25+84.346+.73+275.937.

$$\begin{array}{r} .25 \\ 84.346 \\ .73 \\ 275.937 \\ \hline 361.263 \end{array}$$

The sum is 361 and 263 *thousandths.*

Having set tenths under tenths, hundredths under hundredths, &c.,—this order also causing *units* to fall under *units,* tens under tens, &c., when mixed numbers are to be added—we add up the several columns of figures as in integers; and make three decimal figures .263 in the sum, this being the greatest number of decimal figures in any one of the given numbers.

EXERCISES

1. Find the Sum, 215.5+23.56+32.32. *Ans.* 271.38.
2. Find the Sum, .273+913.4+37.62. *Ans.* 951.293.
3. Find the Sum, .035+346.1+873.4. *Ans* 1219.535.
4. Find the Sum, .874+2.763+7.390. *Ans* 11.027.
5. Find the Sum, 387+534.6+83.81. *Ans* 1005.41.
6. Find the Sum, 2.25+.7373+.7849. *Ans* 3.7722.
7. Find the Sum, 48.36+8370+.0051. *Ans.* 8418.3651.
8. Find the Sum, 8.773+974.6+2703. *Ans.* 3686.373.
9. Find the Sum, 74.03+3737+4301. *Ans.* 8112.03.
10. Find the Sum, .9346+203.7+.7376. *Ans.* 205.3722

11. Find the sum of 100 dollars 72½ cents, 25 dollars 6¼ cents, 34 dollars 5 cents, and 119 dollars 48¾ cents.

$$\begin{array}{r} \$100.72\tfrac{1}{2} \\ 25.06\tfrac{1}{4} \\ 34.05\phantom{\tfrac{1}{4}} \\ 119.48\tfrac{3}{4} \\ \hline \$279.32\tfrac{1}{2} \end{array} \quad 279 \textit{ dollars } 32\tfrac{1}{2} \textit{ cents.}$$

Having made each number of cents a *decimal* of a dollar (§ 140) and placed *tenths* under *tenths*, &c., we first add up the fractions of a cent, namely, ¾, ¼ and ½, and find the sum to be $\frac{6}{4}=1\frac{1}{2}$. We set down the ½, and carry the 1 to 8.

In the sum we point off two decimal figures for *cents*, or *hundredths* of a $; (§ 132.)

12. What sum should be paid for a hat, at 5 dollars 87½ cents; a vest, at 3 dollars 18¾ cents; and a pair of shoes, at 2 dollars 62½ cents? *Ans.* $11.68¾.

13. What should be paid for a quarter of beef, at $7; a barrel of flour, at 4 dollars 56¼ cents; a lot of groceries, at 13 dollars 37½ cents; and a lot of butter, at 2 dollars 6¼ cents? *Ans.* $27.00.

14. Find the sum that must be paid for a quire of paper, at 25 cents; a bottle of ink, at 12½ cents; a dozen books, at 1 dollar 18¾ cents; and a bunch of quills, at 37½ cents. *Ans.* $1.93¾.

15. Find the sum that should be paid for a set of chairs, at $18; a pair of tables, at 35 dollars 50 cents; a looking-glass, at 5 dollars 18¾ cents; and a bedstead, at 9 dollars 31¼ cents. *Ans.* $68.00.

16. Bought a cord of wood, for 2 dollars 50 cents; a ton of hay, for 12 dollars 68¾ cents; a barrel of apples, for 2 dollars 56¼ cents; and quarter of beef, for 5 dollars 75 cents; required the sum paid. *Ans.* $23.50.

17. Sold a barrel of sugar, for \$15; a sack of coffee, for 13 dollars 5 cents; a keg of rice, for 5 dollars 43¼ cents; and a box of candles, for 9 dollars 8 cents; required the sum received. *Ans.* \$42.56¼.

18. A merchant's bill was as follows: for 3½ yards of cloth, \$21; for 3 pair of stockings, 1 dollar 87½ cents; for a dozen skeins of silk, 75 cents; required the amount of the bill. *Ans.* \$23.62½.

19. A farmer sold produce as follows, namely: wheat, for \$300; corn, for 97 dollars 93¾ cents; hay, for 56 dollars 12½ cents; and oats, for 18 dollars 6¼ cents; required his amount of sales. *Ans.* \$472.12.

20. Bought a quantity of flour, for 75 dollars; a quantity of bacon for 57 dollars 18¾ cents; and a quantity of corn, for 42 dollars 6¼ cents; for what sum must the whole be sold to make a profit of 25 dollars? *Ans.* \$199.25.

SUBTRACTION OF DECIMALS.

RULE XXX.

§ **150.** *For the subtraction of decimals.*

1. Set the less value under the greater, with *tenths* under *tenths*, *hundredths* under hundredths, &c., and subtract as in integers; observing to make in the right of the remainder as many decimal figures as will be equal to the *greatest number of decimal figures* in either of the given numbers.

2. When the minuend has *no decimal* figures, or not so many as the subtrahend, conceive the deficient places to be occupied by decimal 0s.

EXAMPLES.

1. To find the difference between 23.0623 and 380.75.

$$\begin{array}{r} 380.75 \\ 23.0623 \\ \hline 357.6877 \end{array}$$

Having set the less quantity under the greater, with *tenths* under *tenths*, &c., we suppose the two vacant places over 23 to be occupied by 00, (§ 135;) and subtract as in integers; thus, 3 from 10 leaves 7, &c. Four decimal figures are made in the right of the remainder.

2. To find the difference between 525 and 9.87534.

$$\begin{array}{r} 525 \\ 9.87534 \\ \hline 515.12466 \end{array}$$

The places for decimals over .87534 must be regarded as occupied by 0s; thus, 4 from 10 leaves 6, &c.

EXERCISES.

1. Find the Difference between 35.804 and 360.75. *Ans.* 324.946.
2. Find the Difference between 734.06 and 2.7431. *Ans.* 731.3169.
3. Find the Difference between 57.863 and 40.736. *Ans.* 17.127.
4. Find the Difference between 12.683 and 940.05. *Ans.* 927.367.
5. Find the Difference between 734.11 and .38703. *Ans.* 733.72297.
6. Find the Difference between 78396 and 476.24 *Ans.* 77919.76.
7. Find the Difference between 57.609 and 4.7305. *Ans.* 52.8785.
8. Find the Difference between 93430 and 800.34. *Ans.* 92629.66.
9. Find the Difference between 72137 and 131.004. *Ans.* 72005.996.
10. Find the Difference between 90000 and 900.192. *Ans.* 89099.808.

11. Find the Difference between \$325 and 93 dollars $6\frac{1}{4}$ cents.

$$\begin{array}{r} \$325.00(\tfrac{4}{4}) \\ 93.06\ \tfrac{1}{4} \\ \hline \$231.93\ \tfrac{3}{4} \end{array} \quad 231\ \textit{dollars}\ 93\ \tfrac{3}{4}\ \textit{cents.}$$

Expressing the $6\frac{1}{4}$ cents in a decimal of a dollar,—supplying the vacant decimal places in the upper number with 00,—and annexing, mentally, $\frac{4}{4}$, equal to a *unit*,—we say $\frac{1}{4}$ from $\frac{4}{4}$ leaves $\frac{3}{4}$; then 1 to 6 makes 7, and 7 from 10 leaves 3, &c. (§ 34).

Two decimal figures are made in the remainder, for cents or *hundredths* of a \$.

12. If a person having 95 dollars $6\frac{1}{4}$ cents, pay 43 dollars $18\frac{3}{4}$ cents for fuel, what sum will he have remaining? *Ans.* \$$51.87\frac{1}{2}$.

13. If a lot of goods were purchased for $579, and sold for 650 dollars 87½ cents, what sum would be gained?

Ans. $71.87½.

14. What would be made on a quantity of lumber, bought for 225 dollars 18¾ cents, and sold for 300 dollars 50 cents?

Ans. $75.31¼.

15. Required the loss on a lot of flour, purchased for 372 dollars 12½ cents, and sold for 321 dollars 56¼ cents.

Ans. $50.56¼.

16. A merchant bought a piece of cloth for 120 dollars, and a piece of silk for 85 dollars 68¾ cents. He sold the whole of both pieces for 316 dollars 56¼ cents; what profit did he make?

Ans. $110.87½.

17. A grazier bought cattle for $160, and sheep for 56 dollars 50 cents. He sold the cattle for 225 dollars 37½ cents, and the sheep for 83 dollars 93¾ cents; what did he make by these transactions? *Ans.* $92.81¼.

18. A manufacturer purchased a quantity of raw cotton for $400, which he made into cloth at an expense of 132 dollars 6¼ cents. What profit will he make by selling the cloth for $700? *Ans.* $167.93¾.

19. A speculator purchased wheat for $344, and bacon for 88 dollars 18¾ cents. He sold his wheat for 300 dollars 75 cents, and his bacon for 100 dollars 12½ cents; what was his gain or loss by the speculation? *Ans.* Loss, $31.31¼.

20. Having on hand 125.5 tons of coal; if I sell 13.75 tons to A, 34 tons to B, 42.125 tons to C, and 5 tons to D, how many tons will I have left? *Ans.* 30.625 tons.

21. Having purchased 575.75 yards of cotton; if I sell to A and B each 125 yards, and to C and D each 93.125 yards—how many yards will I have remaining? *Ans.* 139.5 yards.

22. Bought of two persons each 1575.5 pounds of pork; of which I sold to three persons each 234.125 pounds. How many pounds of the pork purchased are still on hand?

Ans. 2448.625 pounds.

23. A farmer bought, at one time, 375 acres of land; at another time, 233.3 acres; and at another, 136.75 acres. He wishes to make his purchases amount to 1000 acres; how much land does he still want? *Ans.* 254.95 acres.

24. Bought of A 300 bushels of salt, for $137.50, and of B 275.5 bushels for $125.87½;—of which, I sold to C 325 bushels, for $200, and the remainder to D for $150.61¼. What quantity was sold to D, and what was my entire profit or loss?

Ans 250.5 bushels to D; entire profit $87.23¾.

25. From \$3780.93¾ subtract \$2100.12½.

Instead of ¾ and ½ we may take their equivalent decimals.

We shall find .93¾=.9375, and .12½=.125; (§ 144,) and \$3780.9375—\$2100.125=\$1680.8125.

In like manner perform the following exercises:

26. From \$300.18¾+\$25.50 subtract \$16.12½. *Ans.* \$309.5625.

27. From \$578.03½+\$37.25 subtract \$20.06¼. *Ans.* \$595.2225.

28. From \$400.50 +\$9.125 subtract \$100.10. *Ans.* \$309.525.

29. From \$175.56¼+\$1.87½ subtract \$ 75.33. *Ans.* \$102.1075.

30. From \$1000.4 +\$2.03 subtract \$ .06¼. *Ans.* \$1002.3675.

31. Going out to collect money, I received of one person 37 dollars 25 cents, and of another 93 dollars 56¼ cents. Out of these sums having paid a debt of 99 dollars 18¾ cents, what sum have I remaining? *Ans.* \$31.62½.

32. Having deposited in bank \$1000, and having drawn out at different times 74 dollars 50 cents, 390 dollars 87½ cents, and 213 dollars 68¾ cents, what sum have I still in bank? *Ans.* \$320.93¾.

33. Bought a house and lot in a city for \$3000, and paid for improvements on the same 316 dollars 93¾ cents. If the property be sold for \$4500, what amount of profit will be realized? *Ans.* \$1183.06¼.

34. A gentleman who had a journey of 145¼ miles to make, traveled for three days at the rate of 39.5 miles per day. What distance then remained to be traveled? *Ans.* 26.75 miles.

35. A farmer has in one plantation 400 acres, in another 119.25 acres, and in another 230⅛ acres. If he sell 50½ acres from each, how many acres will he have left in the three plantations together? *Ans.* 597.875 acres.

36. A speculator purchased cattle for \$100, mules for 79 dollars 62½ cents, and sheep for 57 dollars 31¼ cents. He sold the whole for 400 dollars 6¼ cents; what did he gain by the speculation? *Ans.* \$163.12½.

37. A merchant bought cloth for \$300, linen for 71 dollars 25 cents, and silk for 112 dollars 6¼ cents. He sold the cloth at a profit of 49 dollars 18¾ cents, the linen at a profit of 18 dollars 87½ cents, and the silk at a loss of 50 dollars; what did he gain or lose on the whole? *Ans.* Gained \$18.06¼.

MULTIPLICATION OF DECIMALS.

RULE XXXI.

§ **151.** *For the multiplication of decimals.*

1. Multiply as in integers, and in the right of the product make as many decimal figures as there are *decimal figures in both the factors;* prefixing 0s to the product when necessary to make up the number.

2. *Integral* 0s *in the right of the multiplier* may be omitted, provided the same number of *decimal* figures be made *integral* in the multiplicand,—0s being annexed to the multiplicand, when necessary to make up the number. When the multiplier is 10, 100, &c., the product is thus immediately obtained.

EXAMPLES.

1. To multiply .19 by .5; that is, to find .5 of .19. (§ 116.)

$$\begin{array}{r} .19 \\ .5 \\ \hline .095 \end{array} \quad 95 \textit{ thousandths.}$$

Multiplying as in integers, we find the product 95; to which we prefix the 0 and the decimal point, to make three decimal figures for the three in the multiplicand and multiplier.

2. To multiply 236 by 3.4.

$$226 \times 3.4 = 802.4.$$

In the product 802.4 we have one decimal figure for the one in the multiplier, there being no decimal in the multiplicand.

3. To multiply 48.5 by 300.

$$48.5 \times 300 = 14550.0.$$

Or, rejecting the *two integral* 0s in the right of the multiplier, and making *two more integral* figures in the multiplicand,

we have $4850 \times 3 = 14550$; as before.

In like manner, $3.45 \times 100 = 345$

$3.45 \times 1000 = 3450$; and so on;

in which cases the products are immediately obtained by making as many *additional* integral figures in the multiplicand, as there are *integral* 0s in the right of the multiplier;—annexing 0s to the multiplicand, when necessary to make up the number.

In the first example, if we multiply the two quantities together under the form of *vulgar fractions*, we shall have

$$\tfrac{19}{100}\times\tfrac{5}{10}=\tfrac{95}{1000}=.095.$$

To express the product $\frac{95}{1000}$ by a *decimal*, requires as many decimal figures as there are 0s in the denominator 1000; (§ 128;) or in the two denominators 100 and 10; that is, as many decimal figures as there are figures in the *two numerators* 19 and 5.

In general terms, the product of two decimal fractions must contain just as many decimal figures as both the factors, because, in using the decimals under the form of *vulgar fractions*, the product will have just as many 0s in its denominator as are in the denominators of both factors, and the number of figures in the decimal numerator must equal the number of 0s in the denominator.

EXERCISES.

1. Multiply .25 by .9, and 1.5 by .03. *Ans.* .225 and .045
2. Multiply 38.3 by 8, and 4.75 by .5. *Ans.* 306.4 and 2.375.
3. Multiply 930 by .1, and 875 by .01. *Ans.* 93 and 8.75
4. Multiply 300 by .04, and .379 by 1.5. *Ans.* 12 and .5685.
5. Multiply 5.76 by 9, and .031 by 80. *Ans.* 51.84 and 2.48.
6. Multiply .003 by 30, and 874 by .03. *Ans.* .090 and 26.22.
7. Multiply .876 by 4.5, and 37.4 by .39. *Ans.* 3.942 and 14.586.
8. Multiply 280 by .02, and 730 by 1.2. *Ans.* 5.6 and 876.
9. Multiply 6.74 by .01, and 89 by .001. *Ans.* .0674 and .089.
10. Multiply 53.7 by 1.4, and 60 by .006. *Ans.* 75.18 and .360.
11. Multiply 100 by .01, and 0.1 by .101. *Ans.* 1 and .0101.
12. Multiply .003 by .05, and .007 by .09. *Ans.* .00015 and .00063.

13. What will 7 cords of wood amount to, at 3 dollars 18¾ cents per cord?

The wood will cost 7 times 3 dollars 18¾ cents.

Expressing the 18¾ cents by a decimal of a dollar, (§ 140,) *we have* $3.18¾ *to be multiplied by* 7.

$$\begin{array}{r} \$3.18\tfrac{3}{4} \\ 7 \\ \hline \$22.31\tfrac{1}{4} \end{array}\qquad 22\ \textit{dollars}\ 31\tfrac{1}{4}\ \textit{cents.}$$

Multiplying the ¾ by 7, we say 7 times ¾ is $\frac{21}{4}$, equal to 5¼; setting down the ¼ and carrying the 5, 7 times 8 is 56, and 5 are 61, &c..

In the product we make two decimal figures for the two in the multiplicand. (§ 132).

14. What should be paid for 9 hundred weight of tobacco, at 10 dollars 37½ cents per hundred weight? *Ans.* $93.37½.

15. What should be paid for 37 head of cattle, at 13 dollars 18¾ cents a head; and 7 mules, at 40 dollars 50 cents a head? *Ans.* $771.43¾.

16. What should be paid for 8 yards of cloth, at 9 dollars 56¼ cents a yard; and 12 yards of linen, at 87½ cents a yard? *Ans.* $87.

17. What should be paid for 25 bushels of wheat, at 1 dollar 6¼ cents a bushel; and 30 bushels of corn, at 43¾ cents a bushel? *Ans.* $39.68¾.

18. A merchant bought 50 yards of cloth, at $4 a yard; and sold the same at 6 dollars 87½ cents a yard. How much did he gain? *Ans.* $143.75.

19. Bought 100 sheep at 1 dollar 31¼ cents a head, and sold the same at 2 dollars 93¾ cents a head: what was the gain per head, and what on the whole?

Ans. Gained per head, $1.62½; on the whole $162.50.

20. Required the sum that must be paid for 3 hundred weight of bacon, at $6 per hundred weight; 4 barrels of flour, at 5 dollars 62½ cents per barrel; and 2 barrels of fish, at 7 dollars 16⅔ cents per barrel? *Ans.* $54.83⅓.

21. Required the sum that must be paid for 3 bushels of potatoes, at 87½ cents a bushel; 17 pounds of butter, at 12½ cents a pound; and 5 dozen eggs, at 8⅓ cents a dozen. *Ans.* $5.16⅔.

22. Required the sum that should be paid for 50 pounds of sugar, at 9½ cents a pound; 20 pounds of coffee, at 15¼ cents a pound; 15 pounds of rice, at 6¼ cents a pound; and 3 gallons of molasses, at 30 cents a gallon. *Ans.* $9.63¾.

23. A merchant bought 30 yards of silk at 93¾ cents per yard, and 40 yards of another kind at 1 dollar 6¼ cents per yard. He sold the first kind at 1 dollar 50 cents per yard, and the other at 1 dollar 87½ cents per yard; what profit did he make on the whole? *Ans.* $49.37½.

24. Required the sum that should be paid for 9 barrels of flour at $5.18¾ per barrel, 11 cords of wood at $2.87½ per cord, 3 tons of hay at $9¾ per ton, and 98 pounds of beef at $.05½ per pound. *Ans.* $112.95¼.

25. A farmer bought 130 acres of land at $27.25 per acre, and 237 acres at $16.93¾ per acre. He sold the first tract at $25.50 per acre, and the second at $21⅜ per acre; what did he gain or lose on the two tracts together?

Ans. Gained $824.18¾.

26. Multiply \$150.18$\frac{3}{4}$ by 7 $\frac{1}{2}$; that is,

find 7 times \$150.18$\frac{3}{4}$,+$\frac{1}{2}$ of \$150.18$\frac{3}{4}$.

$$\begin{array}{r} \$150.18\frac{3}{4} \\ 7\frac{1}{2} \\ \hline 105131\frac{1}{4} \\ 7509\frac{3}{8} \\ \hline \$1126.40\frac{5}{8} \end{array}$$

1126 *dollars* 40$\frac{5}{8}$ *cents.*

We first multiply by the 7, as in the preceding exercises.

To multiply by the $\frac{1}{2}$ we say, $\frac{1}{2}$ of 15 is 7 with one over; $\frac{1}{2}$ of 10 is 5; $\frac{1}{2}$ of 18 is 9, supplying the vacant place before 9 with 0; $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$.

We then add together the two products, and make two decimal figures, for the two in the multiplicand.

But it will often be preferable to reduce the vulgar fractions in the multiplicand and multiplier to their *equivalent decimals.*

In this example we shall have \$150.1875×7.5=\$1126.40625; which is 1126 *dollars,* 40 *cents,* 6 *mills* and 5 *tenths* of a mill.

27. A farmer sold 15$\frac{3}{4}$ acres of land, at 27 dollars 37$\frac{1}{2}$ cents per acre; required the sum he should receive in payment.
Ans. \$431.156'.

28. What should be paid for 3$\frac{1}{2}$ barrels of flour at 6 dollars 58$\frac{3}{4}$ cents a barrel, and 11$\frac{5}{8}$ bushels of meal at 43$\frac{1}{4}$ cents a bushel? *Ans.* \$28.084'.

29. Bought a piece of cloth containing 39$\frac{3}{4}$ yards, for \$238.50; of which 20$\frac{1}{4}$ yards have been sold at \$7.12$\frac{1}{2}$ per yard. What will be the gain or loss on the whole, if the remainder be sold at \$8.06$\frac{1}{4}$ per yard? *Ans.* Gain \$63.

30. Bought 45$\frac{1}{2}$ hundred weight of hemp, at \$6.25 per hundred weight; which has been made into rope and bagging, at an expense of \$130.18$\frac{3}{4}$. For what sum must the manufactured articles be sold to clear \$50? *Ans.* \$464.5625.

31. A trader bought 120 mules at an average price of \$39.50; of which he has sold 20 head at \$54.62$\frac{1}{2}$, and 33 head at \$59 a head. What will be his entire profit or loss if the rest be sold at \$30.50 a head? *Ans.* Profit \$343.

32. A barters to B 35$\frac{1}{2}$ yards of broadcloth at \$7.5 a yard, for 135 yards of silk at \$.93$\frac{3}{4}$ a yard,—the difference in value between the two commodities to be paid in money. Which of them must receive money, and how much?
Ans. B must pay \$139.6875.

DIVISION OF DECIMALS.

RULE XXXII.

§ 152. *For the division of decimals.*

1. Divide as in integers, and in the right of the quotient make as many decimal figures as there are *decimal figures in the dividend* more than *in the divisor;* prefixing 0s to the quotient, when necessary to make up the number.

2. When the divisor *has more decimal figures* than the dividend, or is greater than the dividend (regarding both as *integers*,) annex *decimal* 0s to the dividend, to supply the deficiency.

3. Ciphers may always be *annexed to the remainder*, and the division continued to any required exactness,—observing that the 0s so annexed must be counted as decimal figures *belonging to the dividend.*

4. *Integral* 0s *in the right of the divisor* may be omitted, provided the same number of *integral* figures be made *decimals* in the dividend,—0s being prefixed to the dividend, when necessary to make up the number. When the divisor is 10, or 100, &c., the quotient is thus immediately obtained.

EXAMPLES.

1. To divide .965 by .5, that is, to find how often .5 is contained in .965.

$$\begin{array}{r} .5\,).965 \\ \hline 1.93 \end{array} \qquad .965 \div .5 = 1.93.$$

Dividing as in integers, we find the quotient 193; in which we make *two decimal figures,* since the dividend has two more decimal figures than the divisor.

2. To divide .375 by 125; that is, to find what part .375 is of 125

$$125\,).375\,(.003$$

Having found the quotient 3, we prefix to it two 0s and the decimal point, to make *three decimal figures;* since the dividend has three decimal figures, while the divisor has none.

3. To divide 7 by 1.2.

$$1.2\overline{)7.0}$$
$$5.83' \qquad \text{Or } 5.83\tfrac{1}{3}.$$

The dividend having no decimal figure in it, while the divisor has one, we annex a decimal 0 to the dividend.

Having found the quotient figure 5, we annex a 0 to the remainder 10, and say 12 in 100, 8 times and 4 over; annexing another 0 to the 4, we say 12 in 40, 3 times, and 4 over.

Thus the division might be continued. Or, we may form a fraction $\frac{4}{12}=\frac{1}{3}$ of the remainder and divisor, and annex it to the quotient.

The three 0s annexed make three decimal figures belonging to the dividend; hence we make two decimal figures in the quotient.

4. To divide 8.4 by 300.

$$300)8.40(.028.$$

The dividend 8.4 being less than the divisor, we annex a decimal 0 to the dividend, and divide 300 into 8.40.

Or, rejecting the two *integral* 0s in the right of the divisor, and making two *integral* figures *decimals* in the dividend,—prefixing a 0 to make up the number,—

$$.084\div3=.028,$$

we divide .084 by 3, and find the same quotient as before

In like manner, $345\div100 = 3.45$;
$345\div1000 = .345$;
$345\div10000=.0345$; and so on:

in which cases the quotient is immediately obtained, by making as many integral figures *decimals* in the dividend, as there are integral 0s in the right of the divisor,—prefixing 0s to the dividend, when necessary to make up the number.

☞ The dividend must contain just as many decimal figures as both the divisor and quotient, because the dividend is equal to the *product* of the divisor and quotient.

Thus in the first example, $.965=.5\times1.93$.

Hence also, the number of decimal places in the dividend cannot be taken less than the number in the divisor; for then the product would have a less number of decimal places than one of its factors,—which is impossible.

We prefix the 0s to the quotient, as in the second example, because in no other position of the 0s would the divisor, multiplied by the quotient produce the dividend. ☜

EXERCISES.

1. Divide 22.36 by 4.3 and 3.25 by 1.3. *Ans.* 5.2 and 2.5.
2. Divide 12.25 by 3.5 and 87.9 by .3. *Ans.* 3.5 and 293.
3. Divide .0425 by .05 and 5 by .0625. *Ans.* .85 and 80.
4. Divide 16.776 by 7.2 and .816 by .04. *Ans.* 2.33 and 20.4.
5. Divide 1.5 by .375 and 75 by 12. *Ans.* 4 and 6.25.
6. Divide 4.9 by 70 and 7.02 by 3. *Ans.* .07 and 2.34.
7. Divide 5.85 by .65 and 5.92 by .08. *Ans.* 9 and 74.
8. Divide 15.57 by .45 and .001638 by .07.
   Ans. 34.6 and .0234.
9. Divide 8 by 3.2 and 234.375 by 25. *Ans.* 2.5 and 9.375.
10. Divide .0276 by 23 and .08 by 32. *Ans.* .0012 and .0025.

In the next exercises, let the quotient be continued to thousandths, *and be expressed* by an *approximate decimal.*

11. Divide 13.29 by 2.8 and .278 by .07.
   Ans. 4.746' and 3.971'.
12. Divide 2.37 by 93 and .0011 by .09. *Ans.* .025' and .012'.
13. Divide .737 by 8.9 and .09 by 8.3. *Ans.* .082' and .010'.
14. Divide 8.641 by 13 and .643 by 1.05. *Ans.* .664' and .612'.
15. Divide .023 by .03 and .013 by .074. *Ans.* .766' and .175'.

Another Method of Reducing a Vulgar Fraction to a Decimal.

§ 153. The quotient of a less integer divided by a greater may be represented by a *proper vulgar fraction;* thus $3 \div 4$ is $\frac{3}{4}$.

If the less integer be then divided by the greater, *decimally,* the vulgar fraction will be reduced to a decimal.

$$3 \div 4 = 3.00 \div 4 = .75\ ;\ \text{or}\ \tfrac{3}{4} = .75.$$

The application of Rule XXXII to this case, is substantially the same with that of Rule XXVIII.

16. Divide 4 by 15, or reduce $\frac{4}{15}$ to a decimal. *Ans.* .266'.
17. Divide 9 by 34, or reduce $\frac{9}{34}$ to a decimal. *Ans.* .264'.
18. Divide 13 by 120, or reduce $\frac{13}{120}$ to a decimal. *Ans.* .108'.
19. Divide 17 by 200, or reduce $\frac{17}{200}$ to a decimal. *Ans.* .085.
20. Divide 25 by 339, or reduce $\frac{25}{339}$ to a decimal. *Ans.* .073'.
21. Divide 7 by 13, or reduce $\frac{7}{13}$ to a decimal. *Ans.* .538'.
22. Divide 10 by 19, or reduce $\frac{10}{19}$ to a decimal. *Ans.* .526'.
23. Divide 21 by 121, or reduce $\frac{21}{121}$ to a decimal. *Ans.* .173'.
24. Divide 73 by 300, or reduce $\frac{73}{300}$ to a decimal. *Ans.* .243'.
25. Divide 99 by 500, or reduce $\frac{99}{500}$ to a decimal. *Ans.* .198.

26. How many gallons of wine, at 1 dollar $37\frac{1}{2}$ cents per gallon, may be bought for 25 dollars and 50 cents?

Expressing the cents *in decimals of a* \$, *the number of gallons is the number of times* \$1.37½ cents *is contained in* \$25.5; *that is,*

$$\$25.5 \div \$1.37\tfrac{1}{2}, = \$25.5 \div \$^{2.75}_{\ 2}.$$

But in the *Division of Federal Money,* instead of *fractions of a cent,* it will generally facilitate the operation to take their *equivalent decimals.* We have then

\$25.5÷\$1.375. *Ans.* 18.545' gallons.

27. What quantity of coal, at 18 dollars and 75 cents per ton, may be purchased for 13 dollars?

\$13 *will buy the same part of a ton that* \$13 is of \$18.75.
Ans. .693' of a ton.

28. How many hundred weight of flour, at 2 dollars $18\frac{3}{4}$ cents per hundred, may be purchased for 25 dollars?
Ans. 11.428' hundred weight.

29. What quantity of land, at the rate of 25 dollars per acre, may be bought for 9 dollars $62\frac{1}{4}$ cents?
Ans. .3849 of an acre.

30. How many bushels of clover seed, at 5 dollars $18\frac{3}{4}$ cents per bushel, may be bought for 30 dollars?
Ans. 5.783' bushels.

31. How many yards of cloth, at 4 dollars and 50 cents a yard, may be purchased for 19 dollars 75 cents?
Ans. 4.388' yards.

32. How many barrels of corn, at 3 dollars and 85 cents per barrel, may be bought for 100 dollars? *Ans.* 25.974' barrels.

33. What quantity of bacon, at 8 dollars $31\frac{1}{4}$ cents per hundred weight, may be purchased for 5 dollars $6\frac{1}{4}$ cents?
Ans. .609' of a hundred weight.

34. What quantity of iron, at 45 dollars 50 cents per ton, may be bought for the sum of \$7.06¼+\$13.5?
Ans. .451' of a ton.

35. How many barrels of apples, at 2 dollars $12\frac{1}{2}$ cents per barrel, may be bought for \$75.83¾—\$20.56¼?
Ans. 26.011' barrels.

36. What should be paid for a ton of hay, when .7 of a ton sells for 13 dollars $12\frac{1}{2}$ cents?

If 7 *tenths* of a ton *cost* \$13.12½, 1 *tenth would be worth* $\frac{1}{7}$ *of* \$13.12½; and a WHOLE TON, or 10 *tenths, would be worth* $\frac{10}{7}$ *of* \$13.12½; that is, *such a part of this sum as is expressed by the* reciprocal *of* .7; or \$13.12½÷.7.

We may also reason thus. *The price of a ton will be such that* .7 of the *price*, or the *price* × .7, *will be* \$13.12½.

Hence \$13.12½ is a *dividend*, or product *given*, and .7 a *divisor*, or one factor given to find the *quotient*, which will be the other factor. *Ans.* \$18.750.

37. If 3½ cords of wood sell for 12 dollars 75 cents, what is the price per cord? *Ans.* \$3.642'.

38. If 5¼ yards of broadcloth cost 21 dollars 25 cents, what should be paid for 1 yard of the same cloth? *Ans.* \$4.047'.

39. If ⅝ of a lot of ground be worth 73 dollars 87½ cents, what is the whole of the lot worth at that rate? *Ans.* \$118.20.

40. If 4½ cords of wood come to \$9, what is the price per cord? What would 7¾ cords amount to? *Ans.* \$2; and \$15.50.

41. What is the price of wheat per bushel when 25⅛ bushels sell for 37 dollars 68¾ cents, and what should be paid for 40 bushels at the same rate? *Ans.* \$1.50; and \$60.

42. What is the price of butter per pound when 13½ pounds sell for 1 dollar 62 cents, and what ought to be paid for 145 pounds of butter at the same price? *Ans.* \$.12; and \$17.40.

43. Bought 50 bushels of salt for \$31.25, and sold it at a profit of 25 cents per bushel. At what price per bushel was it sold? *Ans.* \$.875.

44. Bought 32½ barrels of corn for \$81.25, and sold it at a profit of 61¼ cents per barrel: at what price per barrel was it sold, and what was the whole profit made? *Ans.* \$3.1125; and \$19.906'.

45. A bought of B 1000 bushels of wheat, at \$.87½ a bushel; of which he has sold to C 453½ bushels, at \$.9 a bushel. At what price per bushel must the remainder be sold to produce a profit of \$25 on the 1000 bushels? *Ans.* \$.90.

46. A person having \$300 on hand would disburse it for equal quantities of sugar and coffee. What quantity of each can he purchase, if the sugar be 9 cents and the coffee 15 cents per pound?

9 cents +15 cents =24 cents; then 24 cents will buy 1 pound of each. *Ans.* 1250 pounds of each.

47. A miller wishes to purchase wheat, rye, and corn, in equal quantities. The prices of these commodities being respectively \$1, \$.5, and \$.37½ per bushel, how many bushels of each can he purchase for \$500? *Ans.* 266.666' bushels of each.

EXERCISES ON CHAPTER VI.

1. What is the value of the expression

(3 *hundredths* + 23 *thousandths*) $\times$ 9 *thousandths* ?

Ans. .000477.

2. What is the value of the expression

(8 *tenths*—75 *ten-thousandths*)$\times$21 *hundredths* ?

Ans. .166425.

3. What is the value of the expression

(34 *thousandths*+7 *hundredths*)$\div$3 *hundredths* ?

Ans. 3.466'.

4. What is the value of the expression

(723 *thousandths*—9 *thousandths*)$\div$8 *thousand* ?

Ans. .000089'.

5. What is the value of the expression

(34 *ten-thousandths*+8 *hundredths*)$\div$20 *thousand* ?

Ans. .000004'.

Find the answer to each of the three following questions in a vulgar fraction, in its *lowest terms.*

6. What is the value of the expression

(.25+.125+2.5—.05—.005)$\times$.04 ? *Ans.* $\frac{141}{1250}$.

7. What is the value of the expression

(.1+.34+.09+3.2—1.375)$\div$5 ? *Ans.* $\frac{471}{1000}$.

8. What is the value of the expression

(80—28.5+100—50.4—90.1)$\div$50 ? *Ans.* $\frac{11}{50}$.

Find the answer to each of the two following questions in an integer and decimal thousandths ; *employing a mixed or complex decimal to express the exact value.*

9. What is the value of the expression

$(\frac{3}{4}+5\frac{1}{2}+10+.3\frac{1}{4}+2\frac{1}{10})—(7+\frac{2}{5}+5\frac{1}{3})$? *Ans.* 5.941$\frac{2}{3}$.

10. What is the value of the expression

$(25\frac{3}{8}+3.6\frac{2}{3}+16+100\frac{1}{6})—(4\frac{1}{2}+\frac{1}{8}+10)$? *Ans.* 130.583$\frac{1}{3}$.

11. Find the sum 3840+5.123+3.479+31. *Ans.* 3879.602.
12. Find the sum .7309+ 7834+834.3+.75. *Ans.* 8669.7809.
13. Find the sum 80.01+.1347+.9348+3.5. *Ans.* 84.5795.

14. Find the difference 737.68—7.3473. . . *Ans.* 730.3327.
15. Find the difference 80.893—837.83. . . *Ans.* 756.937.
16. Find the difference 376.84—37.639. . *Ans.* 339.201

11. Find the amount of a merchant's bill for $3\frac{1}{2}$ yards of clot at 7 dollars $68\frac{3}{4}$ cents per yard, $12\frac{1}{4}$ yards of silk at 1 dolla $31\frac{1}{4}$ cents per yard, and 16 skeins of silk thread at $6\frac{1}{4}$ cents a skein. *Ans.* \$43.984'.

12. What quantity of iron at 45 dollars per ton, may be purchased for 28 dollars $62\frac{1}{2}$ cents? *Ans.* .636' of a ton.

13. Bought 20 barrels of flour for 102 dollars and 50 cents, and sold the same at a profit of $87\frac{1}{2}$ cents per barrel: at what price per barrel was it sold, and what was the entire profit made? *Ans.* \$6; and \$17.50.

14. If a ship sail at the rate of 130.75 miles per day, in what time ought she to sail $69.33\frac{1}{3}$ miles? *Ans.* .530' of a day.

15. A merchant bought 25 yards of cloth at 4 dollars $87\frac{1}{2}$ cents per yard, and sold it at an entire profit of 50 dollars $68\frac{3}{4}$ cents: at what price per yard was the cloth sold? *Ans.* \$6.9025.

16. What is the price of sugar per hundred weight when $\frac{3}{4}$ hundred weight costs 6 dollars $37\frac{1}{2}$ cents, and what should be paid for $5\frac{1}{2}$ hundred weight of sugar at the same rate? *Ans.* \$8.5; and \$46.75.

17. A farmer bought a plantation containing 400 acres, at $\$20\frac{1}{2}$ per acre, and sold $\frac{1}{2}$ of it at a profit, on that half, of $\$213.12\frac{1}{2}$. At what rate per acre was the land sold? *Ans.* \$21.565'.

18. If $\frac{5}{8}$ of a yard of silk cost $\$1.12\frac{1}{2}$, what sum should be paid for $15\frac{1}{2}$ yards at the same rate? *Ans.* \$27.90.

19. A person wishes to purchase a quantity of coffee, and as much rice. The coffee is at $13\frac{1}{2}$ cents, and the rice at 5 cents, per pound; what amount of each can he purchase for $\$15\frac{1}{2}$? *Ans.* 83.783' pounds.

20. Allowing $\frac{3}{4}$ of a yard of cloth to cost $\$5.43\frac{3}{4}$, what sum ought to be paid for $13\frac{7}{8}$ yards at the same rate? *Ans.* \$100.59375.

21. A bought of B $122\frac{1}{2}$ bushels of wheat, and of C $75\frac{1}{4}$ bushels, at $93\frac{3}{4}$ cents per bushel. He made 60 bushels into flour, and sold the flour at a profit of \$12.50: if he sells the remainder of the wheat at $81\frac{1}{4}$ cents per bushel, what will be his entire profit or loss? *Ans.* Loss \$4.718'.

22. A speculator bought 50 barrels of flour at $\$4.06\frac{1}{4}$ per barrel, and 75 bushels of wheat at $\$.68\frac{3}{4}$ per bushel. Having sold 20 barrels of the flour at \$5 per barrel, and the whole of the wheat at \$.75 per bushel, at what rate per barrel must the remainder of the flour be disposed of to make his profit \$100 on the whole? *Ans.* \$6.614' per barrel.

REMARK.

The preceding Chapters, which contain the Addition, Subtraction, Multiplication, and Division of Integers and Fractions—with the notations and reductions necessary to these operations—involve the essential principles of Arithmetic; and should be fully mastered by the pupil before he is permitted to advance.

At this stage, therefore, in his arithmetical studies, it is recommended that the pupil be thoroughly examined on what precedes, and, if found deficient, be required to review.

CHAPTER VII.

TABLES OF MONEY, WEIGHTS, AND MEASURES.—REDUCTION OF MONOMIALS, POLYNOMIALS, AND CURRENCIES.

Measuring Units.

§ **154.** The *measuring unit* of a Quantity is the concrete unit, of whatever kind, by means of which the quantity is expressed *numerically*.

Thus *one pound* is the measuring unit of the quantity 5 *pounds*.

What is the measuring unit of the quantity 10 *dollars?* Of 13 yards? Of 24 days? Of 75 tons? Of 100 miles? Of 135 gallons? Of $1000?

Different Orders of Measuring Units.

§ **155.** A Quantity is often expressed by two or more *different orders* of measuring units; each lower unit being, for the most part, contained an exact number of times in the next higher.

Thus 10 *dollars* 25 *cents* is a quantity, or sum of money, expressed by *two different orders* of measuring units.

How many different orders of measuring units in the quantity 5 *pounds* 3 *ounces* of gold? In 4 *days* 7 *hours* 20 *minutes* of time? In 15 *miles* 100 *yards* 2 *feet?* In 100 *pounds* 13 *shillings* 6 *pence* 3 *farthings?*

§ **156.** In Federal Money, as has been seen, (§ 138), the measuring units rise from lower to higher orders by a *tenfold* increase, as in abstract numbers.

In other kinds of quantity, the measuring units rise from lower to higher orders according to the *various scales* of increase given in the following

Tables of Money, Weights and Measures.

§ **157.** English or Sterling Money

Is the national currency of the kingdom of Great Britain.

4 farthings (*qr.*) make 1 penny, (*d.*);
12 pence, ... 1 shilling, (*s.*);
20 shillings, ... 1 pound, (£);

Also, 5 shillings make 1 *crown*, and 21 shillings 1 *guinea*.
The English pound sterling is represented by a gold coin,

called a sovereign, which is valued by law in the United States at $4.84.

One *qr.* is what part of a *d.*? 1 *d.* is what part of a *s.*?
1 *s.* is what part of a £.? 1 *s.* is what part of a *crown*?

§ 158. Troy Weight

Is used in weighing *jewels*, *gold*, *silver*, *liquors*, and, generally, the most valuable commodities.

24 grains (*gr.*)	make	1 pennyweight, (*dwt.* or *pwt.*);
20 pennyweights,	.	1 ounce, (*oz.*);
12 ounces,	. . . .	1 pound, (*lb.*)

One *gr.* is what part of a *dwt.*? 1 *dwt.* is what part of an *oz.*?
One ounce is what part of a *lb.*?

§ 159. Avoirdupois Weight

Is used in weighing *groceries*, all the coarser *metals*, and, generally, all coarse commodities.

16 drams (*dr.*)	make	1 ounce, (*oz.*)
16 ounces,	. . . .	1 pound, (*lb.*)
2000 pounds,	. . . .	1 ton, (*T.*)

Occasionally, for the coarsest commodities, as *plaster*, *coal*, *iron*, *hemp*, &c.,

28 pounds (*lb.*)	make	1 quarter, (*qr.*);
4 quarters,	. . . .	1 hundred weight, (*cwt.*);
20 hundred weight, or 2240 *lb.*,		1 ton, (*T.*)

The *pound Avoirdupois* is 1 *lb.* 2 *oz.* 11 *dwt.* 16 *gr.*, Troy Weight.

One *dr.* is what part of an *oz*? 1 *oz.* is what part of a *lb.*
One *lb.* is what part of a *qr.* of a *cwt.*

§ 160. Apothecaries Weight

Is used in *compounding medicines*, which, however, are bought and sold by Avoirdupois Weight.

20 grains (*gr.*)	make	1 scruple, (℈);
3 scruples,	. . . .	1 drachm, (ʒ);
8 drachms,	. . . .	1 ounce, (℥);
12 ounces,	. . . .	1 pound, (℔);

The *pound* and *ounce* in Apothecaries Weight are the same as in Troy Weight. The difference between the two kinds of weight is only a difference in the divisions and subdivisions of the ounce.

One *gr.* is what part of a ℈? 1 ℈ is what part of a ʒ?
One ʒ is what part of an ℥? 1 ℥ is what part of a ℔?

§ 161. Dry Measure

Is used in measuring *grain, fruit, salt, coal*, and, in general, all such commodities as are estimated in the *heap* or aggregate.

2 pints (*pt.*) make 1 quart, (*qt.*);
8 quarts, ... 1 peck, (*pk.*);
4 pecks,. . . . . 1 bushel, (*bu.*);

The English *quarter*, in Dry Measure, is 8 bushels, and the *chaldron* is a coal measure of 36 bushels; but coal is usually sold by *weight*.

One *pt.* is what part of a *qt.*? 1 *qt.* is what part of a *pk.*?
One *pk.* is what part of a *bu.*?

§ 162. Beer Measure

Is used in measuring *beer, ale*, and, in general, all malt liquors, milk, and water.

2 pints (*pt.*) make 1 quart, (*qt.*);
4 quarts, ... 1 gallon, (*gal.*);
36 gallons,. . . . 1 barrel, (*bar.* or *bbl.*);
54 gallons,. . . . 1 hogshead, (*hhd.*);

The English *firkin* is 9 gallons; also 2 *fir.* make one kilderkin, and 2 *kil.* make 1 barrel.

One *gal.* is what part of a *bbl.*? 1 *gal.* is what part of a *hhd.*?

§ 163. Wine Measure

Is used in measuring *wine*, distilled *spirits*, and, in general, all liquids excepting such as fall under Beer Measure.

4 gills (*gi.*) make 1 pint, (*pt.*);
2 pints, ... 1 quart, (*qt.*);
4 quarts,. . . . 1 gallon, (*gal.*);
31½ gallons, . . . 1 barrel, (*bar.* or *bbl.*);
63 gallons, . . . 1 hogshead, (*hhd.*);
2 hogsheads, . . 1 pipe or butt, (*pi.* or *bt.*);
2 pipes or 4 *hhd.*, 1 tun, (*tn.*);

Also 42 gallons make 1 *tierce*, and 84 *gal.* make 1 puncheon. The *gallon* in Wine Measure is .81' *gal.* in Beer Measure.

One *gi.* is what part of a *pt.* 1 *gal.* is what part of a *bbl.*

§ 164. Linear or Long Measure

Is used in measuring *lines*; that is, *length, distance, height*, &c

12 inches (*in.*) make 1 foot, (*ft.*);
3 feet, ... 1 yard, (*yd.*);
5½ yards,. . . . . . . 1 rod or pole, (*r.* or *p.*);
40 rods, ... 1 furlong, (*fur.*);
8 furlongs or 1760 *yd.* 1 mile, (*m.*);

Also, 3 miles make 1 *league*—used in expressing distances at sea. The term *barley corn* was formerly used for one *third* of an *inch*, and the term *line* is sometimes used for one *twelfth* of an inch.

A *hand* is 4 inches—used in measuring the height of horses; and a *fathom* is 6 feet—used in measuring the depth of water.

One *in.* is what part of a *ft.*? 1 *ft.* is what part of a *yd.*?
One *yd.* is what part of a *r.* or *p.*?

§ 165. Cloth Measure

Is used in measuring *cloth, silk, lace,* &c., being a species of Linear or Long Measure.

$2\frac{1}{4}$ inches (*in.*)	make	1 nail,	(*n.*);
4 nails,	. . . . . .	1 quarter,	(*qr.*);
4 quarters,	. . . .	1 yard,	(*yd.*);

Also, 3 quarters make 1 Flemish ell; 4 *qr.* $1\frac{1}{2}$ *in.* 1 Scotch ell; 5 *qr.* 1 English ell; and 6 *qr.* 1 French ell.

The *yard* in Cloth Measure is the same as in Long Measure.

One *in.* is what part of a *na.*? 1 *na.* is what part of a *qr.*?

§ 166. Square Measure

Is used in measuring *surfaces*, or any extension in *length* and *breadth*, without regard to thickness.

A *square inch* is an inch *long* and an inch *wide*; a square foot is a foot long and a foot wide; and so on.

2 *in.* long and 1 *in.* wide would make how many *square inches*?
2 *in.* long and 2 *in.* wide would make how many *square inches*?
3 *in.* long and 2 *in.* wide would make how many square inches?

144 square inches (*sq. in.*)	make	1 square foot, (*sq. ft.*),
9 square feet,	. . . .	1 square yard, (*sq. yd.*);
$30\frac{1}{4}$ square yards,	. . . .	1 *sq.* rod, pole, or perch, (*P.*);
40 perches or *sq. r.* or *p.*,	. . .	1 rood, (*R.*);
4 roods, or 160 *sq. r.*,	. . . . .	1 acre, (*A.*);

Also, 640 acres make 1 *square mile*, or Section of land; and 6 *miles square*, which is 36 *sq. miles*, make a Township.

An *inch square* is an inch long and an inch wide, being the same as a square inch; o a *foot square* is the same as a square foot.

But *two inches square* is 2 *in.* long and 2 *in.* wide, which makes 4 *sq. in.*; 3 inches square is 3 *in.* long and 3 *in.* wide, or 9 *sq. in.*, &c.

4 *in.* square is how many square inches? 5 *ft.* square is how many square feet? 10 miles square is how many square miles?

§ 167. Cubic or Solid Measure

Is used in measuring solids, or any extension in *length, breadth* and *thickness.*

A *cubic inch* is an inch long, an inch wide, and an inch thick; a cubic foot is a foot long, a foot wide, and a foot thick, and so on.

2 *in.* long, 1 *in.* wide, and 1 *in.* thick would make how many *cubic inches?* 2 *in.* long, 2 *in,* wide, and 1 *in.* thick, would make how many cubic inches? 2 *in.* long, 2 *in.* wide, and 2 *in.* thick, would make how many cubic inches?

1728 cubic inches (*cu. in.*) make 1 cubic foot, (*cu. ft.*)
27 cubic feet, ... 1 cubic yard, (*cu. yd.*);
128 cubic feet,. . . . . 1 cord.

A cord of wood is usually put up 8 ft. long, 4 ft. wide, and 4 feet high. One foot in length of such a pile is called a *cord foot,* and contains 16 cubic feet.

50 cubic feet of timber are allowed to weigh a *ton.* Of round timber such a quantity is allowed for a ton as, when hewn, will make 40 cubic feet.

A *perch* of stone is estimated at 1 rod or perch, which is $16\frac{1}{2}$ ft., in length, $1\frac{1}{2}$ ft. in thickness, and 1 ft. in height; and contains $24\frac{3}{4}$ *cu. ft.*

231 *cu. in.* is the capacity of a *gallon* in Wine Measure, and 282 *cu. in.* is the capacity of a gallon in Beer Measure.

The British Imperial gallon contains 277.274 *cu. in.*, and the Imperial bushel, being 8 *Imp. gal.* contains 2218.192 *cu. in.*

The British Winchester bushel, which is the standard bushel in the United States, contains 2150.4 cubic inches.

§168. Circular Measure

Is used in measuring any part of the *circumference* of a *circle,* in reckoning *latitude* and *longitude,* and the motions of the heavenly bodies.

60 seconds (″) make 1 minute, (′);
60 minutes,. . . . . 1 degree, (°);
360 degrees, the *circumference of any circle.*

A *degree,* it is evident, has no determinate *linear* extent; being always the 360th part of the circumference on which it is taken, it is greater or less as that circumference is greater or less.

A degree on the circumference of the earth, is about 69½ miles.

One *minute* on the circumference of the earth, is called a *geographical* or *nautical* mile; the mile of linear measure being denominated a *statute* mile.

§ 169. Measure of Time.

TIME is measured in *days* by the revolution of the earth around its *axis*, and in *years* by the revolution of the earth around the sun.

60 seconds (*sec.*) make	1 minute, (*min.*);
60 minutes, ...	1 hour, (*hr.*);
24 hours, ...	1 day, (*da.*);
7 days, ...	1 week, (*wk.*);
365 days, or 52 *w.* 1*d.*,	1 common year, (*yr.*);
366 days, ...	1 leap year;
100 years, ...	1 century.

A year also consists of 12 *months*, viz: January, February, March, April, May, June, July, August, September, October, November, and December.

The number of days in each is as follows:

Thirty days has September, April, June, and November;
February has twenty-eight alone, and all the rest have thirty-one;
But Leap Year comes one year in four, when February has one day more.

Or, the fourth, eleventh, ninth, and sixth, have 30 *da.* to each affixed;
And every other 31, except the second month alone,
To which we 28 assign, till Leap Year gives it 29.

Solar, Civil, and Leap Years

§ **170.** The period of the earth's revolution around the sun, is 365 *da.* 5 *hr.* 48 *min.* 49.6 *sec.* This constitutes the *solar* year, being 5 *hr.* 48 *min.* 49.6 *sec.* longer than the common *civil* year of 365 days.

To correct the error which arises from reckoning only 365 days to a year, *one day* is added to February every fourth year; and this makes the *Leap* Year of 366 days. But one day is more than the excess (5 *hr.* 48 *min.* 49.6 *sec.*) of the solar above the civil year, amounts to in 4 years.

To correct this second error, so as to preserve the civil in agreement with the solar years, the following rule has been adopted; viz: *if the number of the year is divisible by* 4 *without a remainder, it is made* LEAP YEAR; but the closing year of a century, as 1700, 1800, &c., *is not made Leap Year, unless the number is divisible by* 400, *without a remainder.*

REDUCTION OF MONOMIALS AND POLYNOMIALS.

§ **171.** A *monomial* quantity, or simply a *monomial*, is a quan tity expressed by a *single name* of measuring units, (§ 154).

Thus 5 *dollars* is a *monomial;* 10 shillings is a monomial.

§ **172.** A *polynomial* quantity, or simply a *polynomial*, is a quantity expressed by *two or more names* of measuring units.

Thus 5 *dollars* 25 *cents* is a *polynomial;* 3 pounds, 10 shillings and 6 pence, is a polynomial.

A polynomial is composed of two or more monomials, which may thence be called the *terms* of the polynomial.

Thus in the first example given, the terms are 5 *dol.* and 25 *c.*; and in the second, 3£, 10 *s.* 6 *d.*

Note.—Monomial quantities have by some been called *denominate numbers*, and *polynomials* have usually been called *compound numbers.*

REDUCTION DESCENDING.

§ **173.** *Reduction descending* consists in finding the value of a given quantity in measuring units of a *lower order*, (§ 155). The quantity is then said to be reduced to a *lower* name or *denomination.*

RULE XXXIII.

§ **174.** *To reduce a Quantity to a* LOWER DENOMINATION.

1. Multiply a *monomial* of a higher denomination, or the *highest term* of a polynomial, by that number of the next lower denomination which makes a *unit* of the higher : the *product* will be in the lower denomination.

2. This product may, in like manner, be reduced to a still lower denomination, and so on, observing that each *lower term* in a polynomial must be added to the product in the *same denomination with itself.*

3. *In reducing a* MONOMIAL FRACTION to lower denominations, the *integers* in the successive products may be reserved, and afterwards arranged as the *terms of a polynomial.*

EXAMPLE.

To reduce 5£, 14*s.* 9*d.* to pence.

£5	14 *s.*	9 *d.*
20		
114 *s.*		
12		
1377 *d.*		

We multiply 5 £ by 20, because 20 *s.* make 1 £; the product is *shillings*—to which adding the 14 *s.*, we have 114 *s.*

We next multiply the 114 *s.* by 12, because 12 *d.* make 1 *s.*; the product is *pence*—to which adding the 9*d.*, we have 1377 *d*

Thus we find 5 £ 14 *s.* 9 *d.* to be equal to 1377 *d.*

EXERCISES.

1. Reduce 4 *lb.* 7 *oz.* 13 *dwt.* to *dwt.*

Recollect that 12 *oz.* make 1 *lb.*, and 20 *dwt.* make 1 *oz.* *Ans.* 1113 *dwt.*

2. Reduce 7 *lb.* 10 *dwt.* 2 *gr.* to *gr.* . . . . . *Ans.* 40562 *gr.*
3. Reduce 3 *T.* 2 *cwt.* 3 *qr.* to *qr.* . . . . . *Ans.* 251 *qr.*
4. Reduce 9 *cwt.* 1 *qr.* 13 *oz.* to *oz.* . . . . . *Ans.* 16589 *oz.*
5. Reduce 14 ℥, 2 ʒ, 12 *gr.* to *gr.* . . . . . *Ans.* 6852 *gr.*
6. Reduce 8 ℔, 1 ʒ, 15 *gr.* to *gr.* . . . . . *Ans.* 46155 *gr.*
7. Reduce 15 *bu.* 2 *pk.* 7 *qt.* to *qt.* . . . . . *Ans.* 503 *qt.*
8. Reduce 9 *bu.* 5 *qt.* 1 *pt.* to *pt.* . . . . . *Ans.* 587 *pt.*
9. Reduce 3 *pi.* 1 *hhd.* 40 *gal.* to *gal.* . . . . *Ans.* 481 *gal.*
10. Reduce 4 *tuns*, 5 *hhd.* 3 *qt.* to *qt.* . . . . . *Ans.* 5295 *qt.*
11. Reduce 13 *m.* 7 *fur.* 25 *r.* to *r.* . . . . . *Ans.* 4465 *r.*
12. Reduce 10 *L.* 16 *fur.* 15 *p.* to *p.* . . . . . *Ans.* 10255 *p.*
13. Reduce 20 *yd.* 3 *qr.* 2 *na.* to *na.* . . . . . *Ans.* 334 *na.*
14. Reduce 31 *yd.* 3 *na.* 2 *in.* to *in.* . . . . . *Ans.* 1124¾ *in.*
15. Reduce 14 *A.* 1 *R.* 20 *P.* to *P.* . . . . . *Ans.* 2300 *P.*

16. Reduce 9 *A.* 13 *P.* 4 *sq. yd.* to *sq. yd.* . . *Ans.* 43957¼ *sq. yd.*
17. Reduce 10 *cu. yd.* 17 *cu. ft.* to *cu. ft.* *Ans.* 287 *cu. ft.*
18. Reduce 4 *cu. yd.* 100 *cu. in.* to *cu. in.* *Ans.* 186724 *cu. in.*
19. Reduce 20 *wk.* 5 *da.* 33 *hr.* 5 *min.* to *min.* *Ans.* 210785 *min.*
20. Reduce 1 *yr.* 100 *da.* 20 *hr.* 5 *min.* to *min.* *Ans.* 670805 *min.*
21. Reduce 7 *T.* 13 *cwt.* 1 *qr.* 4 *lb.* to *oz.* . *Ans.* 274688 *oz.*
22. Reduce 75 ℔, 10 ℥, 7 ʒ, 2 ℈, 11 *gr.* to *gr.* *Ans.* 437271 *gr.*
23. Reduce 3 *hhd.* 40 *gal.* 3 *qt.* 1 *pt.* to *gills.* *Ans.* 7356 *gills.*
24. Reduce 5 *L.* 2 *m.* 4 *fur.* 15 *r.* to *yards.* *Ans.* 30882½ *yd.*
25. *Reduce* 11 *A.* 2 *R.* 25 *P.* 25 *sq.yd.* to *sq.yd.* *Ans.* 56441¼ *sq. yd.*

Monomial Fractions Reduced to Integers.

26. Reduce $\frac{2}{7}$ £ to integers in shillings, &c.

$\frac{2}{7}$£ multiplied by 20 produces $\frac{40}{7}$ *s.*$=5\frac{5}{7}$ shillings, (§ 174).

Reserving the *integer* 5 *s.*, and reducing the *fraction* $\frac{5}{7}$ *s.* to pence,—

$\frac{5}{7}$ *s.* multiplied by 12 produces $\frac{60}{7}$ *d.*$=8\frac{4}{7}$ pence.

Reserving the integer 8 *d.*, and reducing the fraction $\frac{4}{7}$ *d.* to farthings,—

$\frac{4}{7}$ *d.* multiplied by 4 produces $\frac{16}{7}$ *qr.*$=2\frac{2}{7}$ farthings.

Arranging the integers reserved as the *terms of a polynomial*,

we find $\frac{2}{7}$ £$=5$ *s.* 8 *d.* $2\frac{2}{7}$ *qr.*

27. Reduce .23 £ to integers in shillings, &c.

.23 £ multiplied by 20 produces 4.60 shillings, (§ 174).

Reserving the integer 4 *s.*, and reducing the fraction .60 *s.* to pence,—

.60 *s.* multiplied by 12 produces 7.20 pence.

Reserving the integer 7 *d.*, and reducing the fraction .20 *d.* to *qr.*,— 20 *d.* multiplied by 4 produces .80 farthings.

Arranging the integers reserved as the terms of a polynomial, we find .23 £$=4$ *s.* 7 *d.* 0.80 *qr.*

The integers found in reducing are arranged with the quantity in the lowest denomination, whether that quantity be an integer or otherwise.

28. Reduce $\frac{4}{9}$ *lb.* to integers in *oz.*, &c.
Ans. 5 *oz.* 6 *dwt.* 16 *gr.*

29. Reduce .17 *lb.* to integers in *oz.*, &c.
Ans. 2 *oz.* 0 *dwt.* 19.2 *gr.*

30. Reduce $\frac{2}{3}$ *qr.* to integers in *lb.*, &c.
Ans. 18 *lb.* 10 *oz.* $10\frac{2}{3}$ *dr.*

31. Reduce .19 *T.* to integers in *cwt.*, &c.
Ans. 3 *cwt.* 3 *qr.* 5.6 *lb.*

32. Reduce $\frac{4}{15}$ ℥ to integers in ʒ, &c. *Ans.* 2 ʒ, 0 ℈, 8 *gr.*

33. Reduce .35 ℥ to integers in ʒ, &c. *Ans.* 2 ʒ, 2 ℈, 8 *gr.*

34. Reduce $\frac{7}{12}$ *pk.* to integers in *qt.*, &c. *Ans.* 4 *qt.* 1 *pt.* $1\frac{1}{3}$ *gi.*

35. Reduce $\frac{8}{15}$ *pi.* to integers in *hhd.*, &c.
Ans. 1 *hhd.* 12 *gal.* $2\frac{2}{5}$ *qt.*

36. Reduce .31 *bu.* to integers in *pk.* &c.
Ans. 1 *pk.* 1 *qt.* 1.84 *pt.*

37. Reduce .6 *tun*, to integers in *pi.*, &c.
Ans. 1 pi. 25 gal. 1.6 pt.

38. Reduce $\frac{387}{880}$ *m.* to integers in *fur.*, &c.
Ans. 3 *fur.* 20 *r.* 4 *yd*

39. Reduce .4 *L.* to integers in *m.*, &c. *Ans.* 1 *m.* 1 *fur.* 24 *r.*

40. Reduce $\frac{10}{11}$ *yd.* to integers in *qr.*, &c.
Ans. 3 *qr.* 2 *na.* $1\frac{5}{22}$ *in.*

41. Reduce .985 *yd.* to integers in *qr.* &c.
Ans. 3 *qr.* 3 *na.* 1.71 *in.*

42. Reduce $\frac{2}{3}$ *A.* to integers in *R.*, &c.
Ans. 2 *R.* 26 *P.* $20\frac{1}{6}$ *yd.*

43. Reduce .83 *A.* to integers in *R.*, &c.
Ans. 3 *R.* 12 *P.* 24.2 *yd.*

44. Reduce $\frac{7}{12}$ *cu. yd.* to integers in *cu. ft.*, &c.
Ans. 15 *cu. ft.* 1296 *cu. in.*

45. Reduce .3 *cu. yd.* to integers in *cu. ft.*, &c.
Ans. 8 *cu. ft.* 172.8 *cu. in.*

46. Reduce $\frac{5}{14}$ *degree* to integers in *min.*, &c.
Ans. 21 *min.* $25\frac{5}{7}$ *sec.*

47. Reduce .37 *deg.* to integers in *min.*, &c.
Ans. 22 *min.* 12 *sec.*

48 Reduce $\frac{7}{10}$ *wk.* to integers in *da.*, &c.
Ans. 4 *da.* 21 *hr.* 36 *min.*

49. Reduce .85 *wk.* to integers in *da.* &c.
Ans. 5 *da.* 22 *hr.* 48 *min.*

REDUCTION ASCENDING.

§ 175. *Reduction ascending* consists in finding the value of a given quantity in measuring units of a *higher order.* The quantity is then said to be reduced to a higher name or *denomination.*

RULE XXXIV.

§ 176. *To reduce a Quantity to a* HIGHER DENOMINATION.

1. Divide a *monominal* of a lower denomination, or the *lowest term* of a polynomial, by the number of that denomination which makes a *unit* of the next higher denomination: the *quotient* will be in the higher denomination.

2. This quotient may, in like manner, be reduced to a still higher denomination, and so on, observing that each *higher term* in a polynomial must be added to the quotient in the *same denomination with itself.*

3. *In reducing* a MONOMIAL INTEGER to higher denominations, each *remainder* may be reserved in the same denomination with *the dividend whence it is derived,* and the last quotient and the several remainders be afterwards arranged as the *terms of a polynomial.*

EXAMPLE.

To reduce 10 *s.* 6 *d.* 2 *qr.* to the denomination of £.

We take the lowest term 2 *qr.* and divide it by 4, because 4 *qr.* make 1 *d.*, a *unit* of the next higher denomination

$$2\ qr. = \tfrac{2}{4}\ d. = \tfrac{1}{2}\ d.$$

We then add the 6*d.* to the $\frac{1}{2}$ *d.*, and divide by 12, because 12 *d.* make 1 *s.*, a *unit* of the next higher denomination.

$$6\tfrac{1}{2}\ d. = \frac{6\frac{1}{2}}{12}\ s. = \tfrac{13}{24}\ s.$$

We next add the 10 *s.* to the $\frac{13}{24}$ *s.* and divide by 20, because 20 *s.* make 1£, a unit of the next higher denomination.

$$10\tfrac{13}{24}\ s. = \frac{10\frac{13}{24}}{20}\ \pounds = \tfrac{253}{480}\ \pounds.$$

Thus we find 10 *s.* 6 *d.* 2 *qr.* to be equal to $\frac{253}{480}$£.

The same reductions performed *decimally*, will be presented thus;

2 *qr.*÷4=.5 *d.*; 6.5 *d.*÷12=.541' *s.*; 10.541' *s.*÷20=.527' £.

Another method of Reducing a Polynomial to a Fraction of a Higher Denomination.

§ 177. A Polynomial may also be reduced to a *vulgar fraction* of a higher denomination, by reducing the given quantity to its lowest denomination, for a *numerator*, and reducing a *unit* of the higher denomination to the same lowest denomination, for a *denominator*.

Thus to reduce 10 *s.* 6 *d.* 2 *qr.* to the fraction of a £.

10 *s.* 6 *d.* 2 *qr.*=506 *qr.*; and 1 £=960 *qr.*

The fraction will then be $\frac{506}{960}$ £=$\frac{253}{480}$ £. And this fraction reduced to a decimal (§ 153) gives .527' £, the same as in the preceding example.

EXERCISES.

1. Reduce 8 *oz.* 15 *dwt.* 18 *gr.* to a fraction of a *lb.* *Ans.* $\frac{703}{960}$ *lb.*
2. Reduce 10 *oz.* 13 *dwt.* 20 *gr.* to a decimal of a *lb.* *Ans.* .890' *lb.*
3. Reduce 2 *qr.* 14 *lb.* 12 *oz.* to a fraction of a *cwt.* *Ans.* $\frac{283}{448}$ *cwt.*
4. Reduce 9 *cwt.* 1 *qr.* 10 *lb.* to a decimal of a *T.* *Ans.* .466' *T.*
5. Reduce 2 ʒ, 2 ℈, 17 *gr.* to a fraction of an ℥. *Ans.* $\frac{59}{160}$ ℥.
6. Reduce 3 *hhd.* 5 *gal.* 3 *qt.* to a decimal of a *tun.* *Ans.* .772' *tun.*

7. Reduce 4 *yd.* 2 *ft.* 9 *in.* to a fraction of a *r.* *Ans.* $\frac{59}{66}$ *r.*
8. Reduce 6 *fur.* 30 *p.* 4 *yd.* to a decimal of a *m.* *Ans.* .846' *m.*
9. Reduce 2 *qr.* 3 *na.* 2 *in.* to a fraction of a *yd.* *Ans.* $\frac{107}{144}$ *yd.*
10. Reduce 1 *qr.* 2 *na.* $1\frac{1}{2}$ *in.* to a decimal of a *yd.* *Ans.* .416' *yd.*
11. Reduce 8 *sq. ft.* 100 *sq. in.* to a decimal of a *sq. yd.* *Ans.* .966' *sq. yd.*
12. Reduce 3 *R.* 20 *P.* 9 *sq. yd.* to a decimal of an *A.* *Ans.* .876' *A.*
13. Reduce 20 *cu. ft.* 1000 *cu. in.* to a decimal of a *cu. yd.* *Ans.* .762' *cu. yd.*
14. Reduce 40′ 30″ to a fraction of a *deg.* *Ans.* $\frac{27}{40}$ *deg.*
15. Reduce 15 *min.* 15 *sec.* to a decimal of an *hr.* *Ans.* .254' *hr.*
16. Reduce 3 *hr.* 4 *min.* 20 *sec.* to a decimal of a *da.* *Ans.* .155' *da.*
17. Reduce 5 *s.* 10 *d.* 2 *qr.* to a fraction of a £. *Ans.* $\frac{47}{160}$ £.
18. Reduce 10 *oz.* 13 *dwt.* 4 *gr.* to a decimal of a *lb.* *Ans.* .888' *lb.*
19. Reduce 17 *lb.* 14 *oz.* 10 *dr.* to a decimal of a *cwt.* *Ans.* .159' *cwt.*
20. Reduce 5 ℥, 2 ℈, 10 *gr.* to a fraction of a ℔. *Ans.* $\frac{35}{576}$ ℔.
21. Reduce 1 *pk.* 3 *qt.* 1 *pt.* to a decimal of a *bu.* *Ans.* .359' *bu.*
22. Reduce 35 *gal.* 1 *pt.* 1 *gi.* to a decimal of a *hhd.* *Ans.* .558' *hhd.*
23. Reduce 25 *r.* 3 *yd.* 2 *ft.* to a decimal of a *m.* *Ans.* .080' *m.*
24. Reduce 50 *sq. yd.* 5 *sq. ft.* to a decimal of an *A.* *Ans.* .010' *A.*
25. Reduce 200 *da.* 14 *hr.* to a fraction of a *yr.* *Ans.* $\frac{2407}{4380}$ *yr.*
26. Reduce 175 *da.* 23 *hr.* to a decimal of a *yr.* *Ans.* .482' *yr.*

Monomial Integers Reduced to Polynomials

27. Reduce 3531 *qr.* to a polynomial in £, *s.* &c.

$$
\begin{array}{r}
4\,)\,\underline{3531}\ qr. \\
12\,)\,\underline{882}\ d.\ \ 3\ qr. \\
2|0\,)\,\underline{7|3}\ s.\ \ 6\ d. \\
3\,£\ \ 13\ s.\ \ 6\ d.\ \ 3\ qr.
\end{array}
$$

Dividing 3531 *qr.* by 4, we find 882 *d.*, with the remainder 3 *qr.*; dividing the 882 *d.* by 12, we find 73 *s.*, with the remainder 6 *d.*; dividing the 73 *s.* by 20, we find 3 £, with the remainder 13 *s.*

Arranging the last quotient 3 £ and the several remainders as the terms of a polynomial, we find 3531 *qr.* equal to 3 £ 13 *s.* 6 *d.* 3 *qr.*

28. Reduce 530 *d.* to a polynomial in £, &c. *Ans.* 2 £ 4 *s.* 2 *d.*
29. Reduce 874 *dwt.* to a polynomial in *lb.*, &c.
Ans. 3 *lb.* 7 *oz.* 14 *dwt.*
30. Reduce 1000 *dr.* to a polynomial in *lb.*, &c.
Ans. 3 *lb.* 14 *oz.* 8 *dr.*
31. Reduce 785 *lb.* to a polynomial in *cwt.*, &c.
Ans. 7 *cwt.* 0 *qr.* 1 *lb.*
32. Reduce 870 ℈ to a polynomial in ℔, &c. *Ans.* 3 ℔, 2 ℨ.
33. Reduce 748 ℨ to a polynomial in ℔, &c. *Ans.* 7 ℔, 9 ℥, 4 ℨ.
34. Reduce 62 *pt.* to a polynomial in *pk.*, &c. *Ans.* 3 *pk.* 7 *qt.*
35. Reduce 730 *qt.* to a polynomial in *bu.*, &c.
Ans. 22 *bu.* 3 *pk.* 2 *qt.*
36. Reduce 890 *bbl.* to a polynomial in *tuns*, &c.
Ans. 111 *tuns*, 1 *hhd.*
37. Reduce 75 *hhd.* to a polynomial in *tuns*, &c.
Ans. 18 *tuns*, 1 *pi.* 1 *hhd.*
38. Reduce 200 *ft.* to a polynomial in *r.*, &c. *Ans.* 12 *r.* 2 *ft.*
39. Reduce 540 *yd.* to a polynomial in *fur.*, &c.
Ans. 2 *fur.* 18 *r.* 1 *yd.*
40. Reduce 1000 *r.* to a polynomial in *L.*, &c. *Ans.* 1 *L.* 1 *fur.*
41. Reduce 375 *na.* to a polynomial in *yd.*, &c.
Ans. 23 *yd.* 1 *qr.* 3 *na.*
42. Reduce 4750 *sq. in.* to a polynomial in *sq. yd.*, &c.
Ans. 3 *sq. yd.* 5 *ft.* 142 *in.*
43. Reduce 7562½ *sq. yd.* to a polynomial in *A.*, &c.
Ans. 1 *A.* 2 *R.* 10 *P.*
44. Reduce 9374 *cu. in.* to a polynomial in *cu. ft.*, &c.
Ans. 5 *cu. ft.* 734 *in.*
45. Reduce 4034″ to a polynomial in *deg.*, &c. *Ans.* 1° 7′ 14″.
46. Reduce 371′ to a polynomial in *deg.*, &c. *Ans.* 6° 11′.
47. Reduce 3875 *sec.* to a polynomial in *hr.*, &c.
Ans. 1 *hr.* 4 *min.* 35 *sec.*
48. Reduce 4375 *min.* to a polynomial in *da.*, &c.
Ans. 3 *da.* 55 *min*
49. Reduce 3470 *hr.* to a polynomial in *wk.*, &c.
Ans. 20 *wk.* 4 *da.* 14 *hr.*
50. Reduce 4831 *d.* to a polynomial in £, &c.
Ans. 20 £ 2 *s.* 7 *d.*
51. Reduce 3743 *d.* to a polynomial in £, &c.
Ans. 15 £ 11 *s.* 11 *d.*
52. Reduce 335 *lb.* to a polynomial in *cwt.*, &c.
Ans. 2 *cwt.* 3 *qr.* 27 *lb.*
53. Reduce 3735 ℈ to a polynomial in ℔, &c.
Ans. 12 ℔, 11 ℥, 5 ℨ.
54. Reduce 17630″ to a polynomial in *deg.*, &c.
Ans. 4° 53′ 50″.

A Polynomial Reduced to the Denomination of either of its Terms.

§ 178. A Polynomial may be reduced to the denomination of either of its terms, by reducing the other terms to that denomination, and adding together the several parts.

EXAMPLE.

To reduce 7£ 10 *s.* 8 *d.* 2 *qr.* to *shillings*.

By Rule XXXIII, 7£=140 *s.*; and by Rule XXXIV, 8 *d.* 2 *qr.*=.708' *s.*;

then 7£ 10 *s.* 8 *d.* 2 *qr.*=140 *s.*+10 *s.*+.708' *s.*=150.708' *s.*

55. Reduce 9 *lb.* 8 *oz.* 15 *dwt.* 5 *gr.* to *dwt.* *Ans.* 2335.208' *dwt.*
56. Reduce 2 *T.* 15 *cwt.* 3 *qr.* 18 *lb.* to *cwt.* *Ans.* 55.910' *cwt.*
57. Reduce 25 *bu.* 3 *pk.* 3 *qt.* 1 *pt.* to *pk.* *Ans.* 103.437' *pk.*
58. Reduce 5 *tuns*, 3 *hhd.* 20 *gal.* 1 *qt.* to *gal.* *Ans.* 1469.25 *gal.*
59. Reduce 4 *m.* 5 *fur.* 30 *r.* 3 *yd.* to *r.* *Ans.* 1510.545 *r.*
60. Reduce 3 *A.* 2 *R.* 19 *P.* 5 *sq. yd.* to *P.* *Ans.* 579.165' *P.*
61. Reduce 6 *tuns*, 2 *hhd.* 35 *gal.* 1 *pt.* to *tuns.* *Ans.* 6.639' *tuns.*
62. Reduce 12 *A.* 3 *R.* 21 *P.* 25 *sq. yd.* to *A.* *Ans.* 12.886' *A.*
63. Reduce 10 *T.* 15 *cwt.* 1 *qr.* 25 *lb.* to *cwt.* *Ans.* 215.473 *cwt.*
64. Reduce 25 *L.* 2 *m.* 7 *fur.* 30 *p.* to *fur.* *Ans.* 623.75 *fur.*

Cubic Measure Reduced to Gallons, Bushels, &c.

§ 179. Cubic measure may be reduced to *gallons* in Beer or Wine Measure, or to *bushels* in Dry Measure, by dividing the number of cubic inches by the number of cubic inches in a gallon or bushel, respectively. (§ 167).

EXAMPLE.

65. How many bushels of wheat would be contained in a box, the capacity of which is 100 *cu. ft.* 500 *cu. in.*?

100 *cu. ft.* 500 *cu. in.*=173300 cubic inches;

and 2150.4 *cu. in.* make 1 *bushel*; then 173300÷2150.4= 80.589' *bushels.*

66. How many bushels of salt could be put into a receptacle which measures 2160 *cu. ft.* 1000 *cu. in.*? *Ans.* 1736.179' *bu.*

67. How many barrels of water will be contained in a cistern whose capacity is 25000 *cu. ft.* 1500 *cu. in.*?
Ans. 4255.466' *bar.*

68. How many barrels of wine will be contained in a vat whose capacity is found to be 730 *cu. ft.* 49 *cu. in.*?
Ans. 173.364' *bar.*

Coins and Currency.

§ 180. The *coin*, or *specie*, of a country, consists of pieces of metal, usually gold, silver, or copper, but in some countries *platinum*, of fixed value, and stamped by public authority—to be used, and to circulate, as *money*.

The gold coins of the United States are the *eagle*, *half-eagle*, and *quarter-eagle;* the silver coins are the *dollar*, *half-dollar*, *quarter-dollar*, *dime*, and *half-dime;* the copper coins are the *cent*, and *half-cent*.

§ 181. The coins of one country will circulate as money in other countries, at values corresponding, very nearly, to the quantity and purity of the metal they contain. Thus in the United States we have in circulation the English sovereign, the Mexican dollar, and other foreign coins.

§ 182. The *Currency* of a country consists of its circulating coin, together with bank notes considered as representing coin or specie, and, occasionally, notes or bills issued by Government.

The Fineness of Gold—how expressed.

§ 183. The degree of purity or fineness of gold is expressed by *carats*. Thus pure gold being combined with some baser metal, called *alloy*, if 20 parts in every 24 of the compound be pure gold, the alloyed gold is 20 *carats* fine.

In like manner 21 parts of pure gold to 24 of the compound, would produce gold 21 carats fine; and so on.

Standard Purity and Weight of U. S. Coins.

§ 184. By an act of Congress, passed in 1837, the gold and silver coins of the United States must contain 900 parts of *pure metal* to 100 of *alloy*.

The alloy of the gold coins is composed of silver and copper, the silver not to exceed the copper in weight. The alloy of the silver coins is pure copper. The copper coins are pure copper.

The Eagle is required to weigh 258 grains; the Dollar, 412½ grains; and the Cent, 168 grains. The half-eagle, half-dollar, &c., weigh half as much as the eagle, dollar, &c., respectively.

The use of the alloy in gold and silver coins, is, to produce greater hardness in the metal, and thus render them less liable to wear in being often handled.

Reduction of Currencies.

§ **185.** The *Federal Currency* of the United States was established by Congress in 1786. At that time, the currency consisting in the bills or notes of the individual States, in the denominations £, *s.*, &c., had fallen below its nominal value: and in different degrees in different States.

Hence the *pound*, *shilling*, &c., in those depreciated currencies, came to be of different values in Federal money, in different States; and those values have continued in use, to some extent, up to the present time.

Recollecting then that the difference in value belongs to the *shilling*, and not to the *dollar*, we have the following

Table of Values of a Shilling.

In New England, *Va. Ky.* and *Tenn.*	$6\,s.=\$1$,	or $1\,s.=16\frac{2}{3}$ *cts.*
In New York, *N. Car.* and Ohio,	$8\,s.=\$1$,	or $1\,s.=12\frac{1}{2}$ *cts.*
In New Jersey *Penn. Del.* and *Md.*	$7\frac{1}{2}\,s.=\$1$,	or $1\,s.=13\frac{1}{3}$ *cts.*
In South Carolina and Georgia,	$4\frac{2}{3}\,s.=\$1$,	or $1\,s.=21\frac{3}{7}$ *cts.*

In some of the new States, the shilling is valued according to the New England Currency, in others according to the New York currency, and others again adhere exclusively to Federal money.

The Pound Sterling.

§ **186.** The English pound sterling, which is represented by a gold coin called a *sovereign*, is valued by law in the United States at $4.84.

The *intrinsic* value of the sovereign, that is, the value corresponding to the quantity and purity of metal it contains, is $4.861. Its *marketable* value, or the sum for which it passes with money dealers, varies from $4.83 to $4.86.

Taking the sovereign or pound sterling at its legal value, we shall find $4\frac{16}{121}$ shillings sterling=$1, or 1 *s. ster.*=$24\frac{1}{5}$ cts.

Federal and English Money Reduced—the one to the other.

RULE XXXV.

§ **187.** (1.) *To reduce Federal to English Money.*

Multiply Federal money, expressed in the denomination of dollars, by the number of shillings in $1; the product will be *shillings*.

(2.) *To reduce English to Federal Money.*

Divide English money, expressed in the denomination of shillings, by the number of shillings in \$1 ; the quotient will be *dollars.*

EXAMPLES.

1. To reduce \$275.18$\frac{3}{4}$ to sterling money, reckoning the £ *ster.* at \$4.84.

\$1 being $4\frac{16}{121}$ *s. ster.* we have

$$\$275.1875 \times 4\tfrac{16}{121} = 1137.1384' s.\ ster.$$

Reducing to £, &c., we find 1137 *s.*=56£ 17 *s.*; and .1384'*s.* =1 *d.* 2.6' *qr.*

Hence \$275.18$\frac{3}{4}$=56£ 17 *s.* 1*d.* 2.6' *qr.*, sterling.

2. To reduce 3£ 13 *s.* 6 *d.* sterling to Federal money.

By reduction, we find 3£ 13 *s.* 6 *d.*=73.5 shillings, (§ 178), and 73.5 *s.*$\div 4\frac{16}{121}$=\$17.787.

EXERCISES.

1. Reduce 2 *s.* 3 *d.* in New England, to Federal money
Ans. \$.375
2. Reduce 5 *s.* 6 *d.* in New York, to Federal money.
Ans. \$.687'.
3. Reduce 14 *s.* 8 *d.* in Pennsylvania, to Federal money.
Ans. \$1.955'.
4. Reduce 16 *s.* 9 *d.* in Georgia, to Federal money.
Ans. \$3.589'.
5. Reduce 100£ 15 *s.* 10 *d.* sterling to Federal money, according to the legal value of the £ sterling in the United States.
Ans. \$487.828'.

EXERCISES ON CHAPTER VII.

1. What will 2 *lb.* 8 *oz.* 13 *dwt.* of silver ware amount to, at \$.31$\frac{1}{4}$ per pennyweight ?

The whole quantity must be expressed in dwt. and multiplied by \$.31$\frac{1}{4}$. *Ans.* \$204.0625.

2. What will 4 *cwt.* 3 *qr.* 19 *lb.* of hemp amount to, at \$6.87$\frac{1}{2}$ per hundred weight ?

The whole quantity must be expressed in cwt., and multiplied by \$6.87$\frac{1}{2}$. *Ans* \$33.818'.

3. What will 2 *bu.* 3 *pk.* 3 *qt.* of strawberries amount to, at \$.12$\frac{1}{2}$ per quart ? *Ans.* \$11.375.

4. What will 2 *hhd.* 40 *gal.* 3 *qt.* of beer amount to, if retailed at \$.03 per pint ? *Ans.* \$35.70.

5. What will 3 *bbl.* 16 *gal.* 3 *qt.* of brandy amount to, if retailed at \$.06¼ per gill? *Ans.* \$222.5.

6. What will 10 *T.* 13 *cwt.* 2 *qr.* 23 *lb.* of coal amount to, at \$5.37½ per ton? *Ans.* \$57.431'.

7. How many pounds of bread may be bought for \$7.84¼, at the rate of 2½ cents per ounce? *Ans.* 19.606' *lb.*

8. How many barrels of ale may be purchased for \$95.87½, at the rate of 15 cents per quart? *Ans.* 4.438' *bar.*

9. How many pipes of wine may be purchased for \$450.50, at the rate of 18¾ cents per pint? *Ans.* 2.383' *pipes.*

10. Required the sum that should be paid for 1 *m.* 3 *fur.* 20 *r.* of fencing, at \$.75 per rod. *Ans.* \$345.

11. Required the sum that should be paid for 13 *yd.* 1 *qr.* 3 *na.* of gold lace, at \$.93¾ per *yd.* *Ans.* \$12.597'.

12. Required the sum that should be paid for 3 *A.* 2 *R.* 20 *P.* of meadow land, at \$25 per acre. *Ans.* \$90.625.

13. Required the sum that should be paid for excavating 100 *cu. yd.* 500 *cu. ft.* of earth, at \$.91¼ per *yd.* *Ans.* \$108.147'.

14. Required the sum that should be paid a laborer for 3 years 7½ months' wages, at \$125 per annum.
Ans. \$453.125.

15. A townsman bought a lot containing 1000¾ *sq. yd.* of ground, at \$100 per acre; what sum did he pay for it?
Ans. \$20.67'.

16. A merchant invested the profits of five years' business, amounting to \$7349.31¼, in land at \$24.12½ per acre. How much land did he purchase? *Ans.* 304 *A.* 2 *R.* 21.44 *P.*

17. An iron-monger bought a quantity of iron at \$45 per ton, and sold 13 *cwt.* 2 *qr.* 15 *lb.* of the same, at \$62½ per ton. What profit did he make on the quantity sold?
Ans. \$11.928'.

18. How many barrels of water will fill a cistern, the capacity of which is 13000 cubic feet? *Ans.* 2212.765' *bar.*

19. How many bushels of wheat will fill a granary, the capacity of which is 4360 cubic feet? *Ans.* 3503.571' *bu.*

20. A merchant bought in New York 135 *yd.* 3 *qr.* of linen, at 2 *s.* 3 *d.* per *yard,* and 74 *yd.* 2½ *qr.* of silk at 6 *s.* 9 *d.* per yard. Required the whole amount in Federal money.
Ans. \$101.144'.

21. A farmer sold in Philadelphia 400 *bu.* 3 *pk.* of wheat, at 6 *s.* 11 *d.* per bushel, and 175¼ *bu.* of oats, at 3 shillings per bushel. Required the whole amount in Federal money.
Ans. \$439.644'.

CHAPTER VIII.

ADDITION, SUBTRACTION, &C., OF MONOMIALS AND POLYNOMIALS. DUODECIMALS.

ADDITION, &C., OF MONOMIALS.

§ **188.** *Dissimilar* Monomials in the *same kind of measure*, may be reduced to similar monomials, and then added or subtracted.

Thus, to add together $\frac{3}{4}$£ and $\frac{2}{3}$ *s.*

$\frac{3}{4}$£=15 *s.*=180 *d.*; and $\frac{2}{3}$ *s.*=8 *d.* Then 180 *d.*+8 *d.*=188 *d.*

RULE XXXVI.

§ **189.** *To add or subtract* DISSIMILAR MONOMINALS in the same *kind of measure.*

Express the given quantities in the *same denomination*, by means of reduction, and then add or subtract.

EXAMPLE.

To find the sum of 3£, $\frac{1}{3}$ *s.* and 8$\frac{1}{2}$ *d.* in the denomination of *pence.*

By reduction, 3£=60 *s.*=720 *d.*; and $\frac{1}{3}$ *s.*=4 *d.*;

then 720 *d.*+4 *d.*+8$\frac{1}{2}$ *d.*+732$\frac{1}{2}$ *d.*

EXERCISES.

1. Find the sum of $\frac{1}{4}$ *T.*, 2 *cwt.*, and $\frac{1}{2}$ *qr.*, in *lb.* *Ans.* 798 *lb.*
2. Find the difference between 3 *s.* and 9$\frac{1}{4}$ *d.*, in *d.* *Ans.* 26$\frac{3}{4}$ *d.*
3. Find the sum of 10 *bu.*, 3$\frac{1}{2}$ *pk.*, and 2 *qt.*, in *bu.* *Ans.* 10$\frac{15}{16}$ *bu.*
4. Find the difference between 3 *hhd.* and 4$\frac{1}{2}$ *gal.*, in *qt.* *Ans.* 738 *qt*
5. Find the sum of $\frac{1}{4}$ *m.*, $\frac{3}{4}$ *fur.*, and 20 *r.*, in *m.* *Ans.* $\frac{13}{32}$ *m.*
6. Find the difference between 3 *A.* and 30$\frac{1}{2}$ *P.*, in *P.* *Ans.* 449$\frac{1}{2}$ *P*
7. Find the sum of .4 *lb.*, 3 *oz.*, and .5 *dwt.*, in *dwt.* *Ans.* 156.5 *dwt.*
8. Find the difference between .3 *T.* and 7.3 *cwt.*, in *cwt.* *Ans.* 1.3 *cwt*
9. Find the sum of 3.7 *bu.*, 3 *pk.*, and 4 *qt.*, in *bu.* *Ans.* 4.575 *bu.*
10. Find the difference between .45 *m.* and .3 *fur.*, in *m.* *Ans.* .4125 *m.*

11. Find the sum of .2 *A.*, 3.1 *R.*, and 4 *P.*, in *sq. yd.* *Ans.* 4840 *sq. yd.*

12. Find the difference between .75 *A.* and 1 *R.*, in *sq. yd.* *Ans.* 2420 *sq. yd.*

Multiplication and Division of Monomials.

§ **190.** In multiplication, the multiplier can be considered only as expressing *repetitions* of the multiplicand, or a *part* of the multiplicand.

§ **191.** In Division, when the answer to be found is the number of times the dividend *contains the divisor*, or the *part* the dividend is of the divisor, these two terms must be taken in the *same denomination* of units.

For example, to find how many times 15 *s.* is contained in 3£.

Reducing, we find 3£=60 *s.*; then 15 *s.* in 60 *s.*, 4 times.

Or, 15 *s.*$=\frac{15}{20}$£$=\frac{3}{4}$£ ; and $\frac{3}{4}$£ in 3£, 4 times.

13. How many yards of silk, at 5 shillings per yard, can be purchased for $9\frac{1}{2}$ £ ? *Ans.* 38 *yd.*

14. How many pounds of butter, at 9 pence per pound, can be purchased for $21\frac{1}{4}$ shillings ? *Ans.* $28\frac{1}{3}$ *lb.*

15. A person having a lot of ground which contained $1\frac{1}{4}$ acres, sold 39 *P.* of it to his neighbor. What part of the lot did he sell ? *Ans.* .195 of it.

16. A laborer who had 25 rods of ditching to execute, has accomplished $51\frac{1}{2}$ yards of it. What part of the whole work has he accomplished ? *Ans.* .374' of it.

17. A farmer sowed $57\frac{1}{4}$ bushels of rye in a field, and found that it was at the rate of $3\frac{1}{2}$ *pk.* per acre. How many acres did he sow ? *Ans.* $65\frac{3}{7}$ *A.*

18. An agriculturist bought, at one time, 2 *T.* of plaster, and at another, $15\frac{1}{2}$ *cwt.* How many acres of meadow can he sow with the whole, at the rate of 100 *lb.* per acre ?

Ans. $62\frac{4}{25}$ *A.*

19. A wine merchant has 3 *tuns* and 1 *pipe* of wine, which he wishes to put into barrels of $31\frac{1}{2}$ *gal.* each. How many barrels will be required for the whole ? *Ans.* 28 *bbl.*

20. Bought at different times, in adjoining parcels, 3 *A.*, $3\frac{1}{2}$ *R.*, and 20 *P.* of ground. I wish to divide the whole into lots of 40 *P.* each ; how many lots will there be ?

Ans. 16 *lots.*

ADDITION OF POLYNOMIALS.

§ 192. Two or more polynomials in the *same kind of measure*, might be reduced to similar monomials, and, under the monomial form, might be added or subtracted.

In like manner, a polynomial might be multiplied or divided.

But polynomials may be added, &c., under the polynomial form,—such operations being usually called *Compound Addition, Compound Subtraction*, &c.

RULE XXXVII.

§ 193. *For Polynomial or Compound Addition.*

1. Set the polynomials with similar terms one under another, in separate columns.

2. Proceeding from right to left, add up each column of similar terms, and under each set its amount, if less than a unit of the next higher denomination.

3. *If not less than such unit*, divide the amount of the column by that number of its own denomination which makes a unit of the next higher; set the *remainder*, if any, under the column, and add the *quotient* to the next column of similar terms.

EXAMPLE.

To add together 13£ 7 *s*. 2 *d*., 49£ 18 *s*. 4 *d*., and 84£ 9 *s*. 5 *d*.

£	*s.*	*d.*
13	7	2
49	18	4
84	9	5
147	14	11

Having set *pounds* under *pounds*, shillings under shillings, &c., we add up the column of *d*., and set down the amount 11 *d*., since it is less than 1 *s*., a unit of the next higher denomination.

Adding up the column of *s*., we find the amount to be 34 *s*., which, being more than 1£, we divide by 20, since 20*s*.=1£.

Setting down the remainder 14 *s*. we add the quotient to the column of £, and find the whole sum to be

147£ 14 *s*. 11 *d*.

By this Rule, an amount in a lower denomination is reduced,

when practicable, to units of the next higher denomination; and all units of the same denominaton are collected into one sum. (§ 23).

EXERCISES.

1. Add together 125£, 13 *s.* 5 *d.*, 19£, 4 *s.* 10 *d.* 2 *qr.*, and 12£, 16 *s.* 8 *d.* 3 *qr.* *Ans.* 157£, 15 *s.* 0 *d.* 1 *qr.*

2. Add together 23 *lb.* 8 *oz.* 16 *dwt.*, 36 *lb.* 5 *oz.* 8 *dwt.* 16 *gr.*, and 300 *lb.* 2 *oz.* 9 *dwt.* 13 *gr.* *Ans.* 360 *lb.* 4 *oz.* 14 *dwt.* 5 *gr.*

3. Add together 3 *T.* 9 *cwt.* 2 *qr.* 16 *lb.*, 10 *T.* 15 *cwt.* 1 *qr.*, and 54 *T.* 7 *cwt.* 3 *qr.* 20 *lb.* *Ans.* 68 *T.* 12 *cwt.* 3 *qr.* 8 *lb.*

4. Add together 56 ℔, 7 ℥, 5 ʒ, 2 ℈, 13 *gr.*, 25 ℔, 6 ℥, 7 ʒ, and 5 ℔, 10 ℥, 3 ʒ, 1 ℈, 19 *gr.*
Ans. 88 ℔, 1 ℥, 0 ʒ, 1 ℈, 12 *gr.*

5. Add together 13 *bu.* 2 *pk.* 7 *qt.* 1 *pt.*, 150 *bu.* 1 *pk.* 5 *qt.*, and 200 *bu.* 3 *pk.* 5 *qt.* 1 *pt.* *Ans.* 365 *bu.* 0 *pk.* 2 *qt.*

6. Add together 3 *hhd.* 20 *gal.* 3 *qt.*, 29 *hhd.* 13 *gal.* 2 *qt.*, and 200 *hhd.* 12 *gal.* 1 *qt.* *Ans.* 232 *hhd.* 46 *gal.* 2 *qt.*

7. Add together 4 *m.* 5 *fur.* 20 *p.*, 29 *m.* 3 *fur.* 16 *p.* 4 *yd.*, and 34 *m.* 7 *fur.* 13 *p.* 1 *yd.* *Ans.* 69 *m.* 0 *fur.* 9 *p.* 5 *yd.*

8. Add together 15 *yd.* 3 *qr.* 1 *na.*, 75 *yd.* 3 *qr.* 3 *na.* 1 *in.*, and 100 *yd.* 1 *qr.* 2 *na.* 1 *in.* *Ans.* 192 *yd.* 0 *qr.* 2 *na.* 2 *in.*

9. Add together 24 *A.* 3 *R.* 20 *P.*, 100 *A.* 2 *R.* 16 *P.* 4 *sq. yd.*, and 95 *A.* 1 *R.* 29 *P.* 20 *sq. yd.*
Ans. 220 *A.* 3 *R.* 25 *P.* 24 *sq. yd.*

10. Add together 200 *A.* 1 *R.* 24 *P.* 20 *sq. yd.*, 50 *A.* 2 *R.*, and 500 *A.* 3 *R.* 19 *P.* 16 *sq. yd.*
Ans. 751 *A.* 3 *R.* 4 *P.* 5¾ *sq. yd.*

11. A farmer raised from one field 150 *bu.* 3 *pk.* of wheat, from another 75 *bu.* 1 *pk.* 7 *qt.*, and from another 200 *bu.* 5 *qt.* What quantity did he raise in all? *Ans.* 426 *bu.* 1 *pk.* 4 *qt.*

12. A merchant has in one piece 34 *yd.* 3 *qr.* of cloth, in another 21 *yd.* 2 *qr.*, and in two others each 19 *yd.* 3½ *qr.* How many yards has he in the four pieces? *Ans.* 96 *yd.*

13. An agriculturist sold at one time 3 *T.* 19 *cwt.* 2 *qr.* of hemp, at another 5 *T.* 13 *cwt.*, and at another 2 *T.* 16 *cwt.* 3 *qr.* 20 *lb.* What amount did he sell?
Ans. 12 *T.* 9 *cwt.* 1 *qr.* 20 *lb.*

SUBTRACTION OF POLYNOMIALS.

RULE XXXVIII.

§ **194.** *For Polynomial or Compound Subtraction.*

1. Set the less polynomial under the greater, with similar terms one under the other.

2. Proceeding from right to left, subtract each lower term from the one above it, and underneath set the remainder.

3. *If the lower term exceed the upper*, add to the upper term that number of its *own denomination* which makes a *unit* of the next higher; from the sum subtract the lower term, and add 1 to the next lower term, before subtracting it.

EXAMPLES.

1. To subtract 85£, 13 *s.* 7 *d.* from 250£, 9 *s.* 10 *d.*

£,	*s.*	*d.*
250	9	10
85	13	7
164	16	3

Having set the less polynomial under the greater, with *pounds* under *pounds*, &c., we subtract 7*d.* from 10*d.*, and set down the remainder 3 *d.*

The next lower term 13 *s.* being greater than the upper term 9 *s.*, we add 20 *s.* to 9 *s.*, since 20 *s.*=1£. From the sum 29 *s.* we subtract 13 *s.*, and set down the remainder 16 *s.*

Then adding 1 to the 5, we say 6 from 10 leaves 4, &c.

2. To subtract 45£, 7 *s.* 3 *d.* from 150£.

£,	*s.*	*d.*
150	0	0
45	7	3
104	12	9

As we cannot take 3 *d.* from 0 *d.*, we add 12 *d.*=1 *s.*, and

say 3 *d.* from 12 *d.* leaves 9 *d.* Then 1 to 7 *s.* makes 8 *s.*, and since we cannot take 8 *s.* from 0 *s.*, we add 20, and say 8 *s.* from 20 *s.* leaves 12 *s.* Then 1 to 5 makes 6, and 6 from 10 leaves 4, &c.

The difference between two polynomials is not changed by adding, to any upper term, a quantity equal to the unit added to the next lower term.

EXERCISES.

1. From 60£, 17 *s.* subtract 35£, 13 *s.* 6 *d.*
   Ans. 25£, 3 *s.* 6 *d.*
2. From 200 *lb.* 9 *oz.* 1 *dwt.* subtract 180 *lb.* 10 *oz.*
   Ans. 19 *lb.* 11 *oz.* 1 *dwt.*
3. From 150 *T.* 13 *cwt.* subtract 75 *T.* 3 *cwt.* 1 *qr.*
   Ans. 75 *T.* 9 *cwt.* 3 *qr*
4. From 433 ℔, 3 ℥, 2 ʒ, subtract 93 ℔, 10 ℥.
   Ans. 339 ℔, 5 ℥, 2 ʒ
5. From 100 *bu.* 2 *pk.* subtract 21 *bu.* 1 *pk.* 1 *qt.*
   Ans. 79 *bu.* 0 *pk.* 7 *qt*
6. From 21 *T.* 2 *hhd.* 3 *gal.* subtract 3 *T.* 13 *gal.*
   Ans. 18 *T.* 1 *hhd.* 53 *gal*
7. From 275 *L.* 2 *m.* subtract 75 *L.* 1 *m.* 5 *fur.*
   Ans. 200 *L.* 0 *m.* 3 *fur*
8. From 150 *yd.* 3 *qr.* 2 *na.* subtract 2 *qr.* 3 *na.*
   Ans. 150 *yd.* 0 *qr.* 3 *na*
9. From 123 *A.* 2 *R.* subtract 30 *A.* 3 *R.* 13 *P.*
   Ans. 92 *A.* 2 *R.* 27 *P.*
10. From 200 *A.* 3 *R.* 20 *P.* subtract 30 *A.* 50 *P.*
    Ans. 170 *A.* 2 *R.* 10 *P.*

11. A jeweler purchased 34 *lb.* 9 *oz.* 13 *dwt.* of silver ware, of which he has sold 19 *lb.* 4 *oz.* 18 *gr.* What quantity has he remaining? *Ans.* 15 *lb.* 5 *oz.* 12 *dwt.* 6 *gr.*

12. An agriculturist raised 30 *T.* 13 *cwt.* 1 *qr.* of hemp, of which he has sent to market 21 *T.* 15 *cwt.* 21 *lb.* What quantity of hemp has he still on hand?
    Ans. 8 *T.* 18 *cwt.* 0 *qr.* 7 *lb.*

13. A farmer raised 500 *bu.* 3 *pk.* 7 *qt.* of wheat. Having sold 300 *bu.* 2 *pk.* 5 *qt.* of this crop, what quantity of wheat has he still unsold? *Ans.* 200 *bu.* 1 *pk.* 2 *qt.*

14. A vintner purchased 3 *T.* 1 *hhd.* 40 *gal.* 2 *qt.* of wine, of which he has sold 1 *T.* 3 *hhd.* 47 *gal.* 1 *pt.* What quantity of wine has he yet unsold? *Ans.* 1 *T.* 1 *hhd.* 56 *gal.* 1 *qt.* 1 *pt.*

15. A merchant bought a bale of cotton containing 400 *yd.* 3 *qr*. Having sold 139 *yd.* 3 *qr.* 2 *na.* of this purchase, how many yards of the cotton remain on hand?

Ans. 260 *yd.* 3 *qr.* 2 *na.*

16. A speculator bought a tract of land containing 960 *A.* 2 *R.* 26 *P.* Having sold from the tract to the amount of 509 *A.* 3 *R.*, how many acres of it remain unsold?

Ans. 450 *A.* 3 *R.* 26 *P.*

17. A merchant bought at one time 4 *T.* 19 *cwt.* of hemp, at another 3 *T.* 2 *qr.*, and at another 1 *T.* 13 *cwt.* 3 *qr.* 10 *lb.* Having sold at different times to the amount of 5 *T.* 10 *cwt.* 1 *qr.* 20 *lb.*, how much hemp has he still unsold?

Ans. 4 *T.* 2 *cwt.* 3 *qr.* 18 *lb.*

18. A person who undertook a journey of 900 miles, traveled the first day 39 *m.* 3 *fur.*, the second 40 *m.* 7 *fur.*, the third and fourth each 43 *m.* 6 *fur.* How many miles of his journey then remained to be traveled? *Ans.* 732 *m.* 2 *fur.*

19. A manufacturer put into one bale 335 *yd.* 3 *qr.* of cotton, into another 400 *yd.*, and into two others each 421 *yd.* 1 *qr.* having sold to one person 100 *yd.* 2 *qr.* from the first bale, and 200 *yd.* from each of the others, how many yards remain?

Ans. 877 *yd.* 3 *qr.*

20. A gentleman's fortune is estimated at 10000£ sterling. If he give to each of his three sons 2000£, 15 *s.*, and to his only daughter the remainder, what will be the daughter's portion? *Ans.* 3997£, 15 *s.*

21. A planter has one tract of land containing 3000 *A.*, and two others containing each 1500 *A.* 1 *R.* 16 *P.* If he sell from the first tract 400 *A.* 2 *R.*, and from the other two together 305 *A.* 3 *R.* 25 *P.*, how many acres will remain to him?

Ans. 5294 *A.* 1 *R.* 7 *P.*

22. A miller bought at one time 200 *bu.* 3 *pk.* of wheat, at another 313 *bu.* 1 *pk.*, and at another 194 *bu.* Having manufactured 405 *bu.* 1 *pk.* of these purchases into flour, what quantity of wheat has he still on hand? *Ans.* 302 *bu.* 3 *pk.*

23. A grocer bought from one distillery 34 *gal.* 3 *qt.* 1 *pt.* of brandy, from another 40 *gal.*, and from another 31 *gal.* 1 *qt.* Having sold to the amount of 50 *gal.* 1 *qt.* 1 *pt.*, what quantity remains unsold? *Ans.* 55 *gal.* 3 *qt.*

24. Farmer A has 300 *A.* 1 *R.* 40 *P.* of land; B has 100 *A.* 2 *R.* 13 *P.* more than A, and C has 39 *A.* 10 *P.* more than B, while D has 75 *A.* 2 *R.* 20 *P.* less than the other three together. How much land have B, C, and D, respectively?

Ans. B, 401 *A.* 13 *P.*; C, 440 *A.* 23 *P.*; D, 1066 *A.* 16 *P.*

Interval of Time between two given Dates.

§ **195.** *In subtracting a prior from a later date,* add to the number of days elapsed in the month of the later date (when requisite,) as many as make the month *of the prior date,* and allow 12 months to a year.

EXAMPLE.

To find the interval between the 20th day of March, 1823, and the 10th day of April, 1848.

y.	*m.*	*da.*
1848	4	10
1823	3	20
25	0	21

March being the 3*d*, and April the 4*th* month in the year, we designate them by these numbers, respectively.

Since March has 31 days, 11 days of it remained after the 20th. Adding these 11 *da.*, to the 10 *da.* of April, we have 21 *da.*; and it is plain that whatever number of months might intervene between the months of *the two given dates,* the surplus days would be found in a similar manner.

But $31-20+10=10+31-20$; hence the 21 *da.* (and the surplus days in every case) will be found, most readily, by the direction above given. (§ 195).

§ **196.** The interval of time, found as above, will include the particular day of the later date, but not that of the prior one.

Thus in the Example given, the 10th of April is included in the interval 25 *y.* 21 *da.*, but not the 20th of March,—as is obvious from the explanation.

25. Find the interval of time between May 16th, 1834, and September 4th, 1848. *Ans.* 14 *y.* 3 *m.* 19 *da.*

26. A person was born on the 3d of April, 1807: required his age on the 15th of December, 1848.
Ans. 41 *y.* 8 *m.* 12 *da.*

27. How long was it from the discovery of America, October 21st, 1492, to the founding of Jamestown, May 23d, 1607?
Ans. 114 *y.* 7 *m.* 2 *da.*

28. How long was the founding of Jamestown prior to the birth of Washington, February 22d, 1732; and what was Washington's age at his death, December 14th, 1799?
Ans 124 *y.* 8 *m.* 30 *da.*, and 67 *y.* 9 *m.* 21 *da.*

MULTIPLICATION OF POLYNOMIALS

RULE XXXIX.

§ 197. *For Polynomial or Compound Multiplication.*

1. Proceeding from right to left, multiply each term of the polynomial separately, and under each set its product, if less than a unit of the next higher denomination.

2. *If not less than such unit*, divide the product by that number of its own denomination which makes a unit of the next higher; set the remainder, if any, under the term, and add the quotient to the product of the next term.

EXAMPLE.

To multiply 25£, 16 *s.* 3 *d.* by 3.

£,	*s.*	*d.*
25	16	3
		3
77	8	9

Proceeding from right to left, we say 3 times 3 *d.* is 9 *d.*; 3 times 16 *s.* is 48 *s.*, which, being more than 1£, we divide by 20, since 20 *s.* make 1£. Setting down the remainder 8 *s.*, we add the quotient 2 to the product of the next term; 3 times 25 is 75, and 2 makes 77.

The product is thus found to be 77£, 8 *s.* 9 *d.*

This Rule depends on the same principles as the Rule for polynomial or compound addition.

EXERCISES.

1. What should be paid for 4 yards of broadcloth, at 1£, 3 *s.* 8 *d.* per yard? *Ans.* 4£, 14 *s.* 8 *d.*

2. Required the aggregate weight of 5 silver goblets, each weighing 1 *lb.* 9 *oz.* 13 *dwt.*? *Ans.* 9 *lb.* 5 *dwt.*

3. Bought 6 loads of hay, each of which weighed 19 *cwt.* 3 *qr.* 23 *lb.* Required their entire weight.
Ans. 119 *cwt.* 2 *qr.* 26 *lb.*

4. An apothecary sold 7 bottles of quinine, each weighing 12 *oz.* 13 *dr.* What was the weight of the whole?
Ans. 5 *lb.* 9 *oz.* 11. *dr.*

5. A farmer filled 23 sacks of corn, each containing 5 *bu.* 3 *pk.* 1 *qt.* What quantity did they all contain?

Ans. 132 *bu.* 3 *pk.* 7 *qt.*

6. A brewer sold to each one of 9 men 18 *gal.* 3 *qt.* 1 *pt.* of beer. What quantity did he sell in all?

Ans. 169 *gal.* 3 *qt.* 1 *pt.*

7. A vintner bought of 10 persons each 3 *hhd.* 24 *gal.* 2 *qt.* of wine. How much did he buy from them all?

Ans. 33 *hhd.* 56 *gal.*

8. If a man travel at the rate of 33 *m.* 7 *fur.* 30 *r.* per day, what number of miles will he travel in 11 *da.*?

Ans. 373 *m.* 5 *fur.* 10 *r.*

9. A merchant sold 19 pieces of linen, each piece containing 16 *yd.* 3 *qr.* 2 *na.* How many yards did he sell?

Ans. 320 *yd.* 2 *qr.* 2 *na.*

10. A farmer has 13 fields, each containing 24 *A.* 3 *R.* 10 *P.* What quantity of land do all the fields contain?

Ans. 322 *A.* 2 *R.* 10 *P.*

11. If a steamboat run at the rate of 12 *m.* 3 *fur.* 19 *r.* per hour, what distance will it run in 14 hours? *Ans.* 174 *m.* 26 *r.*

12. If a ship sail at the rate of 30 *L.* 2 *m.* 4 *fur.* 20 *r.* per day, what distance will she sail in 15 *da.*?

Ans. 462 *L.* 2 *m.* 3 *fur.* 20 *r.*

13. A teamster hauled 16 loads of coal, averaging 2 *T.* 17 *cwt.* 1 *qr.* each. What was the entire weight?

Ans. 45 *T.* 16 *cwt.*

14. A manufacturer made 17 pieces of cloth, measuring 39 *yd.* 3 *qr.* each. How much cloth was there in all?

Ans. 675 *yd.* 3 *qr.*

15. An astronomical year being 365 *da.* 5 *hr.* 48 *min.* 51 *sec.*, required the number of days in 20 such years.

Ans. 7304.84' *days.*

16. An agriculturist had 15 acres of ground in hemp, and found his crop to be at the rate of 17 *cwt.* 3 *qr.* 10 *lb.* per acre; what was the entire crop? *Ans.* 13 *T.* 7 *cwt.* 2 *qr.* 10 *lb.*

17. A brewer filled 3 hogsheads with beer, out of which he has sold to the amount of 75 *gal.* 3 *qt.* 1 *pt.* What quantity remains of the 3 *hhd.*? *Ans.* 1 *hhd.* 32 *gal.* 0 *qt.* 1 *pt.*

18. A merchant bought 7 pieces of silk containing, on an average, 47 *yd.* 2 *qr.* each. Having sold to one lady 11 *yd.*, and to three others each 10 *yd.* 3 *qr.*, how many yards of the silk remain on hand? *Ans.* 289 *yd.* 1 *qr.*

19. A bought of B 100 *A.* 2 *R.* 21 *P.* of land, of C 5 times as much as from B, wanting 10 *A.*, and from D as much as from C, wanting 3 *A.* 2 *R.* 30 *P.* What was the amount of his purchase? *Ans.* 1083 *A.* 1 *R.* 1 *P.*

DIVISION OF POLYNOMIALS.

RULE XL.

§ 198. *For Polynomial or Compound Division.*

1. *When the divisor is an abstract number, and the dividend a polynomial.*—Proceeding from left to right, divide each term of the polynomial, for the corresponding term of the quotient.

2. *When a remainder occurs,* reduce it to the next lower denomination, add it to the term in that denomination, if any, and divide the result for the quotient term in the same denomination.

3. *When the divisor and dividend are both polynomials.*—Reduce them both to monomials of the same denomination, and find the quotient in an abstract number.

EXAMPLE.

To divide 285£, 17*s.* 5*d.* by 3; that is to find $\frac{1}{3}$ of this polynomial.

	£,	*s.*	*d.*	
3)	285	17	5	
	95	5	9	$2\frac{2}{3}$ *qr.*

Proceeding from left to right, we find 3 in 285, 95 times; 3 in 17, 5 times, with 2*s.* over; reducing the 2*s.* to *pence,* and adding the 5*d.*, we have 29*d.*; then 3 in 29, 9 times, with 2*d.* over; reducing the 2*d.* to *qr.*, we have 8*qr.*; 3 in 8, gives $2\frac{2}{3}$ *qr.*

The quotient is thus found to be 95£, 5*s.* 9*d.* $2\frac{2}{3}$ *qr.*

The operation evidently finds $\frac{1}{3}$ of the given polynomial.

EXERCISES.

1. If 4 yards of cloth sell for 9£, 17*s.* 8*d.*, what is the price per yard? *Ans.* 2£, 9*s.* 5*d.*

2. If 5 silver candlesticks weigh 10*lb.* 7*oz.* 18*dwt.*, what is the average weight of each? *Ans.* 2*lb.* 1*oz.* $11\frac{3}{5}$ *dwt.*

3. If 6 barrels of pork weigh 12*cwt.* 2*qr.* 23*lb.*, what is the average weight of each? *Ans.* 2*cwt.* 0*qr.* $13\frac{1}{6}$ *lb.*

4. If 7 acres of ground produce 150*bu.* 2*pk.* 1*qt.* of wheat, what is the produce per acre? *Ans.* 21*bu.* 2*pk.* $\frac{1}{7}$ *qt.*

5. If 8 casks together contain 250*gal.* 3*qt.* 1*pt.* of spirits, what are the average contents of each? *Ans.* 31 gal. 1 qt. $3\frac{1}{8}$ gi.

6. If a person travel 300 *m.* 2 *fur.* 25 *p.* in 9 days, at what rate will he travel per day? *Ans.* 33 *m.* 2 *fur.* $38\frac{1}{3}$ *p.*

7. A merchant has 10 pieces of cloth, of equal length, and together containing 575 *yd.* 2 *qr.* 3 *na.* What is the length of each piece? *Ans.* 57 *yd.* 2 *qr.* $1\frac{1}{10}$ *na.*

8. A farmer having a tract of land containing 486 *A.* 2 *R.* 30 *P.*, wishes to divide it into 12 fields of equal size. What quantity must there be in each field? *Ans.* 40 *A.* 2 *R.* $9\frac{1}{6}$ *P.*

9. A cellar measuring 1570 *cu. yd.* 18 *cu. ft.* was excavated by a man in 30 days. At what rate did he dig per day? *Ans.* 52 *cu. yd.* $9\frac{3}{5}$ *cu. ft.*

10. A manufacturer sold 13 pieces of cotton cloth, measuring in the aggregate 500 *yd.* 3 *qr.* Required the average length of each piece. *Ans.* 38 *yd.* $2\frac{1}{13}$ *qr.*

When the divisor and dividend are both Polynomials.

11. How many yards of silk at 7 *s.* 6 *d.* per yard may be purchased for 3£, 14 *s.* 10 *d.*?

The number of yards is the number of times that 7 *s.* 6 *d.* is contained in 3£, 14 *s.* 10 *d.*

Applying the 2d part of the Rule, we find,

7 *s.* 6 *d.*=90 *d.*; and £3, 14 *s.* 10 *d.*=898 *d.*;

90 *d.* is contained in 898 *d.*, 9.977' times. *Ans.* 9.977' *yd.*

12. How many hundred weight of iron, at 19 *s.* 8 *d.* per *cwt.* may be bought for 20£, 15 *s.*? *Ans.* 21.101' *cwt.*

13. How many acres of ground can be sown with 75 *bu.* 1 *pk.* of wheat, allowing 1 *bu.* 3 *pk.* to an acre? *Ans.* 43 *A.*

14. In what time will a ship perform a voyage of 1000 *L.* 2 *m.*, if she sail at the rate of 60 *L.* 1 *m.* per day? *Ans.* 16.585' *da.*

15. What number of carpets, each to contain 34 *yd.* 3 *qr.*, can be made out of 2 pieces of carpeting, each measuring 50 *yd.*, and another piece measuring 49 *yd.* 2 *qr.*? *Ans.* 4.302' *carpets.*

16. A silversmith makes 6 *lb.* 7 *oz.* 4 *dwt.* of silver into spoons weighing 3 *oz.* 6 *dwt.*, each, and sells the spoons at 3 *s.* 6 *d.* apiece. What does he get for his spoons? *Ans.* 4£, 4 *s.*

17. A sugar planter makes 113 *T.* 9 *cwt.* 2 *qr.* of sugar, which is to be put into hogsheads that will contain, on an average, 12 *cwt.* 2 *qr.* How many such hogsheads will be requisite? *Ans.* 181.56 *hhd.*

18. A traveler performed a journey of 975 *m.* 7 *fur.* The first 20 days he traveled 31 *m.* 2 *fur.* per day, and during the remainder of the journey 29 *m.* 5 *fur.* per day. How long was he on the journey? *Ans.* 31.843' *da.*

DUODECIMALS,

AND THEIR APPLICATION TO SQUARE AND CUBIC MEASURE.

§ **199.** DUODECIMALS are a kind of polynomial quantities which result from supposing a *linear*, *square*, or *cubic foot*, to be divided into 12 *equal parts*, each of these parts again into 12 equal parts ; and so on.

12*ths* of a *foot* are called *primes ;*
12*ths* of a prime are called *seconds ;*
12*ths* of a second are called *thirds ;* and so on.

Primes are denoted by an *index* of *one accent;* thus, 3′, 3 *primes.*

Seconds are denoted by an index of two accents ; thus 4″, 4 seconds.

Thirds are denoted by an index of three accents ; thus, 7‴, 7 thirds, &c.,

The polynomial 8 *ft.* 2′ 5″ 7‴, for example, is 8 *feet*, 2 *primes*, 5 *seconds*, and 7 *thirds.*

How many *fourths* make one *third ?* How many thirds make 1 *second ?* How many seconds make one *prime ?* How many primes make 1 *foot ?*

Linear, Square, and Cubic Inches expressed in Duodecimals.

§ **200.** In *linear* measure, it is plain that *inches* are *primes*

Thus 3 *ft.* 4 *in.* is 3 *ft.* 4′, 3 *ft.* and 4 *primes.*

§ **201.** In *square* measure, square inches are *seconds.*

For 1 *square inch* is $\frac{1}{144}$ of a square foot, since 144 *sq. in.* = 1 *sq. ft.*; and $\frac{1}{144}$ of a *sq. ft.* is 1″ or $\frac{1}{12}$ of $\frac{1}{12}$ of a square foot.

§ **202.** In *cubic* measure, cubic inches are *thirds.*

For 1 cubic inch is $\frac{1}{1728}$ of a cubic foot, since 1728 *cu. in.* = 1 *cu. ft.*; and $\frac{1}{1728}$ of a *cu. ft.* is 1‴ or $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of a cubic foot.

In 7′, *linear* measure, how many inches ? In 9′ ? In 5′ ? In 11′ ? In 5′, *square* measure, how many *sq.* inches ? In 7′ ? In 8′ 3″ ? In 6′, *cubic* measure, how many *cu.* inches ? In 4′ ? In 7′ 2″ ?

Square and Cubic Measure—how found.

§ 203. Square measure, or measure of *surface*, is found by multiplying together *length* and *breadth*.

For example, 4 *in.* long and 3 *in.* wide, makes (4×3) 12 *sq. inches*.

§ 204. Cubic measure, or measure of *solidity*, is found by multiplying together *length*, *breadth*, and *thickness*.

For example, 4 *in.* long, 3 *in.* wide, and 2 *in.* thick, makes $(4\times3\times2)$ 24 *cubic inches*.

Product of two Duodecimal Terms.

§ 205. The product of any two terms in duodecimals, has for its *index* the sum of the *indices* of the two terms;—a term in *feet* being understood to have *no index*.

For example, take 3 *ft.* in length, and 2′ in breadth.

$$3\,ft.\times2'=3\,ft.\times\tfrac{2}{12}\,ft.=\tfrac{6}{12}\,sq.\,ft.\ (\S\ 203),=6'\,sq.\,ft.$$

Again; taking 3′ in length, and 2″ in breadth.

$$3'\times2''=\tfrac{3}{12}\,ft.\times\tfrac{2}{144}\,ft.=\tfrac{6}{1728}\,sq.\,ft.\ (\S\ 203),=6'''\,sq.\,ft.$$

In these examples, the products 6′ and 6‴ have their *indices* (′ and ‴) equal, respectively, to the *sums* of *the indices* of the two terms multiplied together.

Reduction, Addition, &c., of Duodecimals.

§ 206. Reduction, Addition, &c., are performed in Duodecimals in the same manner as in other polynomials.

We have here, however, a distinct case—that of multiplying one duodecimal polynomial by another—from the manner of performing it, sometimes called *Cross Multiplication.*

RULE XLI.

§ 207. *For Duodecimal or Cross Multiplication.*

1. Multiply each term of the multiplicand, from right to left, by each term of the multiplier, noting the denomination of each product term, by means of indices (§ 205), and setting similar terms one under another.

2. When any product term (excepting *ft.*) is 12 or more, divide it by 12; set down the remainder, if any, and add the quotient to the product of the next term.

3. Add up the similar product terms, as in polynomial addition, for the entire product.

EXAMPLE.

To find the number of square feet in a plank 16*ft.* 8*in.* long, and 2*ft.* 5*in.* wide.

ft.	′	
16	8	
2	5	
6	11′	4″
33	4′	
40 *sq. ft.*	3′	4″

Denoting *inches* or *primes*, by an index ′, and multiplying, we have $8' \times 5' = 40''$, the product 40 having an index ″ equal to the sum of the indices of the two terms 8′ and 5′, (§ 205.)

Dividing the 40″ by 12, since 12″ make 1′, we get 3′ and 4″. We set the 4″ towards the right; and then multiplying the 16*ft.*, we find 16*ft.*$\times 5' = 80'$; adding the 3′, we have 83′; dividing by 12, we get 6*ft.* and 11′, which we set in the places of *ft.* and primes, respectively.

Next, $8' \times 2$*ft.* $=16'=1$*ft.* 4′; setting the 4′ under 11′, and adding the 1*ft.* to 16*ft.* $\times$2*ft.*, we find 33*ft.*

The two rows of products are added together, for the entire product. We thus find 40 *sq. ft.* 3′ 4″. And since, in square measure, *seconds* are *square inches*, (§ 201), by reducing the 3′ to *seconds*,

we find $3' \times 12 + 4'' = 40''$ or 40 *sq. in.*

Without employing duodecimals,—

16*ft.* 8*in.* $=16\frac{8}{12}$*ft.*, and 2*ft.* 5*in.* $=2\frac{5}{12}$*ft.*;

then $16\frac{8}{12} \times 2\frac{5}{12} = 40\frac{5}{18}$ *sq. ft.* $=40$ *sq. ft.* and 40 sq. inches.

Or, 16*ft.* 8*in.* =16.666'*ft.*, and 2*ft.* 5*in.* =2.416'*ft.*;

then 16.666'×2.416'=40.265056' *sq. ft.* =40 *sq. ft.* 38.168' *sq. in.*; the number of square inches falling a little below the true number 40, by reason of the imperfect decimals .666' and .416'.

If the 16*ft.* 8*in.*, and 2*ft.* 5*in.* were reduced to *inches*, and then multiplied together, we should find the product in *square inches*, which, divided by 144, would be reduced to *square feet.*

The measure of a surface, as expressed in *sq. in.*, *sq. ft.*, &c., is called its *area.* Thus the area of the plank in the preceding example, is 40 *sq. ft.* 3′ 4″, or 40 *sq. ft.* 40 *sq. in.*

EXERCISES.

1. How many square feet are there in a pavement 30 *ft.* 10 *in.* long, and 7 *ft.* 5 *in.* wide? *Ans.* 228 *sq. ft.* 8′ 2″.

2. How many square feet of plank will make a close fence 80 *ft.* 8 *in.* long, and 6 *ft.* 4 *in.* high? *Ans.* 510 *sq. ft.* 10′ 8″.

3. How many square feet, and also how many square yards are in a ceiling 18 *ft.* 5 *in.* long, and 12 *ft.* 10 *in.* wide?
Ans. 236 *sq. ft.* 50 *sq. in.*=26 *sq. yd.* 2 *ft.* 50 *in.*

4. How many square yards are contained in a floor which measures 25 *ft.* in length, and 16 *ft.* 7 *in.* in breadth?
Ans. 46 *sq. yd.* 84 *sq. in.*

5. How many square yards of plastering would be required for one side of a wall which is 50 *ft.* 6 *in.* in length, and 20 *ft.* 4 *in.* in height? *Ans.* 114 *sq. yd.* 120 *sq. in.*

6. Required the number of *cubic feet* in a piece of timber 9 *ft.* 10 *in.* long, 3 *ft.* 4 *in.* wide, and 2 *ft.* 6 *in.* thick.

Multiplying the length and breadth together, we get the product 32 *sq. ft.* 9′ 4″.

Multiplying this product by the thickness, we get the product 81 *cu. ft.* 11′ 4″ for the solidity.

Since, in cubic measure, *thirds* are *cubic inches*, by reducing the 11′ 4″ to *thirds*, 11′×12+4″=136″, and 136″×12=1632‴=1632 *cu. in.*

Without employing duodecimals, the given dimensions might be taken in *feet* and *fractions* of a foot. Or the dimensions might all be reduced to *inches*, and the final product divided by 1728, since 1728 *cu. in.* make 1 *cu. ft.*

7. How many cubic feet are there in a hewn log 22 *ft.* 8 *in.* long, 1 *ft.* 10 *in.* wide, and 1 *ft.* 2 *in.* thick?
Ans. 48 *cu. ft.* 5′ 9″ 4‴.

8. How many cubic feet are there in a piece of scantling 15 *ft.* long, 1 *ft.* 2 *in.* wide, and 8 inches thick?
Ans. 11 *cu. ft.* 8′.

9. How many cubic feet were dug from a cellar which measures 42 *ft.* 10 *in.* long, 12 *ft.* 6 *in.* wide, and 8 feet deep? How many cubic yards?
Ans. 4283 *cu. ft.* 4′=158 *cu. yd.* 17 *cu. ft.* 4′.

10. It is required to find how many cubic yards of earth were excavated from a ditch which measures 100 *ft.* in length, 4 *ft.* 8 *in.* in breadth, and 3 *ft.* in depth.
Ans. 51 *cu. yd.* 23 *cu. ft.*

EXERCISES ON CHAPTER VIII.

1. A farmer sold at one time $2\frac{1}{4}$ *cwt.*, at another $3\frac{1}{2}$ *qr.*, and at another $18\frac{3}{4}$ *cwt.* of hemp. Find the entire quantity that he sold, in *cwt.* *Ans.* $21\frac{7}{8}$ *cwt.*

2. A grocer bought, at different times, $3\frac{1}{5}$ *cwt.*, $2\frac{1}{4}$ *qr.* and 49 *lb.* of soap; of which he has sold 2 *cwt.* 1 *qr.* Find the quantity remaining, in *lb.* *Ans.* $218\frac{2}{5}$ *lb.*

3. A merchant had 3 pieces of cloth, containing each 29 *yd.* 3 *qr.*; of which he has sold, to different persons, $5\frac{1}{8}$ *yd.*, $3\frac{1}{2}$ *qr.*, and 10 *yd.* $1\frac{1}{2}$ *qr.* Find the remainder in *yd.* *Ans.* $72\frac{7}{8}$ *yd.*

4. A brewer bought of A 13 *bu.* 3 *pk.*, of B $5\frac{3}{4}$ *bu.*, and of C 11 *bu.* 1 *pk.* $1\frac{1}{2}$ *qt.* of barley. Having consumed $9\frac{1}{4}$ *bu.*, $3\frac{1}{2}$ *pk.*, and $5\frac{3}{4}$ *bu.*, how many *bu.* has he on hand? *Ans.* $14\frac{59}{64}$ *bu.*

5. How many yards of cloth, at 30 *s.* per yard, may be purchased for 9£; and what quantity of iron, at 6£ per ton, may be purchased for 40 *s.*? *Ans.* 6 *yd.*; and $\frac{1}{3}$ of a ton.

6. How many days will a ship be in sailing 750 *L.*, at the rate of $10\frac{1}{2}$ miles per hour; and to what part of a common year will the time required be equivalent?

Ans. $8\frac{13}{14}$ *da.*; $\frac{25}{1022}$ of a *y.*

7. A wine merchant has in one cask 74 *gal.* 3 *qt.*, and in two others each 60 *gal.* 2 *qt.* 1 *pt.*, of wine. What quantity has he altogether? *Ans.* 196 *gal.*

8. A traveler who set out on a journey of 600 *m.* 5 *fur.*, proceeded for 12 days at the rate of 34 *m.* 2 *fur.* 20 *rd.* per day. What distance remained to be traveled? *Ans.* 188 *m.* 7 *fur.*

9. A merchant had 5 pieces of cotton, containing 33 *yd.* 3 *qr.* each; which he sold in equal portions to ten customers. What quantity was bought by each customer?

Ans. 16 *yd.* $3\frac{1}{2}$ *qr.*

10. A planter has a tract of land containing 1250 *A.* 3 *R.* 25 *P.*, and an adjoining one containing 2400 *A.* 1 *R.* 39 *P.* If the whole be divided into 10 plantations, what will be the average size of each? *Ans.* 365 *A.* 0 *R.* $22\frac{2}{5}$ *P.*

11. A stage coach is running at the rate of 7 *m.* 3 *fur.* 30 *rd.* per hour. How long will it be in running 75 *m.* 7 *fur.*?

Ans. 10.158' *hr.*

12. An agriculturist raised 500 *bu.* 3 *pk.* 4 *qt.* of oats from one field, and he found the produce to be at the rate of 29 *bu.* 1 *pk.* 1 *qt.* per acre. How many acres did the field contain?

Ans. 17.105' *A.*

13. From a tract of land containing 560 *A.* 2 *R.* 20 *P.*, a farmer sold to one of his neighbors 7 *A.* 3 *R.*, and to another 100 *A.* 25 *P.* What part of the tract did he sell to each?

Ans. .0138'; and .178'.

14. How many yards of carpeting, one yard in width, will be required to cover the floor of a room measuring 19 *ft.* 5 *in.* long, and 10 *ft.* 6 *in.* wide? *Ans.* 22.652' *yd.*

15. How many cords of wood are there in a pile which is 9 *ft.* 10 *in.* in length, 4 *ft.* in breadth, and 3 *ft.* 11 *in.* in height,—there being 128 cubic feet in a cord? *Ans.* 2.427' cords.

16. How many cords of wood in five loads each 8 *ft.* 10 *in.* in length, 3 *ft.* 6 *in.* wide, and 3 *ft.* 4 *in.* high?
Ans. 4.025' cords.

17. How many square yards of painting on the walls of a room measuring 11 *ft.* in height, and 52 *ft.* 6 *in.* in compass,—deducting four windows each 7 *ft.* by 5 *ft.* 2 *in.*?
Ans. 48.092' *sq. yd.*

18. Bought a load of wood which measured 6 *ft.* 4 *in.*, 3 *ft.* 3 *in.*, and 2 *ft.* 10 *in.*; another measuring 7 *ft.*, 3 *ft.* 11 *in.*, and 3 *ft.*; and another measuring 8 *ft.*, 4 *ft.*, and 3 *ft.* Required the number of cords purchased. *Ans.* 1.849' cords

REMARKS.

The last two Chapters, it may be remarked, contain but little more than an *application* of the essential principles of Arithmetic, to the various *arbitrary measures of quantities* which have been established in the commerce of society.

Facility of operation on such quantities will depend very much on a familiarity with the *Tables of Weights* and *Measures*—of which, with the several Rules in these Chapters, the student should be master, before he is permitted to advance.

CHAPTER IX.

ALIQUOT PARTS—ANALYSIS—CANCELLATION—RATIO—PROPORTION.

ALIQUOT PARTS.

§ 208. An *aliquot part* of a quantity is an exact *half, third,* or *fourth,* and so on, of the quantity.

Thus 10 *s.* is an aliquot part of 1£, being ½ of 1£.

What aliquot part is 6 *gr.* of 1 *dwt.*?	5 *dwt.* of 1 *oz.*?	Of 2 *oz.*?
What aliquot part is 7 *lb.* of 1 *qr.*?	2 *qr.* of 1 *cwt.*?	Of 3 *cwt.*?
What aliquot part is 2 *qt.* of 1 *gal.*?	1 *qt.* of 1 *pk.*?	Of 3 *pk.*?
What aliquot part is 8 *rd.* of 1 *fur.*?	2 *fur.* of 1 *m.*?	Of 4 *m.*?
What aliquot part is 10 *P.* of 1 *R.*?	2 *R.* of 1 A.?	Of 5 *A.*?

It is often convenient to regard the lower orders of units in a *polynomial multiplier*, as aliquot parts of one or more higher units.

ANALYSIS.

§ 209. Analysis, in Arithmetic, consists in determining the operations to be performed in the solution of questions, without the aid of special Rules.

EXAMPLE

Of Analysis by Aliquot Parts.

To find the value of 2 *lb.* 5 *oz.* 12 *dwt.* of silver ware, at $45.12½ per *lb.*

We first find the value of 2 *lb., by multiplying, and then of* 5 *oz* 12 *dwt. by taking* aliquot parts, *as folows :*

$45.125
2

The value of 2 *lb.*	is	90.250,	*twice* the value of 1 *lb.*
" " of 4 *oz.*	is	15.0416,	⅓ of " " of 1 *lb.*
" " of 1 *oz.*	is	3.7604,	¼ of " " of 4 *oz.*
" " of 10 *dwt.*	is	1.8802,	½ of " " of 1 *oz.*
" " of 2 *dwt.*	is	.3760,	⅕ of " " of 10 *dwt.*
		$111.3082,	*value of the whole.*

The values of the several parts, 2 *lb.*, 4 *oz.*, 1 *oz.*, 10 *dwt.*, 2 *dwt.* added together, make the value of the whole quantity 2 *lb.* 5 *oz.* 12 *dwt.*

For convenience in dividing, the aliquot parts should be apportioned, when practicable, so that no divisor, exclusive of 0s annexed, shall exceed 12.

EXERCISES

In Analysis by Aliquot Parts.

1. Find the value of 3 *lb.* 4 *oz.* 17 *dwt.*, of jewelry, at $50.50 per *lb.*

After multiplying the given price by 3, *aliquot parts may be* taken thus: 4 *oz.* is $\frac{1}{3}$ of 1 *lb.*, 10 *dwt.* is $\frac{1}{8}$ of 4 *oz.*, 5 *dwt.* is $\frac{1}{2}$ of 10 *dwt.*, and 2 *dwt.* is $\frac{1}{5}$ of 10 *dwt.* *Ans.* $171.909'.

2. Find the sum that should be paid for 3 *T.* 10 *cwt.* 3 *qr.* of iron, at $30.37½ per ton. *Ans.* $107.45'.

3. Find the sum that should be paid for 13 *cwt.* 2 *qr.* 15 *lb.* of soap, at $3.62½ per *cwt.* *Ans.* $49.422'.

4. A farmer sold 125 *bu.* 3 *pk.* 1 *qt.* of wheat, at $0.87½ per bushel. What did the whole amount to? *Ans.* $110.058'.

5. A market man sold 3 *pk.* 3 *qt.* of cherries, at $0.18¾. per peck. What did his cherries amount to? *Ans.* $0.632'.

6. A merchant sold 15 *yd.* 3 *qr.* 3 *na.* of silk, at $1.37½ per yard, and 5 *yd.* 2 *qr.* of lace, at $2.50 per yard. What did the whole amount to? *Ans.* $35.661'.

7. Find what would be the amount of profit or loss on 139 *yd.* 2½ *qr.* of cloth, if purchased at $5 a yard, and sold at the rate of $1.56¼ per *qr.* *Ans.* $174.531' profit.

8. A townsman bought a lot of ground containing 3 *A.* 2 *R.* 25 *P.*, at $75 per acre. What did he pay for the lot? *Ans.* $274.218'.

9. Find what would be the expense of putting up 150 *rd.* 4 *yd.* of fencing, at $.50 per rod, and 230 *rd.* 2½ *yd.* of another kind at the rate of 62½ *cts.* per rod. *Ans.* $219.397.

10. A speculator purchased 15 *A.* 2 *R.* 30 *P.* of land, at $5 per acre, which he divided into town lots, and sold at the rate of $20 per rood or quarter acre. What did he gain by the speculation? *Ans.* $1176.563'.

CANCELLATION.

§ **210.** CANCELLATION, in Arithmetic, consists in *rejecting equal factors* from a dividend and its divisor;—which is equivalent to dividing the dividend and divisor by the same number (§ 69), and therefore does not alter the *required quotient*. (§ 57).

§ **211.** In the Analysis of questions, the given numbers may often be put as factors in a dividend and divisor, and *cancellation* then applied.

EXAMPLE

Of Analysis and Cancellation.

Allowing $\frac{3}{4}$ of a yard of cloth to cost \$10, what will $\frac{5}{8}$ of a yard cost at the same rate?

Analysis. Since 3 *fourths* of a *yd.* costs \$10,

1 *fourth* of a *yd.* will cost $\frac{1}{3}$ of \$10, which is $\frac{\$10}{3}$;

and 1 *yd.* will cost 4 times as much as $\frac{1}{4}$ *yd.*, $\frac{\$10\times4}{3}$.

Again; $\frac{1}{8}$ *yd.* will cost $\frac{1}{8}$ as much as 1 *yd.* will, $\frac{\$10\times4}{3\times8}$,

and $\frac{5}{8}$ *yd.* will cost 5 times as much as $\frac{1}{8}$ *yd.*, $\frac{\$10\times4\times5}{3\times8}$.

Cancellation. Dividing the numerator and denominator, in other words, the dividend and divisor, both by 4, we have $\frac{\$10\times5}{3\times2}$;

dividing these again by 2, we have $\frac{\$5\times5}{3}=\frac{\$25}{3}=\$8\frac{1}{3}$.

If the factors of the dividend be set, *one under another*, on the right of a line, and those of the divisor on the left, the sign of multiplication may be omitted. In canceling, the rejected factors may be *crossed*, and those to be substituted set respectively right and left.

By this method the result of the preceding Analysis and Cancellation, will be presented thus:

$$\begin{array}{r|l} & \cancel{10}\ \ 5 \\ 3 & \cancel{4}\ \ \ 1 \\ 1\ \ \cancel{2}\ \ \cancel{8} & 5 \end{array} \qquad \textit{Result } \frac{5\times5}{3}=8\frac{1}{3}.$$

Canceling 4 and 8, we substitute 1 and 2; canceling 2 and 10, we substitute 1 and 5. The factor 1 is omitted in multiplying.

But observe that *when each factor of the dividend becomes* 1, the 1 must be retained as the *numerator of a fraction.*

EXERCISES

In Analysis and Cancellation.

1. Allowing $\frac{2}{3}$ of a yard of cloth to cost \$6, what should be paid for $\frac{1}{4}$ of a yard at the same rate? *Ans.* \$$2\frac{1}{4}$.

2. If 5 men can accomplish a certain work in 20.5 days, in what time ought 25 men to perform 3.5 times as much work? *Ans.* 14.35 days.

3. What should be paid for $\frac{3}{8}$ of a ton of hay, when $2\frac{1}{4}$ tons of the same amount to \$18? *Ans.* \$3.00.

4. Allowing a person to walk $\frac{7}{8}$ of a mile in 12 minutes, what distance would he walk in 40 minutes? *Ans.* $2\frac{11}{12}$ miles.

5. If $\frac{1}{2}$ of $\frac{3}{4}$ of an acre of ground be worth \$16, what is the value of $2\frac{1}{2}$ acres at the same rate? *Ans.* \$$106\frac{2}{3}$.

6. A teamster hauled 25 *cwt.* of iron 30.5 miles for a certain sum of money. How far then ought he to haul 6.5 *cwt.* for the same sum? *Ans.* 117.307 miles.

7. How long ought 20 men to subsist on a stock of provisions which would suffice 18 men for 300 days? *Ans.* 270 days.

8. A person who owned $\frac{1}{4}$ of a merchant ship, sold $\frac{2}{5}$ of his share for \$400; what was the whole ship worth at that rate? *Ans.* \$4000.

9. Allowing a person to perform $\frac{2}{3}$ of a certain work in $\frac{3}{8}$ of a day, in what time ought he to perform the entire work? *Ans.* $\frac{9}{16}$ day.

10. The distance from A to B, which is 40 miles, is $\frac{4}{5}$ of the distance from B to C: how far then is it from B to C? *Ans.* 50 miles.

11. If $\frac{2}{3}$ of $\frac{3}{8}$ of a yard of cloth be worth \$1.25, what is $\frac{1}{2}$ of $\frac{5}{6}$ of a yard worth, at the same rate? *Ans.* \$2.083'.

12. A contributed towards building a church \$200, which was $\frac{7}{8}$ of the sum contributed by B, who gave $\frac{2}{5}$ as much as C. What was the amount of C's contribution? *Ans.* \$$571\frac{3}{7}$.

13. If 4.5 cords of wood sell for \$13.5, what should be given for 5 loads, each containing 1.25 cords? *Ans.* \$18.75.

14. Allowing 3 bushels of wheat to be worth as much as $6\frac{1}{4}$ bushels of corn, how many bushels of corn are equal in value to $12\frac{1}{2}$ bushels of wheat? *Ans.* $26\frac{1}{24}$ *bu.*

15. A has $\frac{3}{4}$ as much money as B, and $\frac{4}{5}$ as much as C, who has $\frac{5}{6}$ as much as D, and he has \$1800. What sums are owned by A, B, and C, respectively?

Ans. A, \$1200; B, \$1600; C, \$1500.

RATIO.

§ 212. The *Ratio* of one number or quantity, called the *antecedent*, to another of the same kind, called the *consequent*, is the *quotient* of the former divided by the latter.

Thus the ratio of 15 to 5 is 3, since 15 contains 5, 3 *times;* and the ratio of 4 to 9 is $\frac{4}{9}$, since 4 is $\frac{4}{9}$ of 9.

In these examples 15 and 4 are the *antecedents;* 5 and 9 the *consequents*.

The antecedent and consequent together are called the *terms* of the ratio.

What is the ratio of 12 to 3? Which is the *antecedent*, and which the consequent? Of 3 to 12? Of 20 to 4? Of 5 to 30?

What is the ratio of 10 miles to 5 miles? Of 3 yards to 6 yards? Of 6 hours to 13 hours? Of 25 pounds to 8 pounds? Of 9 *s.* to 20 *s.*?

Ratio of Monomials and Polynomials.

§ 213. To find the ratio between two quantities, the antecedent and consequent must be in the *same denomination.*

Thus the ratio of 2 *ft.* to 5 *yd.* is the ratio of 2 *ft.* to 15 *ft.* $=\frac{2}{15}$.

What is the ratio of 3 *in.* to 2 *ft.*? Of 4 *yd.* to 6 *ft.*? Of 2 *hr.* to 1 *da.*? Of 10 *s.* to 2£. 10 *s.*? Of 5 *m.* to 3 *L.* 3 *m.*? Of 1 *A.* 2 *R.* to 5 *A.* 4 *R.*?

Two quantities of *different kinds*, that is, such that one can form no *part of the other*, have *no ratio to each other;* as 2 *ft.* and 5 *hours.*

Sign of Ratio.

§ 214. A *colon* (:) placed between two numbers, signifies that the numbers are taken as the *antecedent* and *consequent* of a ratio.

Thus 3 : 5 signifies the ratio of 3 to 5.

Ratio is also expressed by making the antecedent the *numerator*, and the consequent the *denominator*, of a fraction; thus 3 : 5 is $\frac{3}{5}$.

What fraction expresses the ratio of 3 to 13? Of 25 to 17? Of 4 to 19? Of 21 to 7? Of 3 *quarters* to 7 *yd.* 3 *qr.*?

What fraction expresses the ratio of 5 to 17? Of 4 to 19? Of 5 to 25? Of 7 to 100? Of 5 *bushels* to 7 *bu.* 3 *pk.*

Direct and Inverse Ratio.

§ 215. The *direct* ratio of the *first* of two quantities to the *second*, is the quotient of the first divided by the second.

The *inverse* ratio of the first to the second, is the same as the direct ratio of the *second to the first.*

For example, the *direct* ratio of 6 to 3 is $\frac{6}{3}=2$; the *inverse* ratio of 6 to 3 is the direct ratio 3 to $6=\frac{3}{6}=\frac{1}{2}$.

What is the direct ratio of 8 to 2? Of 3 to 15? Of 10 to 4?
What is the inverse ratio of 9 to 3? Of 5 to 20? Of 21 to 7?
What is the inverse ratio of 4 to 16? Of 25 to 5? Of 9 to 90?

Ratio of Reciprocals.

§ 216. The *inverse* ratio of the *first* of two quantities to the *second*, is equal to the direct ratio of the *reciprocals* of those quantities.

Thus, the inverse ratio of 6 to 3 is $\frac{3}{6}=\frac{1}{2}$;

and the direct ratio of the reciprocals $\frac{1}{6}$ and $\frac{1}{3}$ is $\frac{1}{6}\div\frac{1}{3}=\frac{3}{6}=\frac{1}{2}$.

The *inverse* ratio of 4 to 2 equals the *direct* ratio of what fractions? Of 3 to 5? Of 4 to 9? Of $\frac{2}{3}$ to $\frac{3}{4}$? Of $\frac{3}{5}$ to $\frac{6}{7}$? Of $\frac{3}{8}$ to 10?
The inverse ratio of 3 to 8 equals the direct ratio of what fractions? Of 5 to 7? Of 11 to 4? Of $\frac{1}{4}$ to $\frac{5}{7}$? Of $\frac{3}{7}$ to $\frac{5}{12}$? Of $\frac{4}{9}$ to $\frac{3}{16}$?

Hence *inverse* is sometimes called *reciprocal* ratio. The term ratio, when used alone, always means *direct* ratio.

Ratio of Fractions having a Common Term.

§ 217. The ratio of two fractions having a *common denominator*, is the same as the ratio of their *numerators;* and

The ratio of two fractions having a *common numerator*, is the same as the *inverse ratio* of their *denominators.*

Thus, the ratio of $\frac{4}{15}$ to $\frac{2}{15}=\frac{4}{15}\div\frac{2}{15}=\frac{4}{2}=2$, which is the ratio of the numerators 4 and 2.

And the ratio of $\frac{3}{8}$ to $\frac{3}{16}=\frac{3}{8}\div\frac{3}{16}=\frac{16}{8}=2$, which is the inverse ratio of the denominators 8 and 16.

The ratio of $\frac{3}{7}$ to $\frac{4}{7}$ is equal to the ratio of what *integral* numbers? Of $\frac{13}{12}$ to $\frac{5}{12}$? Of $\frac{7}{16}$ to $\frac{15}{16}$? Of $\frac{20}{27}$ to $\frac{13}{27}$? Of $\frac{31}{60}$ to $\frac{29}{60}$?
The ratio of $\frac{3}{7}$ to $\frac{3}{13}$ is equal to the ratio of what integral numbers? Of $\frac{5}{8}$ to $\frac{5}{20}$? Of $\frac{9}{13}$ to $\frac{9}{25}$? Of $\frac{12}{17}$ to $\frac{12}{19}$? Of $\frac{20}{41}$ to $\frac{20}{53}$?

PROPORTION.

§ **218.** PROPORTION consists in an *equality of ratios.*

Four quantities are in proportion, when the *first* has the same ratio to the *second,* that the *third* has to the *fourth.*

Thus, the numbers 6, 3, 8, 4, are in *proportion;* since the ratio of 6 to 3 is $\frac{6}{3}=2$, and the ratio of 8 to 4 is $\frac{8}{4}=2$.

The *first* and *third* terms, 6 and 8, are the *antecedents* of the ratios; the second and fourth are the *consequents.*

The *first* and *fourth* terms are called the two *extremes;* the second and third, the two *means.*

The *fourth* term is called a *fourth proportional* to the other three taken in order; thus, 4 is a fourth proportional to 6, 3, and 8.

What is the *fourth proportional* to 9, 3, and 12; that is, the number to which 12 has the same ratio that 9 has to 3?

Which are the *antecedents,* and which the consequents? Which are the *extremes,* and which the means?

What is the *4th proportional* to 16, 4, and 20? To 4, 8, and 10?
To 3, 6, and 8? To 4, 12, and 10? To 5, 20, and 12?
To 2, 1, and 10? To 2, $\frac{1}{2}$, and 4? To $\frac{1}{2}$, 1, and $2\frac{1}{2}$?

Direct and Inverse Proportion.

§ **219.** A *direct proportion* consists in an equality between two *direct ratios.* An *inverse* or *reciprocal* proportion consists in an equality between a *direct* and an *inverse* ratio. (§ 215).

Thus, the numbers 6, 3, 8, 4, are in *direct* proportion, since the direct ratio $\frac{6}{3}$ of 6 to 3 is equal to the direct ratio $\frac{8}{4}$ of 8 to 4.

The same numbers, in the order 6, 3, 4, 8, are in *inverse proportion,* since the *direct ratio* $\frac{6}{3}$ of 6 to 3 is equal to the *inverse ratio* $\frac{8}{4}$ of 4 to 8.

What is the *inverse fourth proportional* to 12, 6, and 8; that is, the number to which 8 has the *inverse ratio* of 12 to 6?

What is the inverse fourth proportional to 8, 2, and 3? To 3, 9, and 15? To 4, 20, and 30? To 24, 3, and 4?

The term *proportion,* used alone, always means *direct* proportion.

Variation or General Proportion.

§ **220.** VARIATION expresses the *dependence* of one term or quantity on another, according to some constant ratio, whatever *new value* either of the terms may assume.

One term *varies directly* as another, when both *increase* or *decrease* together in the same ratio.

Thus the *value* of a particular commodity *varies directly* as the *quantity*, since the value will be *multiplied* by 2, or 3, &c., if the quantity be *multiplied by the same number*. More briefly, we say, the value *is directly as* the quantity; or, the value *is as* the quantity.

One term *varies inversely* as another, when one of them *increases* in the same ratio in which the other *decreases*.

Thus the *time* required for a laborer to earn a given sum, *varies inversely* as his *rate of wages*, since the time will be *multiplied* by 2, or 3, &c., if his rate of wages be *divided* by the same number. More briefly, we say, the time is *inversely* as his rate of wages.

Say whether the two *italicised terms* would vary *directly*, or *inversely*, with each other, in each of the following instances:

1. The *value* and the *quantity* of a piece of cloth?—The *time*, and the *number* of men required for a given amount of labor?

2. The *weight* and the *number* of gallons of water?—The *number* of men and the *amount* of provisions that will serve them for a given period of time?

3. The *weight* of an article and the *distance* it may be carried for a given sum of money?—The *length* and the *breadth* of a garden to contain a given area?

4. The *weight* of the five-cent loaf of bread and the *price* of flour?—A *sum* of money and the *number* of laborers that may be hired with it for a given time?

Variation is sometimes called *General Proportion:* being, indeed, an abridgement of a proportion containing four terms without respect to any *particular values* of those terms.

Thus when we say that the *weight* of water *is as* the *number of gallons*, it is understood that,

The *weight* of any *number* of gallons *is to* the weight of any other number of gallons, *as* the *first* of those numbers *is to* the second.

Sign of Proportion.

§ **221.** A Proportion is denoted by a *double colon* (: :), or the sign = *equal to*, between the two ratios of the proportion.

Thus 6 : 3 : : 8 : 4
or 6 : 3 = 8 : 4 denotes

that 6, 3, 8 and 4 are *in proportion;* and is read, 6 *is to* 3 *as* 8 *is to* 4.

The two ratios of a proportion may also be expressed *fractionally*, and the first be put equal to the second.

Thus $\frac{6}{3}=\frac{8}{4}$ denotes that the ratio 6 to 3 is equal to the ratio of 8 to 4, or that 6 *is to* 3 *as* 8 *is to* 4.

No sign has hitherto been adopted by Mathematicians for *inverse proportion.* We shall employ the *sign* $\neq$ between the two ratios of such proportion.

Thus $6 : 3 \neq 4 : 8$ denotes that 6, 3, 4, and 8 are in *inverse* proportion; and must be read, 6 *is to* 3 *inversely as* 4 *is to* 8.

Inverse Converted into Direct Proportion.

§ **222.** An *inverse* is converted into a *direct* proportion by *interchanging* either antecedent and its consequent, that is, by taking the antecedent and its consequent *the one for the other.*

Thus $6 : 3 \neq 4 : 8$ is an inverse proportion. By interchanging 6 and 3, we have $3 : 6 = 4 : 8$, which is a direct proportion.

An *inverse* may also be converted into a *direct* proportion, by substituting, for either antecedent and its consequent, the *reciprocals* of those terms.

Thus the inverse proportion $6 : 3 \neq 4 : 8$ is converted into the direct proportion $\frac{1}{6} : \frac{1}{3} = 4 : 8$, by substituting the reciprocals of 6 and 3. (§ 216).

Product of the Extremes = that of the Means.

§ **223.** In every direct proportion, the product of the *two extremes* is equal to the product of the *two means.*

In the proportion $3 : 6 = 4 : 8$, we have two *equal ratios* $\frac{3}{6}$ and $\frac{4}{8}$; and if these ratios be reduced to a *common denominator*, the resulting *numerators* must be equal. One of these numerators, 3×8, is the product of the two extremes, and the other 6×4, of the two means.

In an *inverse* proportion, the product of the two *antecedents* is equal to that of the two consequents.

By interchanging 3 and 6 in the preceding proportion, we have the *inverse* proportion $6 : 3 \neq 4 : 8$; and $6\times4=3\times8$.

Fourth Proportional—how found.

RULE XLII.

§ 224. *To find a* FOURTH PROPORTIONAL *to three given terms.*

1. Multiply the second and third together, and divide the product by the first term; the quotient will be the fourth term, in *direct* proportion.

2. An *inverse fourth proportional* may be found by interchanging the first and second terms, (§ 222), and then proceeding as above.

3. The *first* and *second* terms must be used in the *same denomination*.

4. When the *third* term is a *polynomial*, it will generally be most convenient to reduce it to a *monomial*, or single denomination.

5. The *fourth* term will be found in the *same kind* of quantity and in the *same denomination*, with the *third* term.

EXAMPLE.

To find a fourth proportional to 3 *yd.*, 5 *yd.* 2 *qr.*, and 2£. 10 *s.*

Reducing the 1st and 2d terms to the *same denomination*, we find 3 *yd.* =12 *qr.* and 5 *yd.* 2 *qr.* =22 *qr.*

Reducing the third term to a single denomination, we find 2£. 10 *s.* =50 *s.*

Multiplying the 2d and 3d terms together, and dividing by the 1st term, we have

50 *s.* ×22÷12=1100 *s.* ÷12=91 *s.* 8 *d.* =4£. 11 *s.* 8 *d*

The proportion is,

3 *yd.* : 5 *yd.* 2 *qr.* : : 2£. 10 *s.* : 4£. 11 *s.* 8*d.*

☞ The product of the second and third terms, is equal to the *product of the first and fourth*; (§ 223); and the product of the *first* and *fourth* divided by the *first term*, gives the fourth term. (§ 66).

The first and second terms must be used in the *same denomination*, to be in the *proper ratio* to each other, in multiplying and dividing. (§ 213).

The third term, multiplied and divided, produces the fourth term in the same denomination. Thus, in the example, 50 *s.* ×22 produces 1100 *s.* and this, divided by 12, gives $\frac{1}{12}$ of 1100 *s.* =91 *s.* 8 *d.* ☜

EXERCISES.

Find the *fourth term* in each of the following proportions.

1. 4 *yd.* : 7 *yd.* 2 *qr.* : : $26 : 4*th term*. . . *Ans.* $48.75.
2. 3 *oz.* : 4 *lb.* 15 *dwt.* = $4 : 4*th term*. . . *Ans.* $65.
3. 5 *men* : 12 *men* $\neq$ 18 *days* : 4*th term*. . . *Ans.* 7½ days.
4. 10 *bu.* : 20 *bu.* 3 *pk.* : : $15 : 4*t term*. . *Ans.* $31⅛.
5. 9 *days* : 4½ *days* $\neq$ 5 *bu.* 1 *pk.* : 4*th term*. *Ans.* 10 *bu.* 2 *pk.*
6. $15 : $2.25 = 3 *A.* 10 *P.* : 4*th term*. *Ans.* 1.8375 *R.*

APPLICATION OF PROPORTION.

§ **225.** The applications of Proportion are very numerous. It is involved in every multiplication and division of numbers.

For, a *unit* : the *multiplier* : : the *multiplicand* : the *product* ;
and, the *divisor* : a *unit* : : the *dividend* : the *quotient*.

In practical questions in Proportion, we shall always have given a term of *supposition* and a like term of *demand*, with a third term *dependent* on the term of *supposition*, and in ratio to the *answer* required, directly or inversely *as* the term of supposition *is to* that of demand.

These several particulars will be exemplified under

RULE XLIII.

§ **226.** *For solving questions by Proportion.*

1. Make the term of *supposition* the *first* term, the like term of *demand* the *second* term, and the *dependent* term the *third* term.

2. Consider whether the *answer* required would *vary directly* or *inversely* with the term of *demand*, and find it, accordingly, as a direct or inverse *fourth proportional*.

EXAMPLES.

1. If 3 yards of cloth cost $19, what will 15 yards cost?

The term of supposition is 3 yards, and the like term of *demand* is 15 yards; while $19 is the *dependent* term, since this term depends for its value on the 3 yards.

The *Proportion* is, 3 *yd.* : 15 *yd.* : : $19 : the *cost* of 15 *yd.*

The required cost of 15 *yd.* would *vary directly* with the 15 *yd.*, since these two terms would both *increase* or decrease together in the same ratio (§ 220).

The proportion is therefore *direct;* and the fourth *proportional* is

$$\$19 \times 15 \div 3 = \$285 \div 3 = \$95, \text{ (§ 224).}$$

Ans. $95.

2. Allowing 6 men to perform a certain work in 30 days, in what time ought 13 men to perform the same work?

The term of supposition *is* 6 *men, and the like term of* demand *is* 13 *men:* while the 30 days is *dependent on the* 6 *men.*

The *Proportion* is, 6 *men* : 13 *men* ≠ 30 *days* : the *time required.*

The *time* required for 13 men would vary *inversely* with the 13 men, since one of these two terms would *increase* in the same ratio in which the other would *decrease.*

The proportion is therefore *inverse:* and the inverse fourth proportional is

30 days×6÷13=180 *da.*÷13=$13\frac{1}{13}$ days, (§ 224—2).
Ans. $13\frac{1}{13}$ days.

Inverse Proportion Discarded.

The preceding Rule recognizes *inverse proportion;* and this kind of proportion is employed in many of the higher applications of the subject. The following Rule will arrange the terms always in a *direct* proportion, and is the one now most commonly employed in Arithmetic.

RULE XLIV.

§ **227.** *For solving questions by Direct Proportion.*

1. Make the term which is of the same kind with the required fourth term, or *answer,* the *third term.*

2. If the nature of the question requires the answer to be greater than the 3d term, make the *greater* of the two remaining terms the *second term,*—otherwise, make the *less* of those terms the 2d term: the term still remaining will be the *first* term.

3. Find the direct *fourth proportional* for the answer.

EXAMPLE.

If 6 men perform a certain work in 30 days, in what time ought 13 men to perform the same work?

30 days *is the term of the* same kind *with the answer, and the answer will be less than* 30 *da.,* since 13 men would require less time than 6 men. Hence the direct proportion will be

13 *men* : 6 *men* : : 30 *da.* : the *time required.*

The fourth proportional, or *Ans.* is 30 *da.*×6÷13=$13\frac{1}{13}$ *days.*

EXERCISES.

The following questions may be put into *proportions*, or may be solved *analytically* (§ 209). Cancellation may be employed in many of them, to abbreviate the operations of multiplying and dividing (§ 210 and 211).

If the last Rule be preferred, it is recommended that the student be required to express, in general terms, the *kind of proportion* naturally involved in the question; as under Ex. 1 and 2.

1. If 9 acres of land sell for \$230.62½, what should 5 acres bring at the same rate?

The value of the land is directly *as the quantity*.

Ans. \$128.125.

2. If 1.5 *T.* of iron be hauled 40 miles for a given sum, how far ought 3 *T.* to be hauled for the same sum?

The distance is inversely *as the weight* to be hauled.

Ans. 20 miles.

3. What will 14 pounds of tea amount to, when 6 pounds of the same kind cost \$7.50? *Ans.* \$17.5.

4. What quantity of cloth may be bought for \$73.75, when 4.25 yards of the same kind cost \$12.75? *Ans.* 24.583' *yd.*

5. If 7 masons can build a house in 28 days, in what time ought 17 masons to build the house? *Ans.* $11\frac{9}{17}$ days.

6. If 35 pounds of coffee cost \$5, how many pounds of the same sort may be purchased for \$100.25? *Ans.* 701.75 *lb.*

7. If a man travels at the rate of 100 miles in 3 days, how many miles ought he to travel in 13½ days? *Ans.* 450 miles.

8. If a quantity of bread will suffice 100 men for 29 days, how long ought it to suffice 84 men? *Ans.* $34\frac{11}{21}$ days.

9. If 5 yards of silk cost \$6.25, what should be paid for 12 *yd.* 3 *qr.* of silk, at the same rate? *Ans.* \$15.937'.

10. Allowing 4 horses to consume 13 *bu.* 3 *pk.* of oats in a week, how much would 9 horses require for a week?

Ans. 30.937' *bu.*

11. Allowing the transportation of 10 *cwt.*, 100 miles, to cost \$25, what should be paid for the transportation of 33 *cwt.* 2 *qr.* the same distance? *Ans.* \$83.75.

12. Allowing a person, by traveling 10 hours a day, to perform a journey in 31 days; in how many days ought he to perform the same journey, if he travel 13 hours a day?

Ans. $23\frac{11}{13}$ days.

13. What sum should be paid for $\frac{3}{4}$ of a yard of cloth, when 9 *yd.* of the same kind cost \$54? *Ans.* \$4½.

14. If 7 men can do a certain work in $\frac{3}{8}$ of a day, in what time ought 9 men to do the same work? *Ans.* $\frac{7}{24}$ of a day

15. If 17 *bu.* of salt cost $8.50, what should be paid for 20 *bu* 3 *pk.* of salt at the same rate? *Ans.* $10.375.

16. If 5 *cwt.* of goods be carried 100 miles for a given sum, how far ought 20 *cwt.* 1 *qr.* to be carried for the same sum? *Ans.* $24\frac{56}{81}$ miles.

17. If 10 head of cattle require 20 *A.* 2 *R.* of pasture ground, for a summer, how many acres ought 25 head to have, for the same length of time? *Ans.* 51 *A.* 1 *R.*

18. Allowing the transportation of 15 *T.*, a given distance, to cost $$25\frac{1}{2}$, what should be charged for the transportation of $3\frac{1}{4}$ *T.* the same distance? *Ans.* $5.525.

19. If a ship sail 194 *L.* in $5\frac{1}{2}$ days, in how many days would she sail 3000 miles, at the same rate? *Ans.* $28\frac{34}{97}$ days.

20. An army of 5000 men had provisions for 3 months. One-eighth of the men having been killed in battle, how long ought the same provisions to last the remainder? *Ans.* $3\frac{3}{7}$ months.

21. If $\frac{3}{4}$ of a *cwt.* of iron cost $16, what will $\frac{7}{8}$ of a *cwt.* cost at the same rate? *Ans.* $$18.66\frac{2}{3}$.

22. What should be paid for $\frac{5}{8}$ of a *yd.* of crape, when $\frac{1}{2}$ *yd.* of the same kind costs $1.75? *Ans.* $2.1875.

23. What should be paid for $15\frac{1}{2}$ *gal.* wine, when 3 quarts of the same sell for $$0.56\frac{1}{4}$? *Ans.* $11.625.

24. If $1\frac{1}{4}$ *A.* of land sell for $34.50, what will 20 *A.* 2 *R.* 10 *P.* amount to, at the same rate? *Ans.* $567.525.

25. A cistern is filled with water, by 2 pipes, in 3 *hr.* 25 *m.* In what time would it be filled by 5 pipes of like size? *Ans.* 1 *hr.* 22 *min.*

26. A sum of money having been equally divided among 19 men, each man received $$3\frac{1}{4}$. If the number of men had been 30, what would have been the share of each? *Ans.* $2.058'.

27. Allowing a person to perform a certain journey in 15 days, when the days are $10\frac{1}{2}$ hours long; in what time ought he to accomplish the same journey, when the days are 13 *hr.* long? *Ans.* $12\frac{3}{26}$ days.

28. If 950 *lb.* of beef be sufficient for the crew of a ship for 57 days; how much would suffice them for $83\frac{1}{2}$ days? *Ans.* $1391\frac{38}{57}$ *lb.*

29. Allowing 15 *A.* 30 *P.* to produce 403 *bu.* 2 *pk.* of wheat, what number of bushels would be raised from a field containing 40 *A.*, at the same rate? *Ans.* $1062\frac{58}{81}$ *bu.*

30. If 5 *T.* 11 *cwt.* of hay amount to $$55.18\frac{3}{4}$, what quantity of hay, at the same price per ton, may be bought for $100? *Ans.* 10.056' *T.*

31. If 25 sacks, each measuring 4 *bu.* will contain a given quantity of grain; how many sacks, each measuring $3\frac{1}{2}$ *bu.*, will contain the same quantity? *Ans.* $28\frac{4}{7}$ sacks.

32. A bought $9\frac{7}{8}$ *yd.* of linen, for $19.75; and B bought 7 *yd.* $1\frac{1}{2}$ *qr.* at the same price per yard? What did the latter pay for the quantity he purchased? *Ans.* $14.75.

33. A post, standing in a stream, has $\frac{1}{5}$ of its length in the earth, $\frac{2}{3}$ in the water, and 5 feet above the water. What is the length of the post?

Analysis. $\frac{1}{5}+\frac{2}{3}=\frac{3}{15}+\frac{10}{15}=\frac{13}{15}$; and $1-\frac{13}{15}=\frac{2}{15}$

The post has $\frac{13}{15}$ of its length below the surface of the water, and, consequently, $\frac{2}{15}$ of its length above the water.

Then $\frac{2}{15}$ of its length is 5 *feet*; $\frac{1}{15}$ of it is $\frac{1}{2}$ of 5 *ft.* $=\frac{5}{2}$; and the whole length is 15 times $\frac{1}{15}$ of it $=\frac{5\times15}{2}=\frac{75}{2}=37\frac{1}{2}$ *ft.*

Proportion. $\frac{2}{15}$: 1 or $\frac{15}{15}$ :: 5 *ft.* : *The entire length.*

The part of the post above the water, *is to* the *whole post,*—represented by a *unit,*—*as* the length of that part *is* to the whole length.

34. A farmer sold $\frac{1}{3}$ of his land to A, $\frac{1}{4}$ of it to B, and the remainder, which was 100 acres, to C. How much land did the farmer own? *Ans.* 240 acres.

35. A merchant who owned $\frac{3}{8}$ of a ship's cargo, sold $\frac{1}{3}$ of his share for $1500. What was the whole cargo worth? *Ans.* $12000.

36. In a certain school, $\frac{1}{4}$ of the pupils are studying Arithmetic; $\frac{3}{8}$ of them study Languages; and the remaining 36 are employed on various other subjects. Required the number in the school. *Ans.* 96 pupils.

37. A person failing in business owes $5000, and is able to pay only $2000. How much can he pay per dollar to his creditors, and how much should that creditor receive to whom he owes $1000? *Ans.* $.40: and $400.

38. A traveler having gone 375.5 miles on his journey, finds that $\frac{3}{8}$ of it remains to be traveled. What was the length of his journey? *Ans.* 600.8 miles.

39. A given quantity of oats is allowed to 15 horses, for 31 days. If 7 horses more be added to the number, for how many days ought the same allowance of food to be made? *Ans.* $21\frac{3}{22}$ days.

40. A cistern whose capacity is 3000 *gal.* is supplied with water by a pipe which pours into it 7 *gal.* per minute. By leakage the cistern will lose, during the time of filling, at the rate of $2\frac{1}{2}$ *gal.* per minute. In what time will the cistern be filled? Ans. 11 hr. $6\frac{2}{3}$ min.

41. A and B depart from the same place, and journey in the same direction. A starts 3 days before B, and goes 30 miles per day; B follows at the rate of $33\frac{1}{3}$ miles per day. In how many days will the latter overtake the former?

Ans. 27 days.

42. If when flour is at \$5 a barrel, the five cent loaf of bread weighs 9 *oz.*, what ought to be its weight when flour is at \$7 a barrel?

The weight of the loaf ought to be inversely as the price of the flour.

Analysis. If flour were at \$1 a barrel, the loaf ought to weigh 5 times as much as if flour were at \$5 a barrel; that is, 9 *oz.* $\times 5$.

And when flour is \$7 a barrel the loaf ought to weigh only $\frac{1}{7}$ as much as when it is \$1 a barrel; that is,

$$\frac{9\,oz. \times 5}{7} = \frac{45\,oz.}{7} = 6\tfrac{3}{7}\,oz.$$

43. If a given store of meat will supply a company of soldiers for 100 days, allowing each man 2 *lb.* per day; what should the daily allowance be, if the time were extended to 135 days? *Ans.* $1\frac{13}{27}$ pounds.

44. A borrowed of B \$500, which he kept $3\frac{1}{2}$ years. On a subsequent occasion, A lends to B \$375; how long ought B to keep this latter sum, in return for the accommodation he had afforded A? *Ans.* $4\frac{2}{3}$ years.

45. A bankrupt owes \$5349.75, and has property amounting to \$2300. In an equitable distribution of his property, how much will a creditor receive whose claim is \$400?

Ans. \$171.970'.

46. A garrison of 90 men has provisions for 42 days. How many of the men must be discharged, that the remainder, without any diminution of rations, may be supported for 60 days? *Ans.* 27 men.

47. A gentleman who owned $\frac{2}{3}$ of a manufactory, sold $\frac{1}{4}$ of his share for \$3000. What was the estimated value of the whole establishment? *Ans.* \$18000.

48. How many miles must a person walk in $5\frac{1}{2}$ days, to accomplish a journey of 500.5 miles, at the same rate, in 15 days? *Ans.* 183.516' miles.

49. If 4 *T.* 13 *cwt.* of iron be carried 50 miles, for \$30, how far should 9 *T.* 5 *cwt.* 3 *qr.* be carried for the same sum?

Ans. 25.033' miles.

50. If 31 *A.* 3 *R.* of ground produce 1000 *bu.* 3 *pk.* of wheat, how many bushels will 51 *A.* 2 *R.* 24 *P.* produce, at the same rate? *Ans.* 1627.991' *bu.*

51. Allowing a man to do a certain work in 3 days, and a boy to do it in 5 days, in what time ought both together to do the work? *Ans.* $1\frac{7}{8}$ days.

Analysis. The man could do $\frac{1}{3}$ of the work, and the boy $\frac{1}{5}$ of the work, in 1 day; and $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$.

Hence, both together could do $\frac{8}{15}$ of the work in 1 day.

Then $\frac{1}{15}$ of the work would be done in $\frac{1}{8}$ of a day; and, consequently, the entire work would be done in 15 times $\frac{1}{8}$ *da.* $=\frac{15}{8}$ *da.* $=1\frac{7}{8}$ *days.*

Proportion. $\frac{8}{15}$: 1 (the whole work) : : 1 *da.* : Time required.

The *part* that both could do in 1 *da.*, *is to* the whole work,—represented by a *unit*,—*as* 1 *day* is to the time required.

52. A can dig a ditch in 5 days, B in 6 days, and C in 8 days. In what time could the three together dig the ditch?

Ans. $2\frac{2}{59}$ days.

53. Two masons together built a wall in 10 days. One of them could have built the wall himself in 15 days; in how many days could the other have done it? *Ans.* 30 days.

54. If 37 *cwt.* 3 *qr.* of coal be hauled 20 miles for $5.75, what sum should be paid for hauling 3 *T.* of coal three times that distance? *Ans.* $27.417'.

55. A merchant bought three pieces of cloth, each containing 25 *yd.* 2 *qr.* for $500; and sold 50 *yd.* of it at cost. What did the 50 yards amount to? *Ans.* $326.797'.

56. A could mow a meadow in 7 days, B in 9 days, and C in 11 days. In what time could the three together mow the meadow? *Ans.* $2\frac{215}{239}$ days.

57. If $\frac{3}{4}$ of $\frac{4}{5}$ of an acre of land sell for 18.18\frac{3}{4}$, what would a lot containing 7 *A.* 2 *R.* 13 *P.* bring at that rate?

Ans. $229.806'.

58. A farmer sold $\frac{2}{5}$ of his whole amount of land, at $25 an acre, and received for it $10000. What amount of land did the farmer own? *Ans.* 1000 acres.

59. A testator bequeathed $\frac{1}{2}$ of his estate to his only son, $\frac{2}{7}$ of it to his only daughter, and the remainder, which was $5000, to his widow. What was the value of his estate?

Ans. 23333\frac{1}{3}$.

60. A, B, and C together can excavate a reservoir in 5 days. A and B together can do it in 9 days; in what time could C alone do the work? *Ans.* $11\frac{1}{4}$ days.

61. Allowing a person to perform a certain journey in $13\frac{1}{2}$ days, by traveling 10 hours a day, in what time ought he to perform the journey if he travel $11\frac{1}{4}$ hours per day?

Ans. 12 days.

62. How many yards of linen $\frac{1}{2}$ *yd.* wide, will be equivalent to 30 *yd.* of another kind, which is $\frac{3}{4}$ *yd.* wide?

The quantity in square measure being the same, the *length will be inversely as the breadth.*

Observe that $\frac{1}{2}$ *yd.* is the term of *demand:* also, that 30 *yd.* is that term which is of the same kind with the required 4th term, since both express *length.*

Analysis. The length of each kind $\times$ the *breadth,* will produce the *square measure;* which is, therefore,

$$30\times\frac{3}{4}=\frac{30\times3}{4}\ sq.\,yards.$$

This divided by the *breadth* of the first kind, will give the *length* of the first kind; which is, therefore,

$$\frac{30\times3}{4}\div\frac{1}{2}=\frac{30\times3\times2}{4}.$$

Canceling 2, we have $\frac{30\times3}{2}=\frac{90}{2}=45$ *yards.*

63. How many yards of cotton $\frac{7}{8}$ *yd.* wide, will be required to line $4\frac{1}{2}$ yards of cloth which is $1\frac{1}{2}$ *yd.* wide?

Ans. $7\frac{5}{7}$ yards.

64. How many yards of carpeting which is $\frac{3}{4}$ of a yard wide, will be required to cover a floor that measures 25 feet in length, and 20 feet in breadth? *Ans.* $74\frac{2}{27}$ yards.

65. How many yards of paper $\frac{5}{8}$ *yd.* wide will be sufficient to cover the walls of a room 70 *ft.* in compass, and 10 *ft.* in height, allowing $\frac{1}{30}$ for windows, &c. *Ans.* $120\frac{8}{27}$ yards.

66. A farmer has a field 100 poles in length, and 45.25 poles in width. He wishes to lay off another field to contain the same quantity of ground, and be 80 poles in length; what must be its breadth? *Ans.* 56.5625 poles.

PARTITIVE PROPORTION.

§ **228.** PARTITIVE PROPORTION is proportion applied to dividing a given quantity into *two or more parts*, which shall be in a given *ratio*, one to another. The terms of the given ratio, or ratios, may be called the *proportional terms*.

For example: to divide \$300 between A, B, and C, in the proportion of 2, 3, and 5; that is, so that A's share shall be to B's as 2 to 3, and B's to C's, as 3 to 5.

In this example, 2, 3, and 5, are the *proportional terms*, the *consequent* of the first ratio 2 to 3 being the *antecedent* of the second ratio 3 to 5.

Note. This part of Arithmetic is commonly called PARTNERSHIP OF FELLOWSHIP.

RULE XLV.

§ **229.** *To divide a given quantity into* TWO *or* MORE PARTS *which shall be in a given* RATIO, *one to another.*

1. Add together all the given proportional terms. Then,

2. The *sum* of those terms *will be* to each term, separately, *as* the quantity to be divided *is to* each *corresponding part* of that quantity.

EXAMPLE.

To divide \$300 between A, B, and C, in the proportion of 2, 3, and 5.

The sum of the proportional terms is $2+3+5=10$.

Then, $10 : 2 :: \$300 : \$300\times2\div10=\$60$, A's share;
$10 : 3 :: \$300 : \$300\times3\div10=\$90$, B's share;
and $10 : 5 :: \$300 : \$300\times5\div10=\$150$, C's share.

Analysis. Suppose the whole sum \$300 to be divided into $2+3+5=10$ *equal parts*. Then it is evident that

A must have 2, B 3, and C 5, of those parts; that is

A's share is $\frac{2}{10}$ of $\$300=\$300\times2\div10=\$60$;
B's share is $\frac{3}{10}$ of $\$300=\$300\times3\div10=\$90$;
C's share is $\frac{5}{10}$ of $\$300=\$300\times5\div10=\$150$.

The expressions $\$300\times2\div10$, &c., for the several shares, found by the Analysis, are the same as those found by the Rule. Hence the Analysis *demonstrates the Rule*.

EXERCISES.

1. Divide \$240 between three persons in such a manner that their shares shall be as 5, 4, and 3, respectively.

Ans. \$100 ; \$80 ; and \$60.

2. A gentleman bequeathed to his son and daughter \$5000, the son's share of it to be to the daughter's as 3 to 2. What was the share of each ?

Ans. Son's \$3000 ; daughter's \$2000.

3. A merchant employed 3 clerks, at the annual salaries of \$300, \$400, and \$500, respectively. At the end of the year, the merchant proving bankrupt, has but \$650 to be divided proportionably among them. What will be the portion of each ?

The proportional terms are 300, 400, *and* 500 ; or without altering the ratios, 3, 4, and 5 ; since $\frac{300}{400}=\frac{3}{4}$, and $\frac{400}{500}=\frac{4}{5}$.
Ans. The 1st, \$162.5 ; the 2d, \$216.666'; the 3d, \$270.833'.

4. An insolvent debtor owes to A \$250, to B \$100, and to C \$300. He is able to pay \$420 ; what would each creditor receive of the \$420 ?

Ans. A \$161.538'; B \$64.615'; C \$193.846'.

5. It is required to divide the number 180 into three parts which shall be to one another as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

Analysis. Reducing the proportional terms to a *common denominator*, we have them $\frac{6}{12}$, $\frac{8}{12}$, and $\frac{9}{12}$; and *these are to one another as the numerators* 6, 8, and 9 ; (§ 217).

Hence 6, 8, and 9 may be taken for the *proportional terms*.

Ans. $46\frac{22}{23}$; $62\frac{14}{23}$; and $70\frac{10}{23}$.

6. A father proposed to divide \$100 between his two sons in the ratio of $\frac{1}{3}$ to $\frac{1}{2}$, provided either of them could ascertain the portion offered to him. What would each portion be ?

Ans. The 1st, \$40 ; the 2d, \$60.

7. The sum of \$500 is to be divided among A, B, and C, in the proportion of $\frac{2}{3}$, $1\frac{1}{4}$, and $2\frac{1}{2}$, respectively. What will be the share of each ?

Ans. A's \$75.471'; B's \$141.509'; C's \$283.

8. Two persons form a partnership in trade, with a capital of \$3000, of which the first contributed \$1800, and the second the remainder. They gain \$900 : what is each one's share ?

Ans. The first, \$540 ; the second, \$360.

9. A bankrupt is indebted to A \$425.50 ; to B \$200 ; to C \$100 ; and to D \$85.75. He is able to pay \$500. If this sum be divided among his creditors proportionably to their respective claims, what will be the share of each ? *Ans.* A's \$262.249'; B's \$123.266'; C's \$61.633'; D's \$52.85'.

When each given Ratio has a separate Antecedent and Consequent.

§ 230. In Partitive Proportion, *when each given ratio has a separate antecedent and consequent,*—take, for the proportional terms, the given 1*st* and 2*d* terms, the *fourth proportional* to the 3*d*, 4*th*, and 2*d* terms, the fourth proportional to the 5*th* and 6*th* terms, and *last fourth proportional*,—and so on to the last term.

EXAMPLE.

10. Divide \$1700 between A, B, C, and D, so that A's share may be to B's as 1 to 2, B's to C's as $\frac{1}{3}$ to 1, and C's to D's as 3 to 4.

The given first and second terms are 1 and 2;

finding a *fourth proportional* to the 3d, 4th, and 2d terms, Rule XLII, we have $\frac{1}{3} : 1 :: 2 : 2\times1\div\frac{1}{3}=6$; and finding a fourth proportional to the 5th and 6th terms, and last *fourth proportional* 6, we have $3 : 4 :: 6 : 6\times4\div3=8$.

Now since B's share is to C's as $\frac{1}{3}$ to 1, or as 2 to 6;
and since C's share is to D's as 3 to 4, or as 6 to 8;

the four shares will be to one another as the numbers 1, 2, 6, and 8, which we accordingly take for the *proportional terms*

Then 17 : 1 :: \$1700 : \$100, A's share.

In like manner B's is \$200, C's \$600, and D's \$800.

11. Divide \$70 between A, B, and C, in such a manner that A's share shall be to B's as 2 to 3, and B's to C's as 4 to 5.
Ans. A's share \$16, B's \$24, and C's \$30.

12. Three persons in a joint speculation gain \$1000; which is to be divided so that the first share shall be to the second as 3 to 2, and the second to the third, as 5 to 6. Required the shares. *Ans.* \$405.405', \$270.270', and \$324.324'.

13. A, B, and C, in partnership, lose \$800. A's portion of the capital employed was $\frac{3}{4}$ of B's, and B's was $\frac{2}{3}$ of C's: what amount of the loss should be assigned to each?

Observe that A's capital was to B's as 3 *to* 4, *and B's to C's as* 2 *to* 3; and that their respective losses should be in like proportion.

Ans. To A \$184.615', to B \$246.153', and to C \$369.230'.

14. A farmer divided 500 acres of land between his three sons, giving to the first $1\frac{1}{2}$ times as much as to the second, and to the second $1\frac{1}{4}$ times as much as to the third. How much did he give to each?

The first share was to the second as $1\frac{1}{2}$ *to* 1, *and the second to the third as* $1\frac{1}{4}$ *to* 1.

Ans. To the 1st, $227\frac{9}{33}$, to the 2d, $151\frac{17}{33}$, to the 3d, $121\frac{7}{33}$ *A*.

Partitive Ratios dependent on Time.

§ 231. When *different periods* of *time* are involved in Partitive Proportion, the proportional terms to be used will be found by multiplying each term reckoned with time, by *its time*.

The different periods of time, before multiplying, must all be in the same denomination.

EXAMPLE.

15. Two persons, A and B, trade together ; A ventures $200 for 7 months, and B $300 for 9 months. They gain $100 ; how must it be divided between them?

$200 for 7 *mo.* is equivalent to (200×7) $1400 for 1 *mo.*
and $300 for 9 *mo.* is equivalent to (300×9) $2700 for 1 *mo.*

Having the time the same, that is, 1 month, in both cases, we take 1400 and 2700, or 14 and 27, for the proportional terms. without regard to time.

Ans. A must have $34.146', B $65,853'.

16. Three persons rent a pasture for $20. A put in 20 sheep for 4 months, B 36 sheep for 3 months, and C 45 sheep for 2 months : how much of the rent should accordingly be paid by each?

Ans. A must pay $5.755', B $7.769', C $6,474'.

17. E, F, and G, in partnership, have made $400. What will be the share of each, supposing E's stock in the business to have been $500 for 10 months, F's $900 for 1 *yr.* 3 *m.*, and G's $600 for 2 years?

Ans. E's $60.790', F's $164.133', G's $175.075.

18. A and B invested capital in a joint speculation as follows : A put in at first $1000, and 6 months after $500 more ; B advanced at first $2000, and 4 months after withdrew $600. At the end of 12 months the profits amounted to $800 : what was each one's share of the same?

A employed $1000 for 6 *m.*; $1000×6=$6000 for 1 *m.*;
and $1500 for 6 *m.*; $1500×6=$9000 for 1 *m.*

Then A's capital was equivalent to $15000 for 1 *month.*

In like manner find the equivalent for B's capital.

Ans. A's $350.877', B's $449.122'.

19. A, B, C, and D engaged in partnership for 2 years. At the outset A advanced $2000, B $3000, C and D each $4000. Six months afterwards A added $500 to his stock in the business, B $300, and C and D each withdrew $1000. At the end of the 2 years, the profits were found to be $800 ; to what amount of profit was each one entitled?

Ans. A $157.024', B $213.223', C and D each $214.876'.

MEDIAL PROPORTION.

§ **232.** MEDIAL PROPORTION is Proportion applied to finding in what *ratio* to one another two or more quantities, at *different rates* of value, must be taken, to form a compound of a given *medial or mean rate of value.*

For example, to find in what ratio to each other, rye at 37 *cents* per bushel, and oats at 25 *cents* per bushel, must be mixed together, that the mixture may be worth 30 *cents* per bushel.

Note. This part of Arithmetic is commonly called ALLIGATION.

RULE XLVI.

§ **233.** *To find the ratio of* TWO *or* MORE QUANTITIES *at different rates of value, for a* COMPOUND *of a given mean rate of value.*

1. *For two different rates*—take the quantities *inversely* as the *differences* between their respective rates and the mean rate.

2. *For three or more different rates*—find the ratio for one rate which is *less*, and another which is *greater*, than the mean rate as above; then for one of these two rates and another, or for two others, in like manner; and so on, until all the different rates are included, and add together all the *proportional terms found for the same rate.*

EXAMPLE.

To find in what ratio to each other, rye at 37 *cents*, and oats at 25 *cents* a bushel, must be mixed together, that the mixture may be worth 30 *cents* a bushel.

The differences between the rates of the *two ingredients* and the *mean rate* 30 cents, are

for the *rye* $37-30=7$, and for the *oats* $30-25=5$.

Then the quantity of RYE will be to that of OATS *inversely as* 7 *to* 5; and by converting the *inverse* into a *direct* proportion (§ 222), we find the quantity of RYE to that of OATS as 5 to 7; that is, the mixture must be in the ratio of 5 bushels of rye to 7 bushels of oats.

In other words, since $5+7=12$, $\frac{5}{12}$ of the mixture must be RYE, and $\frac{7}{12}$ must be oats, whatever be the quantity of the mixture.

Analysis. We wish to find in what ratio to each other, rye at 37 *cents*, and oats at 25 *cents* a bushel, must be mixed together, that the mixture may be worth 30 *cents* a bushel.

On 1 bushel of *rye* there is an *excess* of 7 cents *above*, and on 1 *bu.* of *oats* a deficiency of 5 *cts. below*, the *mean rate* 30 *cts.*

Then $\frac{1}{7}$ *bu.* of rye is in *excess* 1*ct.*, and $\frac{1}{5}$ *bu.* of oats is *deficient* 1 *ct.*, in relation to the mean rate.

This equal excess and deficiency *counterbalance each other.* Hence, $\frac{1}{7}$ *bu.* of rye and $\frac{1}{5}$ *bu.* of oats, mixed together, will be at *the mean rate.*

Again; $\frac{1}{7}=\frac{5}{35}$, $\frac{1}{5}=\frac{7}{35}$; and $\frac{5}{35} : \frac{7}{35} :: 5 : 7$ (§ 217).

Hence, the mixture must be in the ratio of 5 bushels of *rye* to 7 of *oats.*

This result being the same as that found by the Rule, the Analysis demonstrates the *first part of the Rule.*

EXERCISES.

1. In what ratio to each other must corn at 40 *cts.* a bushel, and oats at 25 *cts.* a bushel, be taken, to form a mixture worth 33 *cts.* a bushel? *Ans.* 8 *bu.* of corn to 7 *bu.* of oats.

2. In what ratio must one kind of coffee at 9 *cts.* a *lb.* and another at 13 *cts.* a *lb.*, be taken, to form a mixture of the two which shall be worth $12\frac{1}{2}$ *cts.* a *lb.*?
Ans. $\frac{1}{2}$ *lb.* at 9 *cts.* to $3\frac{1}{2}$ *lb.* at 13 *cts.*

3. In what ratio must one kind of wine at 90 *cts.* a gallon, and another at 75 *cts.* a *gal.*, be mixed, that the compound of the two may be worth $87\frac{1}{2}$ *cts.* a gallon?
Ans. $12\frac{1}{2}$ *gal.* at 90 *cts.* to $2\frac{1}{2}$ *gal.* at 75 *cts.*

4. In what ratio must two kinds of tea, at 75 *cts.* and 90 *cts.* a pound, be mixed together, that the mixture may be worth 83 cents a pound? *Ans.* 7 *lb.* at 75 *cts.* to 8 *lb.* at 90 *cts.*

5. In what ratio should two different kinds of sugar, at $9\frac{1}{2}$ *cts.* and 14 *cts.* a pound, be taken, to form a mixture which shall be worth $12\frac{1}{2}$ *cts.* a pound?
Ans. $1\frac{1}{2}$ *lb.* at $9\frac{1}{2}$ *cts.* to 3 *lb.* at 14 *cts.*

6. In what ratio should two different kinds of raisins, worth $12\frac{1}{2}$ *cts.* and $18\frac{1}{4}$ *cts.* a pound, be taken, to form a mixture which shall be worth $16\frac{2}{3}$ *cts.* a pound?
Ans. $1\frac{7}{12}$ *lb.* at $12\frac{1}{2}$ *cts.* to $4\frac{1}{6}$ *lb.* at $18\frac{1}{4}$ *cts.*

7. A farmer wishes to purchase two different qualities of land, rating at \$20 and \$35 per acre, in such quantities that the average rate shall be $\$27\frac{1}{2}$ per acre. In what proportion must the two kinds be purchased?
Ans. $7\frac{1}{2}$ acres, or equal quantities, of each.

8. A goldsmith wishes to form an alloy of gold which shall be 20 carats fine, from two alloys of the same metal, one of which is 23, and the other 18 carats fine. In what proportion must the two ingredients be taken?

Ans. 2 parts of 23, and 3 parts of 18 carats fine.

EXAMPLE,

Of three ingredients at different rates.

9. A farmer wishes to mix rye at 37 *cts.*, oats at 23 *cts.*, and corn at 32 *cts.* per bushel, in such proportions that the mixture shall rate at 31 *cts.* per bushel; what must be the proportions?

Applying the second part of the preceding Rule, we first suppose the *rye* and the *oats* to be formed into a mixture at the *mean rate* 31 *cts.*

This would require 8 bushels of *rye* to 6 bushels of *oats.*

We next suppose the *oats* and the *corn* to be formed into a mixture at the mean rate 31 *cts.*

This would require 1 bushel of *oats* to 8 bushels of *corn.*

The two mixtures thus formed, if *mixed together,* will evidently be at the mean rate; and this would give a mixture of the *three ingredients,* in the proportion of 8 bushels of *rye,* to 8 bushels of *corn,* and $6+1=7$ bushels of *oats.*

Observe that the proportion of *oats* in the mixture of the three ingredients, is found by adding together the proportional terms 6 and 1, found for the *oats* in the mixtures of the same ingredients taken *two and two.*

Since $8+8+7=23$, $\frac{8}{23}$ of the mixture will be rye, $\frac{8}{23}$ of it *corn,* and $\frac{7}{23}$ of it *oats,* whatever be the quantity of the mixture.

10. A merchant wishes to mix three kinds of tea, which rate at 90 *cts.*, \$1, and \$1.50 per *lb.*, in such portions that the compound shall rate at \$1.25 per *lb.* In what ratios must the different kinds be taken?

Ans. 25 *lb.* at 90 *cts.*, to 25 *lb.* at \$1, and 60 *lb.* at \$1.50

11. A grocer mixed brandy at 30 *cts.* per *gal.*, and wine at \$1 per *gal.*, with water, and found the compound to be worth 50 *cts.* per gallon. What ratios of the ingredients did he take,—the water being rated at 0?

Ans. 50 *gal.* of brandy to 70 of wine and 50 of water.

12. A farmer has one tract of land worth \$15 an acre, another worth \$22 an acre, and another worth \$25 an acre. In what proportion must he sell from the several tracts, that the average price received shall be \$20 an acre?

Ans. 7 acres at \$15 to 5 at \$22 and 5 at \$25.

Different Ratios of the same Ingredients may sometimes be found.

§ **234.** In Medial Proportion, when there are two or more rates *greater*, and two or more *less*, than the mean rate, *different ratios for the same ingredients* may be found, according to the different ways in which the ingredients may be taken, two and two, in adjusting them to the mean rate.

EXAMPLE.

13. Four different kinds of sugar, rating at 5 cents, 8 cents, 13 cents, and 14 cents, per *lb.* are to be formed into a mixture which shall rate at 10 cents per pound. What ratios must be taken?

Adjusting the two at 5 cents and 13 cents to the mean rate 10 cents, we find 3 *lb.* at 5 cents to 5 *lb.* at 13 cents.

Adjusting the two at 8 cents and 14 cents to the mean rate 10 cents, we find 4 *lb.* at 8 cents to 2 *lb.* at 14 cents.

Then 3 *lb.* at 5 cents, 4 *lb.* at 8 cents, 5 *lb.* at 13 cents, and 2 *lb.* at 14 cents, will form a mixture at the mean rate 10 cents.

Find other ratios for the four ingredients, according to each of the following different ways of adjusting the four different rates, taken two and two, to the mean rate 10 cents.

1*st*. 5 cents and 13 cents; 5 cents and 14 cents; 8 cents and 13 cents. *Ans.* 7 *lb.* at 5 *cts.*, 7 *lb.* at 13 *cts.*, 5 *lb.* at 14 *cts.*, 3 *lb.* at 8 *cts.*

2*d*. 5 cents and 13 cents; 8 cents and 13 cents; 8 cents and 14 cents. *Ans.* 3 *lb.* at 5 *cts.*, 7 *lb.* at 13 *cts.*, 7 *lb.* at 8 *cts.*, 2 *lb.* at 14 *cts.*

3*d*. 5 cents and 14 cents; 8 cents and 13 cents; 8 cents and 14 cents. *Ans.* 4 *lb.* at 5 *cts.*, 7 *lb.* at 14 *cts.*, 7 *lb.* at 8 *cts.*, 2 *lb.* at 13 *cts.*

4*th*. 5 cents and 14 cents; 5 cents and 13 cents; 8 cents and 14 cents. *Ans.* 7 *lb.* at 5 *cts.*, 7 *lb.* at 14 *cts.*, 5 *lb.* at 13 *cts.*, 4 *lb.* at 8 *cts.*

When the Quantity at one of the different Rates, is given.

§ **235.** In Medial Proportion, when the *quantity at one of the different rates*, is given, and the other quantity or quantities for a compound of a given mean rate, are required,—find the ratios for *all the different rates* as before; then,

The term found for the rate whose *quantity is given, will be to* that given quantity, *as* the term found for any other rate, *is to* the quantity required at that rate.

EXAMPLE.

14. How many pounds of tea at 85 *cts.* a *lb.*, and at 90 *cts.* a *lb.*, must be mixed with 5 *lb.* at $1 a *lb.*, that the mixture may be worth 94 *cts.* a *lb.*?

Finding the ratios at the three different rates, for a mixture at the mean rate 94 *cts.*, we have, 6 *lb.* at 85 *cts.*, 6 *lb.* at 90 *cts.*, and 13 *lb.* at $1.

Then 13 *lb.* : 5 *lb.* : : 6 *lb.* : the quantity required at 85 *cts.*; and in like manner is found the quantity required at 90 cents.

Ans. $2\frac{4}{13}$ pounds of each.

15. How many ounces of gold 23 carats fine, and how many 20 carats fine, must be compounded with 8 ounces 18 carats fine, that the alloy of the three different qualities may be 22 carats fine?

Ans. 48 *oz.* of the first; and 8 *oz.* of the second.

When the Quantities at two or more of the different Rates, are given.

§ **236.** In Medial Proportion, when the quantities at *two or more* of the different rates are given,—find at what rate the *sum* of the given quantities should be estimated, by dividing said sum into the sum of the *values* of those quantities. Then,

By substituting the *sum* of the given quantities, and *its rate* thus found, for those separate quantities, and their respective rates, the question may be solved like those in the preceding section (§235.)

EXAMPLE.

16. How many gallons of vinegar at 20 *cts.* a *gal.*, and at 50 *cts.* a *gal.*, should be mixed with 4 *gal.* at 25 *cts.* a *gal.*, and 2 *gal.* at 16 *cts.* a *gal.*, that the whole may be worth 28 cents a gallon?

The 4 *gal.* at 25 *cts.* a gallon amount to 100 *cts.*, and the 2 *gal.* at 16 *cts.* amount to 32 *cts.* We have then $4+2=6$ *gal.* amounting to $100+32=132$ *cts.* Hence these 6 *gal.* rate at $132\div6=22$ *cts.* per *gal.*

Substituting now 6 *gal.* at 22 *cts.* per *gal.*, for the 4 *gal.*, and the 2 *gal.*, at their respective rates, the question becomes of the same nature as the 14th, preceding.

Ans. 6 *gal.* at 20 *cts.*, and $3\frac{9}{11}$ *gal.* at 50 *cts.*

17. A farmer wishes to mix 10 bushels of corn at 35 *cts.* per bushel, and 8 bushels of rye at 40 *cts.* per bushel, with such a quantity of oats at 25 *cts.* per bushel, that the whole may be worth $.33$\frac{1}{3}$ per bushel. What quantity of oats must be taken? *Ans.* $8\frac{2}{5}$ bushels.

When the Sum of the Quantities at the several different Rates, and the mean Rate or value of the whole, are given,—to find the Quantity at each particular rate.

§ **237.** Find the ratios of the several different kinds, for a compound at the given mean rate. (§ 233). Then,

The *sum* of the terms or quantities thus found, *will be to* the quantity found for either rate; *as* the *given sum* of all the quantities, *is to* the quantity *required for that rate.*

EXAMPLE.

18. A vintner wishes to mix two kinds of wine, which rate at \$.75 and \$1.25 per gallon, in such proportion and quantities as to produce 100 gallons which shall be worth \$.87½ per gallon. What quantities of the two kinds must be taken?

The ratio of the two kinds for a compound at the mean rate \$.87½, will be found to be,

37½ *gal.* at \$.75, and 12½ *gal.* at \$1.25.

Then 37½+12½ : 37½ : : 100 : quantity required at \$.75.

Ans. 75 *gal.* at \$.75, and 25 *gal.* at \$1.25.

19. How many pounds of each of three different kinds of coffee, rating at 12 cents, 13 cents, and 15½ cents, a pound, respectively, must be taken for a mixture of 100 *lb.* which shall rate at 14 cents a pound?

Ans. 25 *lb.* at 12 *cts.*, 25 *lb.* at 13 *cts.*, 50 *lb.* at 15½ cents.

20. How many bushels of each of three different kinds of grain, which rate at \$.20, \$.37½, and \$.75, per bushel, respectively, must be taken to produce a mixture containing 500 bushels, which shall rate at \$.50 per bushel? *Ans.* 135 5/37 *bu.* at \$.20, 135 5/37 *bu.* at \$.37½, and 229 27/37 *bu.* at \$.75.

CONJOINED PROPORTION.

§ **238.** A CONJOINED PROPORTION consists of two or more pairs of *equivalent terms*—each consequent being of the same kind as the next antecedent, and the first and last terms also of the same kind.

Its application will be seen under

RULE XLVII.

§ **239.** *For the solution of questions in Conjoined Proportion.*

1. Set equivalent terms on the left and right of the sign $=$, one under another, and so that each succeeding left hand term shall be of the same kind with the preceding right hand one.

2. If the answer is to be of the same kind with the first term, set the *odd term* on the left;—otherwise, set it on the right.

3. All the terms on the same side with the odd term, must be multiplied together, for a dividend; and all the others, for a divisor. The quotient will be the answer, in the denomination of the given term of the same kind.

4. Each consequent and the next antecedent must be used in the same denomination.

EXAMPLE.

If 3 *qr.* of cloth be worth 4 *gal.* of wine, and 2 *gal.* of wine be worth 5 *lb.* of tea, how many quarters of cloth will be equal in value to 12 pounds of tea?

The equivalent terms are 3 *qr.* and 4 *gal.*; 2 *gal.* and 5 *lb.*; the *odd term* is 12 *lb.*; and the answer required is of the same kind with the 3 *qr.* Hence the arrangement will be,

$$\begin{aligned} 3\,qr. &= 4\,gal.; \\ 2\,gal. &= 5\,lb.; \\ 12\,lb. &= \text{how many } qr. \text{ of cloth?} \end{aligned}$$

The operation is $3\times2\times12\div4\times5=72\div20=3\frac{3}{5}\ qr.$

☞ The terms being *equivalent in value* on opposite sides, if we had the equivalent of the 12 *lb.*, the product of the terms on one side would be *equivalent to the product of the terms on the other.*

Hence the product on the side on which the number of terms is *complete*, divided by the *incomplete product* on the other side, gives the term *wanting on this latter side.* ☜

Analysis. 3 *qr.* of cloth being equal *in value* to 4 *gal.* of wine, and 2 *gal.* of wine to 5 *lb.* of tea, we wish to find how many *qr.* of cloth are equal in value to 12 *lb.* of tea.

Since 5 *lb.* $= 2$ *gal.*, 1 *lb.* $= \frac{1}{5}$ of 2 *gal.*;

and 2 *gal.*, being $\frac{2}{4}$ of 4 *gal.*, $= \frac{2}{4}$ of 3 *qr.*;

hence 1 *lb.* $= \frac{1}{5}$ of $\frac{2}{4}$ of 3 *qr.* $= \frac{2\times3}{5\times4}$ *qr.*;

and 12 *lb.* $= 12$ times $\frac{2\times3}{5\times4}$ *qr.* $= 2\times3\times12\div5\times4$.

By Proportion. 4 *gal.* : 2 *gal.* : : 3 *qr.* : $3\times2\div4=1\frac{1}{2}$ *qr.* $1\frac{1}{2}$ *qr.* of cloth equals in value 2 *gal.* of wine, or 5 *lb.* of tea.

Then 5 *lb.* : 12 *lb.* : : $1\frac{1}{2}$ *qr.* : $1\frac{1}{2}\times12\div5=3\frac{3}{5}$ *qr.*

EXERCISES.

1. If 7 *bu.* of wheat be worth as much as 3 cords of wood; and 9 cords of wood as much as 2 tons of hay; how many bushels of wheat should be exchanged for 5 tons of hay? *Ans.* $52\frac{1}{2}$ bushels.

2. If A can do as much work in 5 days as B can do in 8 days; and B as much in 4 days as C can do in 11 days; in how many days could A do the same that C could do in 20 days? *Ans.* $4\frac{6}{11}$ days.

3. If 3 barrels of corn be given for 7 *bu.* of wheat; and 4 *bu.* of wheat for 13 of rye; and 15 of rye for 20 of oats; how many bushels of oats would be an equivalent for 10 barrels of corn? *Ans.* $101\frac{1}{9}$ bushels.

4. Allowing that in a certain factory 6 girls do as much work in a day as 4 boys; and 8 boys as much as 6 men; how many men would be required to do as much work as 20 girls? *Ans.* 10 men.

5. If $10\frac{1}{2}$ yards of silk cost $15.75; and $6 will purchase 1 yard of broadcloth; and $4\frac{1}{4}$ *yd.* of broadcloth be bartered for 25 *yd.* of Irish linen; how many yards of silk would be an equivalent for 40 yards of the linen? *Ans.* $27\frac{1}{5}$ yards.

6. Supposing A to earn as much money in 4 months as B earns in 6 months; and B as much in 5 months as C in 7 months; and C as much in 10 months as D in 3 months; in what time could D earn the same that A could earn in 12 months? *Ans.* $7\frac{1}{2}\frac{4}{5}$ months.

7. If 12 *lb.* in the United States be equal to 10 *lb.* at Amsterdam; and 100 *lb.* at Amsterdam be equal to 120 *lb.* at Paris; how many pounds in the United States are equal to 100 *lb.* at Paris? *Ans.* 100 pounds

COMPOUND RATIO.

§ **240.** A *Simple Ratio* is the ratio of a single antecedent to its consequent; as the ratio of 3 to 5, equal to $\frac{3}{5}$.

§ **241.** A COMPOUND RATIO is the ratio of the *product* of two or more antecedents to that of their consequents; and is equal to the product of all the *simple ratios*.

Thus the *compound ratio* of 3 and 4 to 5 and 7, is the ratio of

$$3\times4 \text{ to } 5\times7, =\frac{3\times4}{5\times7}=\frac{3}{5}\times\frac{4}{7} \text{ or } \frac{3}{7}\times\frac{4}{5}.$$

The compound ratio $\frac{3\times4}{5\times7}$, we see, is equal to the product of the simple ratios of 3 to 5, and 4 to 7; or of 3 to 7, and 4 to 5

COMPOUND PROPORTION.

§ **242.** A COMPOUND PROPORTION is one in which the *compound ratio* of two or more antecedents to their consequents, is equal to the ratio of *a remaining antecedent* to its consequent.

Thus $\left.\begin{matrix}2:3\\6:8\end{matrix}\right\}$: : 5 : 10, is a Compound Proportion;

in which the compound ratio $\frac{2\times6}{3\times8}$ of 2 and 6 to 3 and $8=\frac{5}{10}$, the ratio of 5 to 10.

Compound Proportion is applied to the solution of questions which involve two or more simple proportions.

RULE XLVIII.

§ **243.** *To solve a question in* COMPOUND PROPORTION.

1. Take any two terms of the same kind, and the one of the same kind with the answer to be found, and dispose them in a *direct proportion*. (§ 227.)

2. Then take two other terms of the same kind, for another proportion whose *third term* is that of the *first proportion;* and so on, until all the terms are included.

3. Multiply the first terms together for a divisor, and the second and third together for a dividend: the quotient will be the answer required.

4. Each antecedent and its consequent must be used in the same denomination; as in simple proportion.

EXAMPLE.

If a footman can travel 150 miles in 5 days, when the days are 12 hours long, in how many days may he travel 275 miles, when the days are 10 hours long?

Taking first 150 *miles*, 275 *miles*, and 5 *days*, the question is,

If 150 *m.* be traveled in 5 days, in how many days may 275 *m.* be traveled? This gives the proportion,

150 *m.* : 275 *m.* : : 5 days : *Time required.*

Taking next 12 *hours*, 10 *hours*, and 5 *days*, the question is,

If when the days are 12 *hr.* long, 5 days be required, how many days will be required when they are 10 *hr.* long? This gives the proportion,

10 *hr.* : 12 *hr.* : : 5 days : Time required.

The *compound proportion* will then be,

$$\left.\begin{array}{l} 150\,m. : 275\,m. \\ 10\,hr. : 12\,hr. \end{array}\right\} : : 5 \text{ days} : \text{Time required.}$$

The operation is,

$$275\times12\times5\div(150\times10)=16500\div1500=11 \text{ days.}$$

The reason of the operation will become more evident if the compound proportion be stated thus;

$$150\times10 : 275\times12 : : 5 \text{ days} : \text{Time required.}$$

From this it appears that the time required, is a fourth proportional to 150×10, 275×12, and 5 days.

Analysis. Recollecting that we are to find a number of *days*, and supposing at first that the days in both cases are 12 *hr.* long,

1 mile would be traveled in $\frac{1}{150}$ of 5 days; that is, $\frac{5\,d.}{150}$;

and 275 *m.* would require 275 times as long as 1 *m.*; $\frac{5\,d.\times275}{150}$.

With regard now to the different length of the days. If the days were but 1 *hour* long, the number of days would be 12 times as great as when they are 12 *hr.* long;

$$\text{that is, } \frac{5\,d.\times275\times12}{150}.$$

And when the days are 10 *hr.* long, the number will be only $\frac{1}{10}$ as many as when they are but 1 *hr.* long,

$$\text{that is, } \frac{5\,d.\times275\times12}{150\times10}.$$

The expression thus found may be canceled, successively, by 5, 6, 5, 2, and 5. This will reduce it to 11 *days.* (§ 212).

EXERCISES.

1. If 5 oxen require an acre of grass for 9 days, how many acres will 20 oxen require for $30\frac{1}{2}$ days? *Ans.* $13\frac{5}{9}$ acres.

2. If 4 men eat 64 pounds of bread in 2 weeks, how many pounds will 16 men eat in 7 weeks? *Ans.* 896 pounds.

3. If a man travel 100 miles in 3 days of 13 hours length, how far might he travel in 33 days of $14\frac{1}{4}$ hours length? *Ans.* $1205\frac{10}{13}$ miles.

4. If 2 yards of cloth $1\frac{1}{2}$ *yd.* wide, cost \$10.25, what should be paid for 13 yards of like quality, which is $1\frac{3}{4}$ *yd.* wide? *Ans.* \$77.72$\frac{11}{12}$.

5. If a family of 10 persons, in 2 weeks, spend \$200, how long ought a family of 13 persons to be in expending \$500? *Ans.* $3\frac{11}{13}$ weeks.

6. If 6000 *lb.* of bread will supply a garrison of 100 men, for 2 months, how long will 12000 *lb.* supply three such garrisons? *Ans.* $1\frac{1}{3}$ months.

7. If the conveyance of 20 *cwt.*, 40 miles, cost \$15.87$\frac{1}{2}$, what should be charged for the conveyance of 50 *cwt.* 3 *qr.*, 100 miles? *Ans.* \$100.707'.

8. Allowing 4 men to mow 19 *A.* 3 *R.* 27 *P.* of meadow, in 5 days, how long ought 7 men to be employed in mowing 45 acres? *Ans.* 6.454' days.

9. If 11 *oz.* 8 *dr.* of bread be bought for $6\frac{1}{4}$ cents, when flour sells at \$5 a barrel, what quantity of bread should be bought for \$.75, when flour sells at \$6 a barrel. *Ans.* 7*lb.* 3*oz.*

10. Allowing the transportation of 15 *cwt.*, 100 miles, to amount to \$45.50, how far ought 37 *cwt.* 1 *qr.* 20 *lb.* to be carried for \$100? *Ans.* 88.079' miles.

11. If 25 men can dig a ditch 80 *ft.* long, 4 *ft.* wide, and 3 *ft.* deep, in 2 days, in what time ought 30 men to dig one 300 *ft.* long, 5 *ft.* wide, and 4 *ft.* deep? *Ans.* $10\frac{5}{12}$ days.

12. Allowing 17 head of cattle to consume 5 *A.* 2 *R.* 10 *P.* of pasture, in 30 days, how many acres would be consumed by 40 head of cattle, in 50 days? *Ans.* 21 *A.* 3 *R.* $10\frac{10}{17}$ *P.*

13. A contractor engaged to pave 15 miles of road in 12 months, and for that purpose employed 100 men. Seven months have now elapsed, and but 6 miles of the road have been completed; how many more men must be employed to finish the work in the time prescribed? *Ans.* 110 men.

EXAMINATION ON CHAPTER IX.

1. Find the ratio of $2\frac{1}{3}$ to 7, and of $\frac{2}{3}$ to $\frac{2}{5}$. *Ans.* $\frac{1}{3}$ and $1\frac{2}{3}$

2. Find the inverse ratio of $\frac{2}{3}$ of $\frac{3}{4}$ to $5\frac{1}{2}$. *Ans.* 11

3. Find the ratio of 3 *qr.* to 1 *cwt.* 2 *qr.* 20 *lb.* *Ans.* $\frac{21}{47}$.

4. Find the inverse ratio of 2 *cwt.* to 3 *qr.* 5 *lb.* *Ans.* $\frac{89}{224}$.

Find the 4th term in each of the following proportions.

5. $3\frac{1}{2}$ days : 10 days : : $\$37\frac{1}{2}$: 4th term. *Ans.* $\$107\frac{1}{7}$.

6. 7 men : 13 men :: $21\frac{1}{2}$ days : 4th term. *Ans.* $11\frac{15}{26}$ days.

7. 3 *cwt.* 1 *qr.* : 7 *cwt.* : : $\$25.12\frac{1}{2}$: 4th term. *Ans.* \$54.115'.

8. $30\frac{3}{4}$ miles : 100 miles :: 14 *T.* 1 *cwt.* : 4th term. *Ans.* 4 *T.* 6.407' *cwt.*

9. If $13\frac{1}{4}$ yards of broadcloth cost \$65.25, what should be paid for 17 *yd.* $2\frac{1}{2}$ *qr.* of cloth at the same rate? *Ans.* \$86.794'.

10. Allowing $9\frac{1}{2}$ barrels of flour to suffice a family for 12 months, how many barrels would the same family require for 2 *y.* 4 *m.*? *Ans.* $22\frac{1}{6}$ barrels.

11. If \$100 will supply a number of horses with oats, for 3 months, when oats is at 25 *cts.* per *bu.*, how long will the same sum supply them with the same article, when it is at $\$0.37\frac{1}{2}$ per bushel? *Ans.* 2 months.

12. Supposing a company of workmen to erect a building in 30 days, by working $8\frac{1}{2}$ hours per day; in how many days ought the work to be accomplished, if they employ $10\frac{1}{4}$ hours per day? *Ans.* $24\frac{36}{41}$ days.

13. A farmer bought 100 *A.* of land for \$5500, and afterwards sold 37 *A.* 3 *R.* 29 *P.* at the same price per acre. What did the portion sold amount to? *Ans.* \$2086.218'.

14. A father dying left his son a fortune, of which he spent $\frac{1}{4}$ in 2 years, $\frac{2}{3}$ of the remainder lasted him 3 years longer, when he had only \$3000 left. What fortune did his father bequeath him? *Ans.* \$12000.

15. Three pipes will separately fill a cistern with water in 3 hours, 4 hours, and 5 hours. If the three pipes discharge into the cistern together, in what time will it be filled? *Ans.* $1\frac{13}{47}$ hours.

16. If A could do a piece of work in 10 days, B in 12 days, and C in 15 days, in what time could A and B together do it? In what time the three together do it?

Ans. A and B in $5\frac{5}{11}$ days; A, B, and C in 4 days.

17. A merchant sold 4 *yd.* 3 *qr.* of cloth, at $5.25 per yard; 13 *yd.* 2 *qr.* of silk, at $1.12½ per yard; and 7 *yd.* 2½ *qr.* of linen, at $0.62½ per yard. What did the whole amount to?
Ans. $44.890'.

18. How many yards of carpeting, which is $\frac{7}{8}$ of a yard wide, will be sufficient for a room 18 *ft.* 9 *in.* long, and 16½ *ft.* wide? *Ans.* 39$\frac{2}{7}$ yards.

19. A grocer exchanged 12 *cwt.* 3 *qr.* 18 *lb.* sugar, at $7.18¾ per *cwt.*, for wheat at 81¾ *cts.* per bushel. How many bushels of wheat were required to pay for the sugar?
Ans. 113.505' bushels.

20. A gentleman has a flower garden 30 *p.* 3 *yd.* in length, and 17 *p.* 4 *yd.* in breadth. Without altering the area, he wishes to increase the breadth to 20 poles; how much must the length be diminished? *Ans.* 3 *p.* 2$\frac{13}{22}$ *yd.*

21. If 5⅜ yards of cloth which is 3 *qr.* wide, cost $10.50, what ought to be paid for 14¼ yards of cloth, of like quality, 5 *qr.* wide? *Ans.* $46.395'.

22. A and B barter as follows, viz., 4 *T.* 17 *cwt.* 3 *qr.* of hay, at $12 per ton, for corn at $1.87½ per barrel. How many barrels of corn will be required as an equivalent for the hay?
Ans. 31$\frac{7}{25}$ barrels.

23. If the transportation of 15 *cwt.* 3 *qr.*, 100 miles, costs $45.50; what ought to be charged for the conveyance of 37 *T.* 1 *cwt.*, 400 miles? *Ans.* $8562.666'.

24. The sum of $500 is to be divided between A, B, and C, in such a manner that A's share shall be to B's as 5 to 3, and B's to C's as 3 to 7. What will be the share of each?
Ans. A's $166.666', B's $100, C's $233.333'.

25. If 5 laborers, working 8 hours a day, can dig a cellar 25 *ft.* long, 19 *ft.* wide, and 8 *ft.* deep, in 15 days, in what time ought 12 laborers, working 9¼ hours a day, to dig a cellar 20½ *ft.* long, 17 *ft.* wide, and 8¼ *ft.* deep? *Ans.* 4.089' days.

26. A person failing in business, is indebted to A $300, to B $250, and to C $400. His assets amount to $550: what will be the proportional share of each creditor?
Ans. A's $173.684', B's $144.736', C's $231.578'.

27. A merchant has coffee at 12 *cts.*, 16 *cts.*, and 18 *cts.*, a *lb.*, and wishes to form, of the different kinds, a mixture worth 14 cents a pound. What must be the ratios of the mixture?
Ans. 6 *lb.* at 12 *cts.*, 2 *lb.* at 16 *cts.*, and 2 *lb.* at 18 *cts.*

28. An estate consisting of 5000 acres of land, is to be divided among three persons, so that A's share shall be to B's as 2 to 5 and B's to C's as 3 to 8. Required the share of each.
Ans. A's 491$\frac{49}{61}$, B's 1229$\frac{31}{61}$, C's 3278$\frac{42}{61}$.

29. Allowing 2 *lb.* of tea to be worth as much as 13 *lb.* of coffee; and 5 *lb.* of coffee, as much as 9 *lb.* of sugar; and $20\frac{1}{2}$ *lb.* sugar, as much as 50 *lb.* of rice; how many pounds of tea will be equivalent to 100 *lb.* of rice? *Ans.* $3\frac{59}{117}$ *lb.*

30. How many gallons of brandy at 25 *cts.* per *gal.*, and how much water must be mixed with 5 gallons of brandy at 40 *cts.* per *gal.*, that the adulterated compound may rate at 30 *cts.* per *gal.*? *Ans.* $1\frac{3}{7}$ *gal.* of each.

31. A, B, and C form a partnership for 12 months. A and B at once advance $2500 each as their portion of the capital stock. At the end of 3 months C advances $3000; and B withdraws $1000. The profits amount to $1500: what is the share of each? *Ans.* A's $576.923', B's $403.846', C's $519.230'.

32. How many ounces of gold 18 carats fine, must be compounded with 5 *oz.* 23 carats fine, and 8 *oz.* 22 carats fine, that the compound may be 20 carats fine? *Ans.* $15\frac{1}{2}$ ounces.

33. In a joint speculation, A had $\frac{1}{4}$ of the capital for 6 months, B $\frac{2}{5}$ of it for 9 months, and C the remainder for 12 months. The loss amounted to $500: what loss should be sustained by each?

Ans. $80.645' by A, $193.548' by B, $225.806' by C.

34. A merchant has four kinds of coffee, worth, respectively, 8 *cts.*, 12 *cts.*, 14 *cts.*, and 20 *cts.* per *lb.* What quantity of each kind must be taken to form a mixture of 100 pounds, which shall rate at 15 *cts.* per pound?

Ans. $42\frac{4}{13}$ *lb.* at 20 *cts.*, $19\frac{3}{13}$ *lb.* at each of the other rates.

35. The sum of $3000 is to be divided between A, B, C, and D in such a manner that A's share shall be to B's as $2\frac{1}{2}$ to 3; B's to C's as 3 to 4, and C's to D's as 5 to 6. What will be the share of each? *Ans.* A's $524.475', B's $629.370', C's $839.160', D's $1006.993'.

36. If 20 bushels of wheat be worth $37\frac{1}{2}$ *bu.* of rye, and $3\frac{1}{2}$ *bu.* of rye be worth 1 barrel of corn, and 2 barrels of corn be worth 9 *bu.* of oats, and 25 *bu.* of oats be worth 1 ton of hay; how many tons of hay should be exchanged for 100 bushels of wheat? *Ans.* $9\frac{9}{14}$ tons

CHAPTER X.

PERCENTAGE, AND ITS APPLICATIONS.

PERCENTAGE.

§ **244.** *Percentage* is an allowance at a certain rate for *every hundred.*

The Latin *per centum*, or its contraction, *per cent.*, signifies *by the hundred.*

One per cent. on any number is *one* for *every hundred; two per cent.* is *two for every hundred;* and so on.

One per cent. on any sum of money is $1 for every $100; *two per cent.* is $2 for every $100; and so on.

What is meant by 3 *per cent.?* By 4 per cent.? By 6 per cent.?
How much is 1 per cent. on $100? On $200? On $400?
How much is 2 per cent. on $100? On $300? On $500?
How much is 3 per cent. on $100? On $400? On $700?

Ratio of Percentage.

§ **245.** The *ratio of percentage* is the rate per cent. divided by 100. It is usually expressed *decimally*, by making two *integral* figures *decimals* in the rate per cent.—prefixing 0s when necessary.

Thus 1 per cent. on any number is $\frac{1}{100}$, or .01, of that number; the fraction $\frac{1}{100}$, or .01, being the *ratio* of percentage for 1 per cent.

What is the *ratio* for 2 per cent.? For 3 per cent.? For 4 per cent.? For 5 per cent.? For 6 per cent.? For 10 per cent.?

§ **246.** When the rate per cent. is a *mixed number*, or a proper *fraction*, the ratio of percentage will be a *mixed fraction*, which may be reduced to a simple fraction.

Thus the ratio for $2\frac{1}{2}$ per cent. is $\frac{2\frac{1}{2}}{100}=.02\frac{1}{2}=.025$; and the ratio for $\frac{3}{4}$ per cent., that is, $\frac{3}{4}$ of 1 per cent. is $\frac{\frac{3}{4}}{100}=.0075$.

What is the ratio for $\frac{1}{2}$ per cent.? For $\frac{1}{4}$ per cent.? For $1\frac{1}{2}$ per cent.? For $1\frac{1}{4}$ per cent.? For $2\frac{3}{4}$ per cent.? For $5\frac{1}{4}$ per cent.?

§ **247.** The *ratio* of percentage multiplied by 100 produces the *rate per cent.* Thus the ratio being .03, the rate per cent. is $.03\times100=3$.

The rate per cent. is readily found from the decimal ratio, by making two *decimal* figures in the ratio *integral.*

Thus the ratio being .0125, the rate per cent. is $1.25=1\frac{1}{4}$.

If the ratio is .01, what is the rate per cent.? If the ratio is .02? If the ratio is .015? If the ratio is .0225? If the ratio is .0375?

Basis of Percentage.

§ **248.** The sum or number on which percentage is computed, at any given rate, may be called, for convenience, the *basis of percentage.*

Thus when we say, 2 per cent. on $300, the basis of percentage is $300.

If the rate per cent. is 2, and the amount of percentage $4, what is the *basis of percentage?* If the rate per cent. is 3, and the amount of percentage $9? If the rate per cent. is 4, and the amount of per centage $12? If the rate per cent. is 5, and the amount of percentage $20? If the rate per cent. is 6, and the amount of percentage $30?

PERCENTAGE ON A GIVEN NUMBER.

RULE XLIX.

§ **249.** *To find an* AMOUNT OF PERCENTAGE *on a given number.*

Multiply the given number, or *basis* of percentage, by the *ratio of percentage;* the product will be the *amount* of percentage.

EXAMPLE.

To find 5 per cent. on $150.

The *ratio* of percentage is $\frac{5}{100}=.05$. (§ 245).

Then $\$150\times.05=\$7.50$, the amount of percentage.

Observe that 5 per cent. on $150, is $\frac{5}{100}$ of $150, and multiplying by a fraction finds such a part of the multiplicand as is expressed by the multiplier.

By PROPORTION. $100 : $150 : : $5 : Amount of percentage.

From a given Amount of Percentage, to find the Basis of Percentage.

From the preceding Rule it follows, that

§ **250.** The amount of percentage divided by the *ratio of percentage,* gives the basis of percentage.

INSURANCE.

§ **251.** Insurance is an obligation assumed by a company, or an individual, to pay for the *loss* or *damage* of property by fire, shipwreck, or other casualty.

The *Policy* is the certificate of insurance issued by the *Insurers,* sometimes called the *underwriters,* to the person *insured.*

The price or *Premium* paid for insurance is usually a specified percentum on the amount insured.

EXERCISES.

1. What would be the annual premium of insurance on a house, valued at \$3500, at 1 per cent.? *Ans.* \$35.

2. What would be the annual premium of insurance on a manufactory, valued at \$20000, at $1\frac{1}{2}$ per cent.? *Ans.* \$300.

3. What would be the insurance on a shipment of goods amounting to \$5200, from New York to New Orleans, at $1\frac{1}{4}$ per cent.? *Ans.* \$65.

4. What would be the insurance on a ship and cargo, from New York to Liverpool, the ship being valued at \$30000, and the cargo at \$42000, at $1\frac{3}{4}$ per cent.? *Ans.* \$1260.

5. The annual insurance on a paper mill, at 2 per cent., amounts to \$165.50. What is the value insured?

Here the amount of percentage is given, to find the basis of percentage. (§ 250). *Ans.* \$8275.

6. The policy of insurance on a shipment of cotton, at $\frac{3}{4}$ per cent, costs \$75. What was the amount insured? *Ans.* \$10000.

7. A gentleman procures an insurance on his house and furniture, valued at \$7500, at $\frac{5}{8}$ per cent. per annum. What is the annual premium to be paid? *Ans.* \$46.875.

8. The annual insurance on a manufactory, at $1\frac{1}{4}$ per cent., amounts to \$250. What is the amount insured? *Ans.* \$20000.

9. What would be the cost of insurance on a store house, valued at \$5000, and a stock of goods amounting to \$7500.50, at $2\frac{1}{2}$ per cent.? *Ans.* \$312.51'.

10. A steamboat is valued at \$35000, and the proprietors of it obtain an insurance on $\frac{3}{4}$ of its value, at $3\frac{1}{4}$ per cent. Required the amount of premium to be paid? *Ans.* \$853.125.

11. An upholsterer has his warehouse, valued at \$8000, insured to the amount of $\frac{4}{5}$ of its value, at $1\frac{1}{2}$ per cent. per annum; and \$10000 worth of furniture insured at $1\frac{3}{4}$ per cent. per annum. What does he pay for insurance annually? *Ans.* \$271.

DUTIES.

§ **252.** DUTIES, in commerce, are certain charges, imposed by Government, on articles imported from foreign countries. These duties are paid in the *Custom Houses* established in the several *ports of entry* of foreign trade.

A *specific duty* is a specified charge on the quantity; as on a *yard*, pound, gallon, &c., of the article.

An *ad valorem* duty is a specified percentum on the cost of the articles in the country from which they were imported (*Ad valorem* is a Latin phrase which signifies, *according to value.*)

An *invoice* is a written statement of articles and their cost.

Tare, Draft or Tret, and Leakage, are allowances made for the weight of the box or cask, &c., containing the articles; or for waste, leakage, &c., before the duty is computed.

12. What would be the amount of duty on an invoice of broadcloth amounting to $5465.75, at 30 per cent.?
Ans. $1639.725.

13. What would be the amount of duty on an invoice of philosophical instruments, amounting to $30200, at 25 per cent.?
Ans. $7550.

14. The duty on a box of goods, at 12½ per cent., amounted to $65; what was the amount of the invoice? *Ans.* $520.

15. A lot of cutlery was purchased in Sheffield for $575.18¾. What would be the amount of duty on it at 9½ per cent.?
Ans. $54.642'.

16. An invoice of Irish linens paid a duty of $330. What was the amount invoiced, the duty being 33 per cent.?
Ans. $1000.

17. A quantity of ready-made clothing was purchased in Paris for $1534.12½. What would be the amount of duty to be charged, at 15 per cent.? *Ans.* $230.118'.

18. An invoice of silk was purchased in Canton for $12000.25, what is the amount of duty to be paid, at 10 per cent.? *Ans.* $1200.025.

19. The duty on an importation of iron, at 20 per cent., amounted to $15000. For what sum then was the iron purchased abroad? *Ans.* $75000.

20. What would be the amount of duty to be paid on 3 *cwt* 3 *qr.* 20 *lb.* of steel, purchased at $21.12½ per *cwt.*,—at 9 per cent.? *Ans.* $7.46'.

ONE GIVEN NUMBER A PERCENTAGE ON ANOTHER.

RULE L.

§ **253.** *To find what* PERCENTUM *one given number is of another.*

Divide the number which is made the *percentage* by that which is made the *basis* of percentage. The quotient will be the *ratio* of percentage; and this quotient multiplied by 100, will produce the required *rate per cent.*

EXAMPLE.

To find what percentum \$9 is of \$150.

$9 \div 150 = .06$, the *ratio* of percentage;

and $.06 \times 100 = 6$, the required *rate per cent.* (§ 247).

This Rule follows from the preceding one. (§ 249).

Proportion. \$150 : \$100 : : \$9 : the required *rate per cent*

TAXES.

§ **254.** TAXES are contributions in money, imposed by Government on property, and frequently on persons, for public purposes.

A *poll* or *capitation* tax is a tax on the person, without regard to property. A tax on property is sometimes *specific*, that is, a specified sum on certain articles; but it is most commonly *ad valorem*, or a specified *percentum* on the value.

EXERCISES.

21. A person paid a tax of \$52.88$\frac{1}{4}$ on \$3525.50 worth of property; at what rate per cent. was the tax assessed?

Ans. 1$\frac{1}{2}$ per cent.

22. The property of a town amounts to \$50000, and the citizens resolve to tax it to the amount of \$1125, for public improvements. At what percentum must the tax be laid?

Ans. 2$\frac{1}{4}$ per cent.

23. Allowing a gentleman to pay a tax of \$1379.16$\frac{2}{3}$, on property amounting to \$82750, at what rate per cent. is the tax assessed? *Ans.* 1$\frac{2}{3}$ per cent.

24. A citizen pays taxes as follows, viz: for three polls at \$1.25; on a carriage \$10; on silver plate valued at \$500, 3 per cent.; on \$15000 of other property, $\frac{3}{4}$ per cent. What amount of tax does he pay? *Ans.* \$141.25.

25. The taxable polls in a State amount to 325830, and are assessed at \$1.12$\frac{1}{2}$. The landed property in the State is valued at \$38400000; at what percentum must the land be taxed, that the revenue from both sources may amount to \$462558.75?

Ans. $\frac{1}{4}$ per cent.

A Required Number, increased by a Percentum of itself, equal to a Given Number.

RULE LI.

§ **255.** *To find a number which, increased by a specified* PER CENTUM OF ITSELF, *shall be equal to a given number.*

Divide the given number by 1 *plus* the *ratio of percentage:* the quotient will be the number required.

EXAMPLE.

What number, increased by 5 per cent. of itself, is equal to 210?

The *ratio of percentage* is .05; and 1+.05 is 1.05.

Then 210÷1.05=200, the *number required.*

Proportion. 100+5 : 100 : : 210 : the *number required.*

☞ The reason of the Rule is evident from considering that, as the *required number* multiplied by 1+ the *ratio* of percentage, would be increased by the specified percentum of itself, and thus be equal to the *given number;* so the given number divided by 1+ the *ratio* of percentage, will give the required number.

In the example, 200×1.05=210, which is 200 increased by .05 of 200; then 210÷1.05 gives 200, the required number. ☜

COMMISSION.

§ **256.** COMMISSION is a compensation to an Agent, Factor, or Commission Merchant, for buying or selling for another; and is usually reckoned at a certain percentum on the amount of purchase or sale. BROKERAGE is a commission charged by Brokers, or dealers in *money, stocks,* &c., on the amount of exchange, purchase or sale, which they effect for another.

EXERCISES.

26. An agent receives $500 to purchase goods—himself to retain a commission of 2 per cent. on the amount of purchase. What is the amount of purchase to be made?

Ans. $490.196'.

27. A factor receives a remittance of $1200 to purchase cloth, at a commission of 1¼ per cent. on the purchase. What will be the amount of purchase? *Ans.* $1185.185'.

28. A commission merchant sold goods amounting to $1785.81¼; at a commission of 1½ per cent. What sum must the merchant pay to the owner of the goods?

Ans. $1759.025'.

29. An agent is intrusted with $450 to purchase iron. His commission being ¾ per cent. on the purchase, how many tons of iron can he buy at $30 per ton? *Ans.* 14.888' *T.*

30. A, as factor for B, sells 50 bales of cotton, averaging 450 pounds, at $7\frac{1}{2}$ *cts.* per *lb.*—commission $1\frac{3}{4}$ per cent. With the proceeds A purchases for B, a supply of provisions,—commission $\frac{1}{2}$ per cent. on the purchase. What sum is expended for provisions? *Ans.* $1649.71'.

A Required Number, diminished by a Percentum of itself, equal to a Given Number.

RULE LII.

§ **257.** *To find a number which, diminished by a specified percentum of itself, shall be equal to a given number.*

Divide the given number by 1 *minus* the *ratio of percentage*. the quotient will be the number required.

EXAMPLE.

What number diminished by 5 per cent. of itself, is equal tc 190?

The ratio of percentage is .05; and 1—.05 is .95.

Then, 190÷.95=200, the *number required.*

Proportion. 100—5 : 100 : : 190 : the *number required.*

☞ The reason of the Rule is evident from considering that, as the *required number* multiplied by 1 *minus* the *ratio* of percentage, would be diminished by the specified per centum of itself, and thus be equal to the *given number;* so the given number divided by 1 *minus* the *ratio* of percentage, will give the required number.

In the example, 200×1—.05=190, which is 200 diminished by .05 of 200; then 190÷1—.05 or .95=200, the required number. ☜

STOCK.

§ **258.** STOCK, or *Capital,* is money or other property employed in any way to produce a profit; as in manufactures, banking, &c. Bonds of the Government are also called Government Stock.

The stock of a Company is divided into shares, usually of $100 each; and the owner of one or more shares is called a *Stockholder.*

The *nominal value,* or *par* value of a share of stock, is its *first cost,* or the sum originally invested in it.

Stock is said to be *above par,* at a *premium,* or an *advance,* when it sells for more than its nominal value; and *below par,* or at a *discount,* when it sells for less than its nominal value.

The rise or fall of stock is expressed by a *percentum* on its *nominal or par value.*

EXERCISES.

31. When bank stock sells at $5\frac{1}{2}$ per cent. below par, what nominal amount of stock can be bought for \$945 ?

The nominal amount is such, that, when diminished by $5\frac{1}{2}$ per-centum of itself, it will be equal to \$945. *Ans.* \$1000.

32. When rail road stock sells at a discount of $7\frac{1}{2}$ per cent., what nominal value of it can be bought for \$2775 ?
Ans. \$3000.

33. What nominal amount of stock in an insurance office, at an advance of 5 per cent., can be purchased for \$2100 ? and what amount in another, at a discount of 5 per cent., can be purchased for \$1900 ? *Ans.* \$2000 in each.

34. What amount of stock in the capital of a manufacturing company, at a discount of $3\frac{1}{2}$ per cent., can be purchased for \$1930 ? and what amount in another, at an advance of 4 per cent., can be purchased for \$3120.
Ans. \$2000 ; and \$3000.

35. A merchant ships, from New York to Charleston, a stock of goods amounting to \$5000. He wishes to insure for a sum which shall cover both the value of the goods, and the premium for insurance. For what amount must the policy be taken, at $\frac{1}{2}$ per cent.?

The amount of the policy, diminished by $\frac{1}{2}$ per cent. of itself, will be \$5000. *Ans.* \$5025.125'.

36. A manufactory valued at \$2500, is insured, at $1\frac{1}{4}$ per cent., in such a sum, that, in case of a total destruction of the establishment, the proprietors may claim, at the insurance office, the value of the property, together with the premium paid for insurance. What was the amount insured ?
Ans. \$2531.645'

PERCENTAGE OF PROFIT AND LOSS.

The preceding Rules of Percentage are applicable to various questions relating to *profit* and *loss* in trade. This application of the subject will be seen in the following Exercises.

It must be carefully observed that,

§ **259.** In calculating the Percentage of Profit, or Loss, in trade, —— the *cost* of the commodity is always regarded as the *basis* of percentage (§ 248).

The amount of Profit or Loss found from the Cost, and the percentum of Profit or Loss.

§ **260.** The amount of profit or loss is the amount of percentage on the cost, at the given *rate* per cent. of profit or loss. (§ 249).

When the rate per cent. is an aliquot part of 100, the amount of percentage will often be found, most readily, by taking *such part of the cost,* or basis of percentage.

EXERCISES.

1. A merchant bought a quantity of cloth for \$330, and sold the same at a profit of $33\frac{1}{3}$ per cent. What amount of profit did he make.

The amount of profit is $\$330\times.33\frac{1}{3}$; or since the rate per cent. $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, the answer will be found, more readily, by taking $\frac{1}{3}$ of \$330. *Ans.* \$110.

2. A grocer bought a hogshead of sugar for \$55.75, and sold it at a profit of $12\frac{1}{2}$ per cent. What amount of profit did he make? $12\frac{1}{2}$ is $\frac{1}{8}$ of 100. *Ans.* \$6.968'.

3. A merchant bought silk for \$160, and, on account of its becoming damaged, sold it at a loss of $5\frac{1}{2}$ per cent. What amount of loss did he sustain? *Ans.* \$8.80.

4. A flour dealer bought 130 barrels of flour, at $\$4.12\frac{1}{2}$ per barrel, and sold it at a profit of 10 per cent. What amount of profit did he make? *Ans.* \$53.625'.

5. A person purchased 25 barrels of dried apples, at $\$2.12\frac{1}{2}$ per barrel. On account of damage received, he must sell them at a loss of 15 per cent.; what amount of loss will he sustain? *Ans.* \$7.968'.

6. A grocer purchased 100 *gal.* 3 *qt.* of wine, at $\$.61\frac{1}{4}$ per gallon. He sold one half of the wine at a profit of 40 per

cent., and the other half at a profit of $33\frac{1}{3}$ per cent.; what amount of profit did he make? *Ans.* \$22.625'.

The price in selling found from the Cost, and the percentum of Profit or Loss.

§ **261.** The price in selling at a *profit* is equal to the cost + the percentage on the cost; and the price in selling at a *loss* is equal to the cost *minus* the percentage on the cost. (§ 249).

7. A farmer bought land at \$44.75 per acre, and immediately sold it at an advance of 25 per cent. At what price per acre was the land sold? *Ans.* \$55.937'.

8. A quantity of Irish linen was bought at \$.$62\frac{1}{2}$ per yard; but, becoming damaged, is to be sold at a loss of $12\frac{1}{2}$ per cent. Required the selling price per yard? *Ans.* \$.$54\frac{11}{16}$.

9. A manufacturer sold cotton cloth at a profit of 20 per cent. on the cost of making it, which was \$.$12\frac{1}{2}$ per yard. At what price was the cotton sold? *Ans.* \$.15.

10. A grocer purchased coffee at \$10.50 per *cwt.* From a depreciation in value, the coffee must be sold at a loss of $3\frac{1}{2}$ per cent. What will be the selling price per *cwt.*?

Ans. \$10.132'.

The Cost found from the amount of Profit or Loss, and the percentum of Profit or Loss.

§ **262.** The cost is the sum or basis on which the amount of profit or loss is the specified *per centum*. (§ 250.)

11. By selling a lot of iron at an entire profit of \$22.50, I made 9 per cent. on the cost. What did I pay for the iron?

Ans. \$250.

12. An agent sold a consignment of flour at a profit of \$.35 per barrel, which was 10 per cent. on the cost. What did the flour cost per barrel? *Ans.* \$3.50.

13. A merchant sold broadcloth at an advance of $33\frac{1}{3}$ per cent., and in so doing realized a profit of \$2 per yard? What did the cloth cost him per yard? *Ans.* \$6.

14. A commission merchant sold a consignment of hats at a loss to the manufacturer of \$12 per dozen, and 25 per cent. What was the cost per dozen to the manufacturer?

Ans. \$48.

15. A speculator purchased a quantity of pickled pork, which he disposed of at a profit of 20 per cent.—this being a profit of \$2.50 per barrel. What did he pay per barrel?

Ans. \$12.50

16. A broker purchased railroad stock for $450 less than its nominal value; and that was at a discount of 9 per cent. What was the par value of the amount of stock purchased?

Ans. $5000.

The percentum of Profit or Loss found from the Cost, and the amount of Profit or Loss.

§ **263.** The percentum of profit or loss is the percentum that the amount of profit or loss is of the *cost*, regarded as the basis of percentage. (§ 253).

17. If silk were purchased at $1.50 per yard, and sold at $2 per yard, what would be the gain per cent.?

Ans. $33\frac{1}{3}$ per cent.

18. If a lot of books be purchased at $2.50 per dozen, and sold at $3 per dozen, what will be the gain per cent.?

Ans. 20 per cent.

19. A merchant bought hats at $36 per dozen, and sold them at $4.62 apiece. What was his gain or loss per cent.?

Ans. Gain 54 per cent.

20. A shop-keeper bought shoes at $18.75 per dozen, and sold them at $1.37½ a pair. What was his percentum of profit or loss. *Ans.* Loss 12 per cent.

If the shoes had been sold at $1.93¾ a pair, what would have been the percentum of profit or loss?

Ans. Profit 23.733' per cent.

The Cost found from the Price in selling and the percentum of Profit or Loss.

§ **264.** The cost is such that, when increased by the specified percentum of itself, it will equal the price in selling at a *profit;* or diminished by the specified percentum of itself, will equal the price in selling at a *loss.* (§ 255 and § 257).

21. A grocer sells sugar at $.12½ per *lb.*, and in so doing makes a profit of 25 per cent. What did the sugar cost per pound? *Ans.* $.10.

22. A merchant sold a lot of damaged flour at $3.75 per barrel,—sustaining a loss of 12½ per cent. What did the flour cost per barrel? *Ans.* $4.285.

23. A drover sold a lot of cattle for $900; which was 20 per cent. more than he gave for them. What did he pay for them? *Ans.* $750.

24. What does a shop-keeper pay for tea, when, by selling it at \$1.25 per pound, he makes a profit of 25 per cent.?

Ans. \$1.

25. A manufacturer sold a lot of shoes for \$400; which was $33\frac{1}{3}$ per cent. advance on the cost of making them. What amount of profit did the manufacturer realize? *Ans.* \$100.

26. A gentleman sold his stock of household furniture for \$3200.50; which was at a loss of $22\frac{1}{2}$ per cent. on the cost of it. What amount of loss was sustained on the furniture?

Ans. \$929.177'.

27. A merchant bought a piece of cloth containing 34 *yd.* 3 *qr.*, at $\$4.87\frac{1}{2}$ per yard, and sold the same at a profit of 30 per cent. What amount of profit did he make?

Ans. \$50.821'.

28. An upholsterer bought a piece of carpeting containing 41 *yd.* $3\frac{1}{2}$ *qr.*, at $\$1.12\frac{1}{2}$ per yard. For what sum must he sell the whole to realize a profit of $33\frac{1}{3}$ per cent.?

Ans. \$62.812'.

29. A grocer having sold a lot of liquors for \$375.50, finds that his profit is at the rate of 25 per cent. For what sum were the liquors purchased?

The cost is such that, when increased by 25 per cent. of itself, it is equal to \$375.50. *Ans.* \$300.04'.

30. A speculator sold a quantity of flour for \$800.25, and found that he had sustained a loss on it of $3\frac{1}{2}$ per cent. For what sum was the flour purchased? *Ans.* \$829.274'.

31. A manufacturer made 300 *yd.* $2\frac{1}{4}$ *qr.* of woolen cloth, at a cost of $\$3.18\frac{3}{4}$ per yard. For what sum must the whole be sold to clear 15 per cent.? *Ans.* \$1101.748'.

32. A farmer bought a tract of land for \$3000. Having put improvements on the land at an expense of $\$735.37\frac{1}{2}$, for what sum must the whole property be sold to realize a gain of 20 per cent.? *Ans.* \$4482.45'.

33. A merchant bought 30 *yd.* of silk at $\$.87\frac{1}{2}$ per yard. Having sold one half of the silk at \$1.25 per yard, what will be his percentum of profit on the whole if the other half be sold at $\$1.18\frac{3}{4}$ per yard? *Ans.* 39.285' per cent.

34. Sold a quantity of bacon at $\$.12\frac{1}{2}$ per pound, and in so doing made a profit of 25 per cent. If it had been sold at \$.09 per pound, what would have been the percentum of profit or loss? *Ans.* 10 per cent. loss.

First find the cost, and thence the percentum of profit or loss *at the latter price.*

EXERCISES ON CHAPTER X.

1. A person insured his house and furniture in the sum of \$5000, for five years, at $2\frac{1}{4}$ per cent. annually. What amount of premium did he pay? *Ans.* \$562.50.

2. An invoice of foreign manufactured articles, amounting to \$1300.87, is chargeable at the custom house with a duty of 20 per cent. What is the amount of duty to be paid? *Ans.* \$260.174.

3. The tax on a certain landed estate amounts to \$734.25. What is the estimated value of the estate, the tax being levied at $\frac{3}{4}$ per cent.? *Ans.* \$97900.

4. A broker's commission on a purchase of bank bills amounting to \$1000, was \$2.50. At what percentum was commission charged? *Ans.* $\frac{1}{4}$ per cent.

5. An agent receives \$3000 to purchase merchandise,—$1\frac{1}{4}$ per cent. on the amount of purchase to be retained. What amount of purchase can the agent make? *Ans.* \$2962.96'.

6. What nominal amount of stock in the capital of a road company, at $10\frac{1}{2}$ per cent. below par, could be bought for \$1342.50? *Ans.* \$1500.

7. A merchant bought 75 yards of cloth for \$262.50. Its transportation cost $\$5.18\frac{3}{4}$, and the insurance on its transportation $\frac{1}{4}$ per cent. At what price per *yd.* must it be sold to clear $33\frac{1}{3}$ per cent.? *Ans.* \$4.770:.

8. The property of a city amounting to \$5000000, is to be taxed to the amount of \$100000, for the purpose of constructing a railroad. At what percentum must the tax be laid? *Ans.* 2 per cent.

9. Bought a quantity of wheat for $\$700.62\frac{1}{2}$. Paid for transportation, and other charges on it $\$43.06\frac{1}{4}$; and sold it at a profit of 15 per cent.; what was the amount of profit made? *Ans.* \$111.553.

10. A grocer bought coffee at \$.10 per pound, and sold the same at $\$.12\frac{1}{2}$ per pound. What percentum of profit did he make on the coffee? *Ans.* 25 per cent.

11. A merchant sold a lot of cloth at \$6.50 per yard, and thus realized a gain of 25 per cent. If the cloth had been sold at \$5 per yard, what would have been his percentum of gain or loss? *Ans.* $3\frac{11}{13}$ per cent. loss.

12. Find the amount of stock in an insurance office, at 10 per cent. advance, that could be bought for \$440, and the amount in a canal company, at 10 per cent. discount, that could be bought for \$1000. *Ans.* \$400; and \$900.

13. A bought 200 head of cattle, for \$1700. He sold to B one fourth of the number, at \$13.25, and the remainder to C,

at $15.37½ a head. What per centum of profit or loss did A realize on his cattle? *Ans.* Profit 74.632' per cent.

14. A commission merchant is to sell 12000 *lb.* of cotton, and invest the proceeds in sugar,—retaining 1¾ per cent. on the sale, and the same on the purchase. Cotton selling at 7 *cts.* and sugar at 5 *cts.*, a *lb.*, what quantity of sugar can the merchant buy? *Ans.* 16222.11' pounds.

15. On a stock of leather, sold at 18¾ per cent. profit, a merchant clears $237.50. At what cost was the leather bought? *Ans.* $1266.666'

16. A church which cost $20000, is insured at ¼ per cent., in such a sum, that, in case of its being destroyed by fire, the insurance company shall be liable for the cost of the building, and the premium of insurance. For what amount is the insurance taken? *Ans.* $20050.125'.

17. By selling a piece of damaged silk at $1.25 a yard, a merchant sustains a loss of 16⅔ per cent. At what price was the silk purchased, and what was the amount of loss on 20 yards? *Ans.* $1.50; and $5.

18. A broker purchased for another $3000 of canal stock, at an advance of 3 per cent.; and charged a commission of ¾ per cent. on the sum disbursed. What did the broker pay for the stock, and what is his commission? *Ans.* $3090; and $23.175.

19. A commission merchant sold a lot of cloth for $200; at a profit to the owner of 20 per cent. If the cloth ha been sold for $175, what would have been the percentum of profit or loss to the owner,—the commission in each case being 2 per cent.? *Ans.* Profit, 5 per cent.

20. Put into the hands of a broker $1000, in bank bills at a discount of 10 per cent. The broker was to purchase stock in the capital of a mining company, and have 1¼ per cent. on the nominal amount of the stock purchased. The stock is bought at 55 per cent. below par; what amount of brokerage do I have to pay? *Ans.* $25.

CHAPTER XI.

INTEREST.—DISCOUNT.—EQUATION OF PAYMENTS.—COMPOUND INTEREST.

INTEREST.

§ **265.** INTEREST is the price or premium paid for the *use of money*, and is reckoned at a certain percentum, *annually*, on the sum for which it is paid.

Thus, if A lends B $500 at 6 per cent., for 3 years; at the end of that time, B must return the $500, and pay to A, as *interest*, $6 on each $100, for each of the 3 years.

The *Principal* is the sum for which Interest is paid—the *Amount* is the sum of the Principal and Interest.

What is the interest of $100, for one year, at 6 per cent.? For 2 years? For 3 years? For 5 years? For 9 years?

What is the interest of $200, for one year at 7 per cent.? For 3 years? For 4 years? For 8 years? For 10 years?

What is the amount of $100, for one year, at 6 per cent.? Of $200 for 2 years? Of $300 for 3 years? Of $1000 for 4 years?

Legal Interest.

§ **266.** The *legal rate* of interest is the rate prescribed by law.

On debts in favor of the United States, interest is computed at 6 per cent.

In the individual States, the legal rate of interest is 6 per cent., with the exceptions that

In New York, South Carolina, Michigan, Wisconsin, and Iowa, it is 7 per cent.; in Georgia, Alabama, Mississippi, and Florida, it is 8 per cent.; in Texas, 10 per cent.; in Louisiana, 5 per cent., excepting that in the Louisiana banks it is 6 per cent.

Usury.

§ **267.** *Usury* is any rate of interest above the rate *allowed by law.* The taking of usury is prohibited by law, in the different States, under various penalties; such as a forfeiture of double the usury, a forfeiture of the usury and a part of the debt, &c.

In some of the States, the law sanctions a rate of interest higher than the *prescribed legal rate*, when the higher rate has been stipulated for between the debtor and creditor. The limit to such stipulated rate of interest is usually 10 or 12 per cent.; but in the State of Wisconsin the law sanctions any rate which may be agreed upon between the parties.

RULE LIII.

§ **268.** *To calculate* INTEREST *on any sum of money.*

1. Multiply the Principal by the *ratio* of percentage; the product will be the interest for *one year*.

2. *Interest for any number of years* is found by multiplying one year's interest by that number.

3. *Interest for* MONTHS and DAYS may be found by taking proper *parts of one year's interest*,—in which case, reckon 12 months to a year, and 30 days to a month.

EXAMPLE.

Find the interest on \$220.12½ for 4 *y.* 7 *m.* 10 *da.* at 6 per cent.

The *ratio of percentage* is .06; (§ 245).

		\$220.125	
		.06	
The int. for	6 *m.* is ½ of	13.20750,	the int. for *one year*.
		4	
		52.8300	the int. for 4 *years*;
" "	for 1 *m.* is ⅙ of	6.6037'	" " for 6 *months*,
" "	for 10 *da.* is ⅓ of	1.1006'	" " for 1 *month*;
		.3668'	" " for 10 *days*.
		\$60.9011	" " for 4 *y.* 7 *m.* 10 *da.*

Finding the interest for *one year*, is the same as finding 6 per cent. on the principal; (§ 249). The interest for 7 *m.* 10 *da.* is found by *aliquot parts*; (§ 208).

The several parts of the interest added together, make the whole interest.

By *Compound Proportion.* The question may be stated thus:

If the interest on \$100 for one year, be \$6; what will be the interest on \$220.12½, for 4 years, 7 months, and 10 days?

$$\text{Hence, } \left.\begin{matrix}\$100 : \$220.12\tfrac{1}{2} \\ 1\,y. : 4\,y.\ 7\,m.\ 10\,da.\end{matrix}\right\} :: \$6 : \textit{Interest required.}$$

Note. In calculating interest for *months* and *days*, strict accuracy would require that the number of days in those months be taken, and 365 days allowed to a year. The method by the preceding Rule is commonly used, for convenience, though it allows but 360 days to a year.

EXERCISES.

1. A borrowed of B $500, at 6 per cent. interest, for 2 years 5 months, and 12 days. What did the interest amount to? *Ans.* $73.50.

2. C loaned to D $750.50, for 1 year, 8 months, and 20 days, at 7 per cent. interest. What did the interest amount to? *Ans.* $90.476'.

3. Borrowed of my neighbor $175.25, at 6 per cent. int est. What amount will I owe him, if the money be kept 3 *y.* 11 *m.*? *Ans.* $216.433'.

4. Loaned to a friend $436.75, at 5 per cent. interest. What amount of money will discharge the debt, at the end of 1 *y.* 2 *m.* 15 *da.*? *Ans.* $463.136'.

5. An account with a merchant, amounting to $75.87½, bore interest, at 6 per cent., for 2 *y.* 4 *m.* 10 *da.* What sum was then required to pay off the principal and interest? *Ans.* $86.623'.

6. A farmer hired a laborer 12 months, for $125. Having deferred the payment of his wages for 3 *y.* 1 *m.* 25 *da.*, what amount should the farmer now pay, allowing interest at 6 per cent.? *Ans.* $148.645'.

7. Bought a plantation for $6000; of which one fourth is to be paid in hand, and the remainder in 2 *y.* 6 *m.*, with interest at 8 per cent. What will be the amount of the remainder, at the expiration of the credit? *Ans.* $5400.

8. A person buys a house for $3600, to be paid in 3 equal instalments, in one, two, and three years, with interest at 6 per cent. from the time of purchase. What will be the entire amount of principal and interest? *Ans.* $4032.

9. A speculator borrowed $5000, which he immediately invested in land, and in 6 months sold the land for $7500, on a credit of 12 months, with interest from the time of sale. Money being at 6 per cent. interest, what is the speculator's gain or loss at the end of the 12 *m.*; at which time he pays the $5000? *Ans.* Gained $2500.

10. A planter consigned to a commission merchant 45000 *lb.* of cotton, which the latter sold at 9½ *cts.* per *lb.*—commission ½ per cent. The proceeds due the planter were not paid until the expiration of 1 *y.* and 10 *m.*; what was the amount at that time, allowing interest at 7 per cent.? *Ans.* $4799.506'.

A Concise Method of Computing Interest at 6 per cent. for Months and Days.

Reckoning 12 months, of 30 days each, to a year.

§ **269.** *To find the interest at 6 per cent. for a number of months*—multiply the principal in *dollars* by the number of months, and divide the product by 200; and

For a number of days—multiply the principal in dollars by the number of days, and divide the product by 6000.

The divisions may be abbreviated by rejecting the 0s from the divisors; (§ 152). The quotient, in each case, will be the interest in *dollars.*

EXAMPLES.

1. To find the interest of \$50 for 11 months, at 6 per cent.

$\$50 \times 11 = \$550$; and $\$550 \div 200 = \$2.75$.

The two integral 0s may be omitted in the divisor 200, if we make two integral figures *decimals* in the dividend \$550.

We shall then have $\$5.50 \div 2 = \$2.75$.

2. To find the interest of \$50 for 19 days, at 6 per cent.

$\$50 \times 19 = \$950$; and $\$950 \div 6000 = \$.158'$.

We may omit the three 0s in the divisor 6000, by making three integral figures *decimals* in the dividend \$950.

We shall then have $\$.950 \div 6 = \$.158'$.

3. The interest of 50 cents, for 11 *months,* would be

$\$.50 \times 11 \div 200 = \$5.50 \div 200$, or $\$.0550 \div 2, = \$.027'$;

and for 19 *days,* it would be

$\$.50 \times 19 \div 6000 = \$9.50 \div 6000$, or $\$.00950 \div 6, = \$.001'$.

☞ The interest for 1 *year,* or 12 months, is $\frac{6}{100}$ of the principal; for *one month* it is $\frac{1}{12}$ of that, that is, $\frac{1}{12}$ of $\frac{6}{100} = \frac{6}{1200} = \frac{1}{200}$ of the principal; and the interest for 1 *m.* multiplied by the given number of months, will give the interest for that number of months; hence we find 200 for a *divisor,* and the given number of months for a *multiplier.*

Again. The interest for 1 *day* is $\frac{1}{30}$ of the interest for 1 *month,* that is, $\frac{1}{30}$ of $\frac{1}{200} = \frac{1}{6000}$ of the principal; and the interest for 1 *da.* multiplied by the given number of days, will be the interest for that number of days; hence we find 6000 for a divisor, and the given number of days for a multiplier. ☜

When the Rate of Interest is greater or less than 6 per cent.

§ 270. Having found the interest at 6 per cent., $\frac{5}{6}$ of that interest will be the interest at 5 per cent.; $\frac{7}{6}$ of it will be the interest at 7 per cent.; and in like manner for any other rate.

We may here remark that, as 365 days make a year, the interest found from allowing but 360 days to a year, is *greater than* the *true interest* in the same ratio in which 360 is less than 365.

360 is less than 365 by $\frac{5}{365}=\frac{1}{73}$ of 365; hence the interest found for days by the preceding methods, should be diminished by $\frac{1}{73}$ of itself for the true interest for the given time. These methods are, however, in general use; and, having thus the general consent, are to be considered as correct.

By these methods perform the following

EXERCISES.

11. Find the interest on $54 for 6 *m.* 20 *da.*, at 6 per cent.

First find the interest for 6 *m.*, and then for 20 *da.*, and add the two together for the whole interest. *Ans.* $1.80.

12. Find the interest on $100 for 9 *m.*, 18 *da.*, at 6 per cent. *Ans.* $4.80.

13. Find the interest on $230 for 11 *m.* 15 *da.*, at 6 per cent. *Ans.* $13.22½.

14. Find the interest on $1234.75 for 120 *da.*, at 6 per cent. *Ans.* $24.69½.

15. Find the interest on $2300.25 for 137 *da.*, at 5 per cent. *Ans.* $43.768'.

16. Find the interest on $4360.12½ for 54 *da.*, at 5 per cent *Ans.* $32.700'.

17. Find the interest on $1385.50 for 23 *da.*, at 7 per cent *Ans.* $6.196'.

18. Find the interest on $3879.06¼ for 9 *da.*, at 7 per cent. *Ans.* $6.788'.

19. Find the interest on $7351.18¾ for 13 *da.*, at 7 per cent. *Ans.* $18.582'.

20. Find the interest on $2360.25 for 7 *m.*, at 8 per cent. *Ans.* $110.14½.

21. Find the interest on $1500.31¼ for ~~21~~ *da.*, at 6 per cent. *Ans.* $5.251'.

22. Find the interest on $9000.87½ for 17 *m.*, at 7 per cent. *Ans.* $892.586'.

23. Find the interest on $8730.62½ for 5 *m.* 23 *da.*, at 6 per cent.:—also at 5 per cent., and at 8 per cent.
Ans. $251.733'; $209.777'; $335.644'.

When the Time of Interest is the Interval between two Dates.

§ **271.** When the time for which interest is to be computed, is the interval between two given dates, it is customary to include only one of those two days in the time of interest.

The interval found by the method of § 195, will be the proper one, since that interval will include but one of the days of the two given dates. (§ 196).

When the time is *years*, *months*, and *days*, we may employ the method of Rule LIII, or find the interest for the *months*, and *days*, by § 269, § 270, and add such interest to that found for the given number of *years*.

24. What is the interest on $250 from January 15th, 1840, to June 10th, 1842, at 6 per cent.? *Ans.* $36.083'.

25. What is the interest on $192.25 from February 20th, 1841, to May 12th, 1843, at 5 per cent.? *Ans.* $21.361'.

26. What is the interest on $370.12½ from September 5th, 1842, to December 1st, 1845, at 7 per cent.? *Ans.* $83.915'.

27. What is the interest on $500.18¾ from October 28th, 1844, to January 16th, 1847, at 8 per cent.? *Ans.* $88.811'.

28. What is the interest on $734.62½ from November 13th, 1845, to August 29th, 1848, at 10 per cent.?

Ans. $205.286'

Partial Payments.

§ **272.** No one method has been universally approved for computing the *remainder due* on a debt at interest, on which *partial payments* have been made.

The method which has been adopted by the Supreme Court of the United States, and by the courts in most of the individual States, may be stated thus:

Whenever a PAYMENT, or the AGGREGATE of *payments made*, will discharge the interest then due, add said interest to the principal, and from the amount subtract the payment, or the aggregate of payments up to that time.

The remainder is to be treated as a *new principal*, dating from the time of the last payment, and payments on this principal are to be credited as before; and so on.

Note. When no particular rate of interest is stipulated for between the debtor and creditor, it is understood that the *prescribed legal rate* (§ 266) at the *place where the debt is contracted*, is the one intended; as that is the only rate which, in such *case, the* law will enforce.

EXAMPLE.

$4000. *Washington, January* 1st, 1840.

On demand, I promise to pay to James Wealthy, Four thousand dollars, with interest at 6 per cent.; for value received.

John Ready.

This note was *endorsed* as follows:

July 1st, 1841, received $300
April 11th, 1843, recieved $700

Settlement was demanded, and full payment made, August 20th, 1845; what was the remainder or balance then due?

From the date of the note, January 1st, 1840, to that of the first payment, July 1st, 1841, was 1 *year* and 6 *months.* (§ 195).

Interest on $4000 for that time, at 6 per cent., $360.

The payment $300 was not sufficient to discharge the *interest then due.*

From the date of the note to that of the 2d payment, is 3 *y.* 3 *m.* 10 *da.*

Interest on $4000 for that time, $786.66.

The whole payment, $300+$700=$1000, up to the date of the 2d payment, will discharge the interest then due.

Adding the interest to the principal, $4000+$786.66= $4786.66.

Subtracting the whole payment made, from the amount, we find $3786.66, for a *new principal,* dating April 11th, 1843.

Amount of $3786.66 from April 11th, 1843, to August 20th, 1845,—which was the balance due at the time of settlement. —$4317.13'.

Were there other payments on the note, we should treat them and the new principal according to the same method.

EXERCISES.

$950. *Philadelphia, June* 26th, 1844.

29. On the first day of January next, I promise to pay to Timothy Friend, Nine hundred and fifty dollars, with interest; for value received. *Jacob Faithful.*

This note is endorsed as follows:

March 20th, 1845, received $430.
May 15th, 1845, received $234.75.

What was the balance due, June 1st, 1846?

Ans. $353.172'.

$3000. *New York, May* 1st, 1845.

30. Six months after date, I promise to pay to John Prosperous, Three thousand dollars, with interest; for value received.

William Needy.

The endorsements on this note are as follows:

November 1st, 1845, received $1000.
October 10th, 1846, received $93.75.
December 20th, 1847, received $300.50.

What was the balance due, January 1st, 1848?

Ans. $2030.23'.

Rate per cent., or Time of Interest—how found.

§ **273.** The *Rate per cent. is found* by dividing the given Interest by the interest of the Principal, at 1 per cent., for the given Time; and

The Time, in the denomination of *years, months*, or *days, is found* by dividing the given interest by the interest on the principal for *one year*, month, or day, respectively.

☞ These methods are evident from considering, *first*, that, as the interest at 1 per cent. × any given *rate*, would produce the interest at that rate; so the given interest ÷ the interest at 1 per cent., will give the required rate; and

Secondly, that, as the interest for 1 year, &c., × the given time, produces the interest for that time; so the given interest ÷ the interest for 1 year, &c., will give the required time. ☜

EXERCISES.

31. At what rate per cent. must $250 be put at interest, to amount to $287.50, in 2 *yr.* 6 *m.*?

The given interest is $287.50—$250=$37.50.

Ans. 6 per cent.

32. In what time will $300 amount to $373.50, if the rate of interest be 7 per cent.? *Ans.* 3 *yr.* 6 *m.*

33. At what rate per cent. must $1000 be put at interest, to amount to $1120, in 1 *yr.* 6 *m.*? *Ans.* 8 per cent.

34. In what time will $475.37½ amount to $532.42, if the rate of interest be 6 per cent.? *Ans.* 2 years.

35. In what time will $100, or any other principal, double itself, if put on interest at 6 per cent.? and in what time would it double itself, at 7 per cent.?

Ans. 16⅔ years; and 14²⁄₇ years.

36. A loaned to B $200 for 2 *yr.* 7 *m.* 25 *da.* At the end of that time B paid the debt, and loaned to A $150 until he received from A as much interest as he had paid him; how long did A keep the money? (*It is immaterial what was the rate of interest.*) *Ans.* 3.537' years.

DISCOUNT.

§ **274.** Discount is a deduction from a debt due at a future time, without interest, on account of the *present payment* of the debt.

The *Present Worth* of such a debt, is that sum which, at interest, would *amount to the debt* by the time the debt becomes due; and the present worth subtracted from the future debt, leaves the *discount.*

For example, the rate of interest being 6 per cent., the *present worth* of $106 due in one year, without interest, is $100, since $100 would amount to $106 in one year; and the *discount* is $6.

The rate of interest being 6 per cent., what is the *present worth* of $112 due in 2 years, without interest? Of $118 due in 3 years? Of $224 due in 2 years? Of $336 due in 2 years?

The Present Worth corresponds with a Principal at interest, &c.

From the above it is evident that,

§ **275.** The present worth of a debt not due, and not bearing interest, may be regarded as a *principal;* the discount, as the *interest* on such principal for the time the debt has to run; and the debt itself, as the amount of such principal and interest.

RULE LIV.

§ **276.** *To find the* principal *when the Amount, Time, and Rate of interest are given.*

Divide the given amount by the amount of $1, found for the given time, at the given rate of interest: the quotient will be the principal required.

EXAMPLE.

A debt of $500 will be due in 3 years, without interest. What is the *present worth* of the debt, allowing the rate of interest to be 6 per cent.?

The required present worth is the principal of which $500 is the amount for 3 years, at 6 per cent.

The interest of $1 for 3 years, at 6 per cent., is 18 cents, and the amount $1.18.

Then $500÷1.18=$423.72, the principal or *present worth.*

☞ The reason of the rule is evident from considering, that, as the amount of $1 *multiplied by* any principal, produces the amount of that principal for the same *time* and *rate* of interest; so, the *amount* of any principal *divided by* the *amount* of $1 for the same time and rate, will give that principal.☜

EXERCISES.

In finding the amount of $1, in the application of this Rule, let the decimal be extended to *four figures*, if it does not become *even* with two or three.

1. What principal would amount to $650 in 2 years, allowing the rate of interest to be 5 per cent.? *Ans.* $590.909'.

2. What is the present worth of $1000, payable in 1 year and 6 months, when money is worth 8 per cent. interest? *Ans.* $892.857'.

3. What principal would amount to $2500.75, in 3 years and 8 months, when the rate of interest is 6 per cent.? *Ans.* $2049.795.

4. What *discount* should be allowed for the present payment of $400, due in 3 years and 5 months,—interest at 6 per cent.? *Ans.* $68.05.

5. What would be the present worth of $3050, payable in 4 years and 9 months, when the rate of interest is 8 per cent.? *Ans.* $2210.144'.

6. A note for $750 will become due in 2 years, 3 months, and 20 days. What would be the discount for present payment, allowing money to be at 7 per cent. interest? *Ans.* $104.173'.

7. Sold property amounting to $3000, on a credit, without interest, of 12 months. What sum in hand would be an equivalent for the debt, allowing money to bring 8 per cent. interest? *Ans.* $2777.777'.

8. A note for $1200 has 1 year, 10 months, and 15 days to run, without interest. What sum in hand would be an equivalent for the note, supposing the rate of interest to be 6 per cent.? *Ans.* $1078.651'.

9. A merchant bought goods amounting to $4200, on a credit of 6 months, without interest. What sum in ready money would discharge the debt, allowing interest to be at 8 per cent.? *Ans.* $4038.461'.

10. A farmer sold a plantation containing 275 acres, at $30 per acre,—to be paid in two equal instalments, in one and two years, without interest. The purchaser immediately proposing

to pay an equivalent in ready money, what sum should the farmer receive, rating money at 6 per cent. interest?

Ans. \$7574.544'.

Discount in Banks.

§ **277.** *Bank Discount* is the interest on a sum loaned, deducted at the time the loan is made.

A note on which money is borrowed from a bank, has usually from two to four months in which to *mature*, that is, become due. The time specified in the note is, by custom, extended three or four days, called *days of grace*, before payment is required.

The bank deducts the interest from the *face of the note*, or sum for which the note is written,—days of grace being included in the calculation; the remainder is the sum paid for the note, and is called the *avails* or *proceeds* of the note.

EXAMPLE.

A note for \$500, payable in 60 days, is discounted in a bank, at 6 per cent. Required the *avails* of the note.

Adding three days of grace, the time is 63 *days.*

The interest on \$500 for 63 days, is \$5.25. This interest is the bank *discount.* Then \$500—5.25=\$494.75, the *avails of the note.*

The borrower receives \$494.75 from the bank, and, at the end of the 63 days, must pay the bank \$500.

It is sometimes desirable to know for what principal a note should be given, that the *avails* of it in bank shall be a given sum. Hence,

RULE LV.

§ **278.** *To find the Principal from which the Bank Discount deducted, will leave a* GIVEN SUM.

Divide the given sum by \$1 *minus* the interest of \$1 for the given time—days of grace included; the quotient will be the principal required.

EXAMPLE.

To find for what principal a note, payable in 60 days, must be given, that, when discounted in bank, at 6 per cent., the proceeds of it shall be \$500.

The interest of \$1 for 63 days is \$.0105; and \$1—\$.0105 =\$.9895.

Then \$500÷.9895=\$505.30', the principal required.

☞ The reason of the Rule appears from considering, that, as the *avails* of $1 for the given time, *multiplied by* any principal, will produce the *avails* of that principal for the same time; so, the avails of any principal, *divided by* the avails of $1 for the same time, will give that principal. ☜

EXERCISES.

In these Exercises add 3 days of grace to the time specified; and in finding the interest of $1, in the application of this Rule, let the decimal be extended to *four figures*, if it does not become *even* with two or three.

11. What would be the proceeds of a note for $1000, due in 90 days, if discounted in bank, at 6 per cent.? *Ans.* $984.5.

12. A note for $300, payable in 4 months, is discounted in bank, at 8 per cent. Required the sum received for it. *Ans.* $291.8.

13. What would be the proceeds of a note for $200.25, having 3 months, or 90 days, to run, if discounted in bank, at 6 per cent.? *Ans.* $197.14'.

14. A note for $430.50, payable in 60 days, is discounted in bank at 7 per cent. Required the amount of the discount. *Ans.* $5.273'.

15. What would be the difference between bank discount and the true discount on a note for $5000, payable in 4 months or 120 days, reckoning interest at 6 per cent.? *Ans.* $4.46.

16. A merchant wishes to borrow in bank $2500, for 90 days. For what principal must his note be drawn, rating interest at 6 per cent.? *Ans.* $2539.36'.

17. A person wishes to pay a debt of $375.25, by having a note discounted in bank, for 60 days, at 6 per cent. For what must the note be made? *Ans.* $379.231'.

-mer owes a debt in bank of $500. He pays the payable in 3 months, which is discounted at receives $125.50 as the balance of proceeds what amount was the note made? *Ans.* $635.347'.

EQUATION OF PAYMENTS.

§ **279.** The *Equation of Payments* consists in reducing two or more different times at which payments of money are to be made, without interest, to one *equitable mean time* for the payment of the whole.

For example. A owes B $200 to be paid in 5 months, and $300 to be paid in 9 months; and it is required to find an *equitable mean* time for the payment of the whole $500.

RULE LVI.

§ **280.** *To find the proper period of credit for the sum of two or more payments due at different times, without interest.*

Multiply each payment not due by its own period of credit, and divide the sum of the products thus obtained, by the sum of all the separate payments. The quotient will be the time required.

The multipliers must all be used in the *same denomination* of time.

EXAMPLE.

A owes B $600; of which, $300 is to be paid in hand; $200 in 6 months; and the remaining $100 in 18 months. If the whole were reduced to *one payment*, what would be the proper *credit* to be allowed?

Multiplying the $200 by its credit, 6 months, we find 1200; and multiplying the $100 by its credit, 18 months, we find 1800.

The sum of these products is $1200+1800=3000$.

Then $3000\div600$, the sum of all the payments, gives 5 *months*, for the credit to be allowed on the $600.

☞ The period of credit for the *sum* of the payments, should be such that the discount on said sum, would be equal to the *sum* of the *discounts* on the separate payments, for their respective credits.

The preceding Rule proceeds on the supposition that the *discount* is equal to the *interest*, for the same time. The discount is, however, less than the interest; being the interest on the *present worth* of a debt, instead of the *principal.* The Rule is, therefore, inaccurate; though, for convenience, it is adopted in business.

In the preceding example,

The interest on $200 for 6 *m.* is the same as on $1 for 1200 months and the interest on $100 for 18 *m.* is the same as on $1 for 1800 months

The whole interest involved is therefore equal to the interest on $1, for 1200+1800=3000 *months;* and the same interest would accrue on the $600 in 3000÷600=5 *months.*

Hence 5 months is the credit to be allowed on the sum $600. ☞

EXERCISES.

1. C is indebted to D $900; of which $200 will be due in 6 months; $300 more in 9 *m.*; and the remainder in 12 months. What would be the proper time for the payment of the whole at once? *Ans.* $9\frac{2}{3}$ months.

2. A merchant bought goods amounting to $5000; of which he was to pay $3000 in hand, and the remainder in 6 months. It is since agreed that the whole shall be paid at one time; what is the proper credit to be allowed? *Ans.* $2\frac{2}{5}$ months.

3. A certain sum of money was to be paid as follows, viz: $\frac{1}{4}$ of it in 2 years, $\frac{1}{2}$ of it in 3 years, and the rest in 4 years and 6 months. The debtor proposing to pay the whole at the same time, it is required to find the proper term of credit.

Ans. $3\frac{1}{8}$ years.

4. A engaged to pay to B, $200 on the 1st day of January; $300 on the 15th of April; and $400 on the 20th of August. They now agree to make but one payment of the whole, and wish to know on what day that payment will be equitably due.

The $200 was due on the 1st of January; when the $300 was entitled to a credit of 3 *m.* 15 *da.*, to the 15th of April; and the $400, to a credit of 7 *m.* 20 *da.*, to the 20th of August.

$300×(3 *m.* 15 *da.*)=$300×105 *da.* =31500;

and $400×(7 *m.* 20 *da.*)=$400×230 *da.* =92000.

Then (31500+92000)÷(200+300+400)=123500÷900= $137\frac{2}{9}$ *da.*

∴ We thus find that the sum of the payments will claim a credit of $137\frac{2}{9}$ days, to be reckoned from the 1st of January.

Disregarding the $\frac{2}{9}$ of a day, and allowing 30 *days to a month*, we shall find that this credit will extend to the 17th of May.

5. On the 5th of September, 1846, a merchant bought goods amounting to $8000; of which $4000 was to be paid in 4 months, $2000 in 6 *m.*; and the remainder in 8 *m.* It was afterwards agreed that one payment might be made of the whole; what was the proper day of payment?

Ans. February 17th, 1847.

6. On the 10th of January, 1848, A bought of B, 100 acres of land, at $24 per acre,—to be paid in three equal instalments, on the 20th of October, 1848, the 1st of June, and 30th of December, 1849. If the whole be converted into one pay*ment,* on what day should that payment be made?

Ans. May 25th, 1849.

COMPOUND INTEREST.

§ **281.** *Simple Interest* is interest on a given *principal* only; (§ 265). *Compound Interest* is interest on both *principal* and *interest*, when the *latter remains unpaid* after it has become due.

The interest is compounded *annually*, *half yearly*, or *quarterly*, &c., according to the time at which it becomes due.

Compound interest is not sanctioned, by law, on money lent, or debts contracted in ordinary commercial transactions.

§ **282.** *To calculate Compound Interest.*—Make the *amount*, at simple interest, for the first year, or period when the interest becomes due, the *principal* for the second; the amount for the second, the principal for the third; and so on. From the *last amount* subtract the original principal; the remainder will be the compound interest.

EXERCISES.

1. What is the compound interest on $200, for 3 years, at 6 per cent., allowing interest to be due annually?
Ans. $38.203'.

2. What is the compound interest on $1000, for 2 years, at 8 per cent., allowing interest to be due half yearly?
Ans. $169.858'.

3. What would $500 amount to in 5 years, at 6 per cent. interest, if the interest be *compounded* annually?
Ans. $669.112.

EXERCISES ON CHAPTER XI.

$2500.18¾. *Philadelphia, June 1st*, 1845.

1. On the 1st day of January, 1846, I promise to pay to William Kind, the sum of Two thousand, five hundred dollars, 18¾ cents, with interest; for value received.

Simon Thankful.

This note was endorsed as follows:

January 1st, 1846, received $1000.
October 10th, 1846, received $35.25.
August 16th, 1847, received $200.

The balance on the note was not paid until the 1st of Jan uary, 1848. What amount was then to be paid?
Ans. $1541.40'

2. A farmer bought of a merchant, goods amounting to \$175.12½, on a credit of 12 months; but paid the debt in 2 months and 10 days. What sum should have been discounted from the debt, allowing the rate of interest to be 8 per cent? *Ans.* \$10.596'.

3. A debt of \$3000.75 will be due in 2 years, 7 months and 18 days, without interest. What sum in hand would be an equivalent for the debt, money being at 7 per cent. interest? *Ans.* \$2533.775'.

4. A held a note against B for \$473.50, due April 3d, 1846; and B held a note against A for \$500.62½, due June 10th, 1846; no interest accruing in either case, until the note is due. Settlement was had May 5th, 1846; what was then the balance between A and B, allowing money to be worth 6 per cent.? *Ans.* A owed B \$21.61'.

5. A merchant bought 43 *cwt.* 3 *qr.* of sugar, at \$5.25 per *cwt.*, which he immediately sold at \$7 per *cwt.*, on 6 months credit. Taking the purchaser's note for the amount, he gets the note discounted in bank, at 6 per cent.; what profit did the merchant make? *Ans.* \$67.22'.

6. Wishing to raise the sum of \$3760.50, I design, for the purpose, to put a note in bank for 4 months. For what principal must the note be drawn,—interest being at 8 per cent.? *Ans.* \$3866.042'.

7. A promissory note for \$350.75 was at interest from the 4th of July, 1844, to the 19th of January, 1847, when it had amounted to \$404.238, at what rate was the interest computed? *Ans.* 6 per cent.

8. In what time will \$400 produce the same amount of interest, at 6 per cent., that would acrue on \$375.18¾, in 5 years, 7 months and 25 days, at 7 per cent.? *Ans.* 6.185' years.

9. A owes B \$5000; of which \$1200 is to be paid in 9 months, \$3000 in 1 year and 3 months, and the remainder in 2 years. In what time might the whole sum be paid at once, without injustice to either? *Ans.* 15 months.

10. A rice plantation was to be paid for as follows, namely; ¼ of the purchase money in hand; ⅓ of it in 12 months; and the remainder in 1 year and 9 months. The parties have since agreed that the whole shall be paid at one time; when should the payment be made? *Ans.* In 12¾ months.

11. A money dealer borrowed \$1000 for 2 years, at 6 per cent. interest; and loaned the same in such a manner as to *compound* the interest every 6 months. What profit did he make in the 2 years, by this proceeding? *Ans.* \$5.508.

CHAPTER XII.

POWERS AND ROOTS.—INVOLUTION.—EVOLUTION.—APPLICATION OF SQUARE AND CUBE ROOT

POWERS AND ROOTS.

§ 283. The *first power* of a number is the number itself.

Thus, the *first power* of 3 is 3.

§ 284. The *second power*, or *square*, of a number, is the product of that number *multiplied into itself*.

Thus, the *second power* or *square* of 3, is $3\times3=9$.

What is the second power, or square, of 4? Of 5? Of 7? Of 10?
What is the second power, or square, of 8? Of 11? Of 12? Of 20?

§ 285. The *second root*, or *square root*, of a number, is that number which, multiplied into itself, produces the given number.

Thus the *square root* of 36 is 6, because $6\times6=36$.

What is the square root of 4? Of 25? Of 49? Of 64? Of 100?
What is the square root of 9? Of 16? Of 36? Of 81? Of 144?

§ 286. The *third power*, or *cube*, of a number, is the product of that number multiplied into its second power, or square.

Thus the *third power*, or *cube*, of 3, is $3\times3\times3=27$.

What is the third power, or cube, of 2? Of 4? Of 5? Of 7? Of 10?

§ 287. The *third root*, or *cube root*, of a number, is that number which, being multiplied into its second power, or square, produces the given number.

Thus, the cube root of 27 is 3, because $3\times3\times3=27$.

What is the cube root of 8? Of 64? Of 125? Of 216? Of 1000?

§ 288. The *4th power* of a number is the product of that number multiplied into its 3d power, or cube. Thus, the 4th power of 2, is $2\times2\times2\times2=16$.

What is meant by the 5th power of a number? By the 6th power?

The *4th root* of a number is that number which, being multiplied into its 3d power, or cube, produces the given number: thus, 2 is the 4th root of 16.

What is meant by the 5th root of a number? By the 6th root?

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From the preceding it is plain, that,

When one number is any *power* of another, the latter is the corresponding *root* of the former. Thus 9 is the *square* of 3; then 3 is the *square root* of 9.

Powers and Roots of Unity.

§ **289.** Any *power* or *root* whatever, of *unity*, is *unity;* since any number of 1s, multiplied together, produce 1.

Thus, $1\times1=1$; $1\times1\times1=1$; and so on.

Powers and Roots of Fractions.

§ **290.** A *power* or *root of a fraction* is found by taking the power or root of the *numerator* and *denominator*, separately.

Thus, the square of $\frac{2}{3}$ is $\frac{2\times2}{3\times3}=\frac{4}{9}$; and the square root of $\frac{4}{9}$ is therefore $\frac{2}{3}$. So the cube of $\frac{2}{3}$ is $\frac{8}{27}$.

What is the square of $\frac{3}{4}$? Of $\frac{5}{7}$? Of $\frac{7}{10}$? Of $\frac{5}{12}$?
What is the square root of $\frac{9}{16}$? Of $\frac{36}{49}$? Of $\frac{25}{100}$? Of $\frac{81}{121}$?
What is the cube of $\frac{3}{4}$? Of $\frac{2}{5}$? Of $\frac{3}{6}$? Of $\frac{4}{10}$?
What is the cube root of $\frac{8}{64}$? Of $\frac{27}{125}$? Of $\frac{125}{1000}$? Of $\frac{216}{1000}$?

Powers and Roots of Mixed Numbers.

§ **291.** A power or root of *a mixed number* may be found by reducing to an *improper fraction*, and taking the power or root of the numerator and denominator, separately.

Thus, the *square root* of $5\frac{4}{9}$= the square root of $\frac{49}{9}=\frac{7}{3}=2\frac{1}{3}$.

What is the square root of $2\frac{1}{4}$? Of $20\frac{1}{4}$? Of $11\frac{1}{9}$? Of $1\frac{11}{25}$?
What is the cube root of $3\frac{3}{8}$? Of $2\frac{10}{27}$? Of $1\frac{91}{125}$?

Perfect and Imperfect Powers.

§ **292.** A *perfect power*, of any *order*, is a number which has an *exact root* of the corresponding order. An *imperfect power* has no exact root of the corresponding order. Thus,

A *perfect square*, or a *square number*, is any number, integral or fractional, which has an exact square root; and a *cube* number is one which has an exact cube root.

Name all the *square numbers*, in succession, from unity to the square of 12. Name several *cube numbers*, beginning with unity.

Name three fractions which are *perfect squares*. Name three which are *perfect cubes*.

An imperfect power is also called a *Surd;* and its root is called an *irrational* number, because its *ratio* to unity cannot be exactly determined.

Exponents of Powers and Roots.

§ **293.** An *exponent* or *index* is an *integer* annexed to a number to denote a *power*, or a *fraction* annexed to denote a *root*, of that number.

Thus, 5^2 denotes the *2d power* or square of 5.
5^3 denotes the *3d power* or cube of 5.
$16^{\frac{1}{2}}$ denotes the *square root* of 16; $27^{\frac{1}{3}}$ the *cube root* of 27, &c.

In these expressions, 2, 3, $\frac{1}{2}$ and $\frac{1}{3}$ are *exponents*. An exponent is always set on the right of the number, and a little elevated, as in the examples.

A root is also denoted by the *radical sign* $\sqrt{\ }$, with an integral exponent or index.

Thus, $\sqrt{9}$ denotes the square root of 9; $\sqrt[3]{27}$, the cube root of 27; $\sqrt[4]{100}$, the 4th root of 100; and so on.

INVOLUTION.

§ **294.** *Involution* consists in raising a given number to any required *power*. Thus, in finding the square of 25, we perform an *involution* on 25.

EXERCISES.

1. Find the square of .14. . . . . *Ans.* .0196
2. Find the square of .16. . . . . *Ans.* .0256.
3. Find the square of $\frac{19}{37}$. . . . . *Ans.* $\frac{361}{1369}$.
4. Find the square of $\frac{49}{163}$. . . . . *Ans.* $\frac{2401}{26569}$.
5. Find the square of $23\frac{1}{9}$. . . . . *Ans.* $534\frac{10}{81}$.
6. Find the square of 14.3. . . . . *Ans.* 204.49.

7. Find the cube of $\frac{5}{8}$. . . . . *Ans.* $\frac{125}{512}$.
8. Find the cube of 124. . . . . *Ans.* 1906624.
9. Find the cube of $13\frac{1}{4}$. . . . . . . *Ans.* $2326\frac{13}{64}$.
10. Find the cube of .25. . . . . . . *Ans.* .015625.

§ **295.** A *higher power* of a given number may be found, most readily, by multiplying together two or more known powers, the *sum of whose exponents* is equal to the exponent of the *required power*.

Thus, the *square* of a number multiplied into itself, produces the 4th *power* of that number. $3^2 \times 3^2 = 3 \times 3 \times 3 \times 3 = 3^4$.

The *square* $\times$ the *cube*, produces the 5th *power*. $3^2 \times 3^3 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$; and so on.

11. Find the 4th power of $2\frac{1}{4}$. . . . . *Ans.* $25\frac{161}{256}$.
12. Find the 5th power of 3.5. . . . . *Ans.* 525.21875.
13. Find the 6th power of 1.2. . . . . *Ans.* 2.985984.

EVOLUTION.

§ **296.** *Evolution* consists in *extracting* any required *root* of a given number regarded as the corresponding *power* of the root to be found.

Thus, in extracting the square root of 625, we perform an *evolution* on 625.

The *extraction of the square root* of a number, consists in finding one of the *two equal factors* which, multiplied together, produce the given number.

Thus we find 25 to be the $625^{\frac{1}{2}}$, because $25\times25=625$.

Extraction of the Square Root

The following principles are involved in the Rule to be given for extracting the square root.

§ **297.** I. The square of any number has, at most, only *twice as many* figures, and, at least, only *one less than twice* as many, as the number itself.

Thus, $9^2=81$; $99^2=9801$; $999^2=998001$; in which examples, the squares 81, &c., have only twice as many figures as the numbers 9, &c., and these numbers are the *largest that can be expressed* by the same number of figures.

Again; $10^2=100$; $100^2=10000$; $1000^2=1000000$; in which examples, the squares 100, &c. have only one less than twice as many figures as the numbers 10, &c., and these numbers are the *smallest that can be expressed* by the same number of figures.

From the preceding, it follows, that,

§ **298.** II. A number has *two figures* for each figure in its *square root*, excepting the left hand one—for which it has one, or two figures, according as said left hand figure *produces one* or *two* figures in *squaring the root*.

§ **299.** III. If a number be divided into *any two parts*, the *square of the number* will be equal to
the *square of the* 1*st part* + *twice the* 1*st* $\times$2*d* + the *square of the* 2*d part*.

For example, $16=10+6$, and the square of $16=$ the square of $(10+6)$.

$$\begin{array}{r}
10+6 \\
10+6 \\
\hline
60+36 \\
100+\ 60 \\
\hline
100+120+36
\end{array} = \text{the square of } (10+6)=16^2$$

In squaring $10+6$, that is, in multiplying it by itself, we first multiply $10+6$ by 6; this gives $60+36$. We then multiply $10+6$ by 10; which gives $100+60$. The sum of the two products is $100+120+36=16^2$.

This square is composed of $10^2=100$, *twice* $10\times6=120$, and $6^2=36$. Thus we see that the square of the number 16 is equal to the square of the first part 10, $+$ twice the product of the two parts 10 and 6, $+$ the square of the second part 6

RULE LVII.

§ 300. *To extract the Square Root of a given number.*

1. Separate the given number into periods of two figures each, from right to left; observing that the last period may sometimes have but one figure.

2. From the left hand period subtract the greatest square number it contains, and set the root of said square for the first figure of the root required.

3. To the remainder affix the next period for a dividend. Divide this dividend, exclusive of its right hand figure, by twice the root already found, and annex the quotient figure to both the root and the divisor.

4. Multiply the divisor thus increased, by the quotient figure; subtract the product from the dividend; to the remainder affix the next period; divide by twice the root already found; and so on, till the operation is completed.

EXAMPLES.

1. To extract the square root of 529.

```
     5'2 9(2 3
     4
43)1 2 9
   1 2 9      529½=23.
```

The left hand period is 5, and the greatest square number it contains is 4, the root of which is 2. Subtracting, and to the remainder 1 affixing the next period, we have for a dividend 129. Excluding its right hand figure 9, we divide 12 by 4, which is *twice* the root 2 already found, and annex the quotient figure 3 to both the root and the divisor. Multiplying the divisor 43 by 3, completes the operation.

2. To extract the square root of 368449.

```
            3 6'8 4'4 9(6 0 7
            3 6
 1 2 0 7)     8 4 4 9
              8 4 4 9      368449½=607.
```

The left hand period 36 being a square number, we set its root 6 for the first figure in the root required; and set down the next period 84, for a dividend.

Excluding its right hand figure 4, and dividing 8 by 12, which is *twice* the root 6 already found, the quotient is 0, which we annex to the root 6 and divisor 12.

To 84 we affix the next period 49, and the dividend is 8449. Excluding its right hand figure, and dividing by 120, which is *twice* the root 60 already found, the quotient is 7. Annexing this to the root and divisor, and multiplying, the operation is completed.

☞ The given number is separated into periods to determine the number of figures in the root; and also whether one or two figures on the left correspond to the first figure in the root; (§ 298). In the first example,

The first figure 2 in the root is 2 *tens* =20, and its square is 400, which, subtracted, leaves the remainder 129.

The given number 529= the square of 2 *tens*, the 1*st part* of its root, + *twice* 2 tens × the *units* or 2d part of the root, + the *square* of the units. (§ 299).

And since the square of 2 tens has been subtracted, the remainder 129 = twice 2 *tens* × the *units* + the *square* of the units.

In taking 4,= twice 2, for a divisor, we omitted the 0 on the right of 2 tens. Omitting, therefore, the 9 in the corresponding place of the dividend 129, we say 4 in 12, 3 times.

The quotient 3 annexed to the 4 in the divisor, makes the 4 become 4 *tens;* and the divisor 4 *tens* +3, multiplied by 3,=4 *tens* $\times 3+3^2$; or twice 2 tens × the *units* in the root, + the *square* of the *units*, =129.

Hence the second figure in the root is correctly found by the Rule.

When the given number contains three or more periods, the first and second figures of the root having been found, these two figures together may be regarded as the 1st part of the entire root, and the remainder of the root as the 2d part. Three figures of the root having been found, these may together be regarded as the 1st part of the root, and the remainder as the 2d part; and so on.

Under these views, the principle on which depends the method of forming the divisors, (§ 299), becomes applicable when there are three or more, as well as when there are but two figures in the root. ☜

EXERCISES.

1. Find the square root of 784, and of 11236.
Ans. 28; and 106.
2. Find the square root of 2025, and of 38809.
Ans. 45; and 197.
3. Find the square root of 7396, and of 75076.
Ans. 86; and 274.
4. Find the square root of 22801, and of 473344.
Ans. 151; and 688.
5. Find the square root of 36100, and of 904401.
Ans. 190; and 951.
6. Find the square root of $\frac{256}{289}$, and of $\frac{1156}{5041}$.
Ans. $\frac{16}{17}$; and $\frac{34}{71}$.
7. Find the square root of $\frac{6561}{9025}$, and of $\frac{10000}{39204}$.
Ans. $\frac{81}{95}$; and $\frac{100}{198}$.
8. Find the square root of $\frac{5929}{7056}$, and of $\frac{11881}{94864}$.
Ans. $\frac{77}{84}$; and $\frac{109}{308}$.
9. Find the square root of $\frac{1024}{1089}$, and of $\frac{1225}{1296}$.
Ans. $\frac{32}{33}$; and $\frac{35}{36}$.
10. Find the square root of $\frac{8281}{8836}$, and of $\frac{9409}{10000}$.
Ans. $\frac{91}{94}$; and $\frac{97}{100}$.

Square Root of Decimals, Imperfect Squares, &c.

§ 301. In extracting the square root, an *integer* is separated into periods from *right to left.*

A *decimal* must be separated into periods of two figures each, from the *decimal point towards the right,* and a 0 annexed when necessary to complete the last period.

The number of decimal figures in the root, will be equal to the number of *periods* in the given decimal.

In finding the root of an imperfect square, (§ 292), a period of 0s may be annexed to the last remainder, and the operation continued in this manner to any required exactness;—observing that each period thus annexed must be counted as a *decimal period* belonging to the given number.

A fraction *will be an imperfect square,* if either of its terms is an imperfect square. Its root in such case will be found, most readily, by extracting the root of its *equivalent decimal.*

A *vulgar fraction annexed to an integer* may be reduced to a decimal, and the root of the mixed number be then extracted.

EXAMPLE.

To extract the square root of $346\frac{1}{8}$.

Reducing $\frac{1}{8}$ to an equivalent decimal, we find $\frac{1}{8}=.125$. We have then to extract the square root of 346.125.

```
           3'46.12'50(18.6044
           1
        28)246
           224
       366)2212
            2196
     37204)165000
             148816
    372084)1618400
             1488336
              130064
```

We separate the *integer* 346 into periods from *right* to *left*, and the *decimal* .125 into periods from the *decimal point towards the right*, and annex a 0 to complete the last period 50.

All the periods in the given number having been included in the operation, we annex 00 to the remainder, and thus continue the operation.

The two decimal periods .12'50, with the two periods of 0s annexed to the remainders, make *four decimal periods;* hence we make *four decimal figures in the root.*

☞ The product of two decimal fractions contains just as many decimal figures as are in both the factors. The *square* of a decimal fraction has, therefore, twice as many decimal figures as the decimal itself. Hence, each decimal *period* must contain *two figures;* and the number of *decimal figures* in the root must equal the number of *decimal periods.* ☜

§ **302.** In extracting the square root of an *approximate decimal,* such decimal should be continued to *twice as many figures* as the number of decimal figures required in the root.

For example, to extract the square root of $\frac{1}{3}$ to *three decimal figures.*

As $\frac{1}{3}$ is an *imperfect* square, we reduce it to a decimal of six figures.

$$\frac{1}{3}=.333333$$

The approximate decimal thus found, having three periods will give three decimal figures in its square root.

EXERCISES.

11. Find the square root of .582169. . . . *Ans.* .763.
12. Find the square root of .454276. . . . *Ans.* .674.
13. Find the square root of .795664. . . . *Ans.* .892.
14. Find the square root of .3478312. . . *Ans.* .5897'.
15. Find the square root of 737.8742. . . *Ans.* 27.16'.
16. Find the square root of 43.73731. . . *Ans.* 6.613'.
17. Find the square root of .0073474. . . *Ans.* .0857'.
18. Find the square root of $\frac{2314}{4000}$. . . . . *Ans.* .7605'.
19. Find the square root of $\frac{7348}{8700}$. . . . . *Ans.* .919'.
20. Find the square root of $23734\frac{23}{100}$. . *Ans.* 154.059'.
21. Find the square root of $74786\frac{346}{1000}$. . *Ans.* 273.47'.
22. Find the square root of 90374376. . *Ans.* 9506.543'.
23. Find the square root of 23473783. . *Ans.* 4844.975'.
24. Find the square root of $847376\frac{31}{270}$. *Ans.* 920.530'.
25. Find the square root of $783703\frac{471}{385}$. *Ans.* 885.27'.

APPLICATION OF THE SQUARE ROOT.

§ **303.** The *area* of a surface is equal to the product of its *length* into its *breadth;* using these dimensions in the *same denomination.*

Since a square has its length and breadth equal to each other, the *area of a square* is equal to either of its sides multiplied into itself; that is, the area of a square is the *square of either of its sides.*

Hence, the area of the square being given, a *side of the square* will be found by extracting the *square root of the area.*

EXERCISES.

26. How long must the side of a square lot be, which shall contain just one acre of ground? *Ans.* 12.649'*p*.

27. How long must the side of a square field be, which shall contain just 10 acres of ground? *Ans.* 40 poles.

28. What must be the side of a square, which shall be equal in area to a surface 320 *yd.* long, and 75 *yd.* wide?

Ans. 154.919'*yd.*

29. A merchant bought a bale of cloth, containing just as many pieces as there were yards in each piece. The whole number of yards was 1089; what was the number of pieces?

Ans. 33 pieces.

30. What must be the sides of two squares, one of which shall contain 2 square miles, and the other 3 square miles, of land? *Ans.* 452.548'*p.*; and 554.256'*p.*

31. A regiment consisting of 5476 men, is to be formed into a solid square. How many men must be placed in rank and file, that is, in a line on the front, and from front to rear?

Ans. 74 men.

32. What would be the expense of enclosing 15 *A.* 2 *R.* 18 *P.* of ground, in the form of a square, at the rate of \$2.12½ per rod for the fencing? *Ans.* \$424.82'.

33. A company of men on a journey expended \$6084; each man expending as many dollars as there were men in the company. What was the number of men? *Ans.* 78 men.

34. A farmer wishes to plant an orchard which shall contain 8464 trees, and have as many rows of trees as trees in each row. What will be the number of trees in each row?

Ans. 92 trees.

35. A city whose corporate area is in the form of a circle, contains 3.1416 square miles, and is 6.2832 miles in circumference. Had the same amount of area been incorporated in the form of a square, what would have been the compass of the city? *Ans.* 7.088' miles.

36. What must be the dimensions of a field to contain 10 acres of ground, and have its length equal to twice its breadth?

One half of the given area will be the area of a *square*, whose size is equal to the breadth of the field.

Ans. 28.284'*p.* in breadth; 56.568'*p.* in length.

37. A garden which shall contain an acre of ground, is to have its breadth equal to one half of its length. What must be its dimensions?

Ans. 8.944'*p.* in breadth; 17.888'*p.* in length.

38. A cemetery containing 15 acres is laid out in such a manner that its length is equal to three times its breadth. What are the dimensions of the cemetery?

Ans. 28.284'*p.* in breadth; 84.852'*p.* length.

39. A warehouse whose base shall occupy 10000 square feet, is intended to have its breadth only one-third of its length. What must be the length and breadth?

Ans. 173.205'*ft.* by 57.735'*ft.*

40. A farmer intending to enclose 50 acres of land, wishes to know what difference in the amount of fencing there would be between having the enclosure in the form of a square, and having it such that its length shall be double its breadth?

Ans. 21.702' poles.

Extraction of the Cube Root.

The following principles form the basis of the Rule for extracting the cube root.

§ 304. I. The cube of a number has, at most, only *three times* as many, and, at least, *only two less than three times* as many figures, as the number itself.

Thus, $99^3=970299$; $999^3=997002999$; in which examples the cubes have only three times as many figures as the respective numbers or roots, and those numbers are the largest that can be expressed by the same number of figures.

Again, $10^3=1000$; $100^3=1000000$; in which the cubes have only two less than three times as many figures as the numbers or roots, and those numbers are the smallest that can be expressed by the same number of figures.

From the preceding, it follows that

§ 305. II. A number has *three figures* for each figure in its *cube root*, excepting the left hand one—for which it has one, two, or three figures, according as said left hand figure produces one, two, or three figures, in *cubing the root.*

§ 306. III. If a number be divided into any two parts, the *cube of the number* will be equal to the *cube* of the *1st part*, + 3 times the *square* of the *1st into the 2d*, + 3 *times the first into the* $2d^2$, + the *cube of the 2d part.*

For example, $16=10+6$, and the *cube* of 16 = the *cube* of $(10+6)$.

The square of 16, equal to the square of $(10+6)$ as heretofore found,

$$\begin{array}{rrrr}
\text{is } & 100+ & 120+ & 36 \\
 & & 10+ & 6 \\
\hline
 & 600+ & 720+ & 216 \\
1000+ & 1200+ & 360 & \\
\hline
1000+ & 1800+ & 1080+ & 216
\end{array}$$

$$=\text{the cube of } (10+6)=16^3.$$

The square of $(10+6)$ multiplied by $(10+6)$, produces the cube of $(10+6)$.

Multiplying, we have $36\times6=216$; $120\times6=720$; and $100\times6=600$; also, $36\times10=360$; $120\times10=1200$; and $100\times10=1000$.

The sum of all these products is equal to $1000+1800+1080+216=16^3$.

This cube is composed of $10^3=1000$; 3 times $10^2\times6=1800$; 3 times $10\times6^2=1080$; and $6^3=216$.

RULE LVIII.

§ 307. *To extract the* CUBE ROOT *of a given number.*

1. Separate the given number into periods of three figures each, from right to left; observing that the last period may sometimes have but one or two figures.

2. From the left hand period subtract the greatest cube number it contains, and set the root of said cube for the first figure of the root required.

3. To the remainder affix the next period for a dividend. Divide this dividend, exclusive of its two right hand figures, by three times the square of the root already found, taken as an *incomplete divisor*, and annex the quotient figure to the root.

4. *Complete the divisor* by annexing to it two 0s, and adding three times the product of the last quotient figure into the previous root with a 0 annexed, and also the square of the last quotient figure.

5. Multiply the divisor thus completed by the last quotient figure; subtract the product from the dividend; to the remainder affix the next period for a dividend; divide by three times the square of the root already found, and so on, as before, until the operation is completed.

In applying this Rule it will be convenient to refer to the following

Table of Roots and Cubes.

ROOTS.	CUBES.	ROOTS.	CUBES.	ROOTS.	CUBES.
1 . . .	1	4 . . .	64	7 . . .	343
2 . . .	8	5 . .	125	8 . . .	512
3 . . .	27	6 . .	216	9 . . .	729

EXAMPLE.

1 To extract the cube root of 91125.

$$\begin{array}{r|l} & 91'125(45 \\ & 64 \\ \hline 48 & 27125 \\ 4800+600+25=5425 & 27125 \end{array} \qquad 91125^{\frac{1}{3}}=45.$$

The left hand period is 91, and the greatest cube number it ontains is 64, the root of which is 4. Subtracting, and to

the remainder 27 affixing the next period, we have for a divi dend 27125.

Excluding the two right hand figures, we divide 271 by 48, which is 3 times 16, the square of the root 4 already found, and annex the quotient figure 5 to the root.

To complete the divisor 48, we annex two 0s, making 4800, and add 600, which is 3 times 200, the product of the last quotient figure 5 into 40, the previous root 4 with a 0 annexed, and add also 25, the square of 5.

The divisor thus completed is 5425, which, multiplied by 5, equals the dividend.

2. To extract the cube root of 223648543.

108	223'648'543(607
	216
10800	7648543
1080000+12600+49=1092649	7648543

The left hand period is 223, and the greatest cube number it contains is 216, the root of which is 6. Subtracting, and to the remainder 7 affixing the next period, we have for a dividend 7648.

Excluding the two right hand figures, we divide 76 by 108, which is 3 times 36, the square of the root 6 already found. The quotient figure is 0. We annex 0 to the root, and affix the next period, obtaining a new dividend 7648543.

☞ The given number is separated into periods, to determine the number of figures in the root, and also whether one, two, or three figures on the left, correspond to the first figure in the root. (§ 305).

In the 1st example, the first figure 4 in the root, is 4 *tens* $=40$; and its cube $=64000$, which, subtracted, leaves the remainder 27125.

The given number $91125=4$ *tens*^{3} $+3\times4$ *tens*^{2} $\times$ the *units* of the root, $+3\times4$ *tens* $\times$ the *units*^{2}, $+$ the *units*^{3}. (§ 306).

And since 4 tens^3 has been subtracted, the remainder $27125=3\times4$ *tens*^{2} $\times$ the *units*, $+3\times4$ *tens* $\times$ the *units*,^{2} $+$ the *units*^{3}.

In taking $48=3\times4^2$ for an incomplete divisor, we omitted two 0s in the right of 4 *tens*^{2}. Omitting, therefore, the 25 in the corresponding places of the dividend 27125, we say 48 in 271, 5 times.

Regarding now the quotient 5 as the *units* to be found in the root, the completed divisor $4800+600+25$ is 3×4 *tens*^{2} $+3\times4$ *tens* $\times5$ *units* $+5$ *units*^{2}; and this divisor multiplied by $5=3\times4$ *tens*^{2} $\times5$ *units*, $+$ 3×4 *tens* $\times5$ *units*^{2}, $+5$ *units*^{3}; $=27125$; and this added to 4 *tens*^{3} makes up the given number.

Hence the given number $91125=(4$ *tens* $+5$ *units*$)^3=45^3$; (§ 306), hence also, the root is correctly found by the Rule.☜

Note. The quotient figure—being found from an *incomplete divisor*—will often be a *unit or two* less than the number of times such divisor is contained in those figures of the dividend, which are taken in dividing.

EXERCISES.

1. Find the cube root of 59319. . . . . . *Ans.* 39.
2. Find the cube root of 103823. . . . . . . *Ans.* 47.
3. Find the cube root of 262144. . . . . . *Ans.* 64.
4. Find the cube root of 2406104. . . . . *Ans.* 134.
5. Find the cube root of 22906304. . . . . *Ans.* 284.
6. Find the cube root of $\frac{19683}{389017}$. . . . . . . *Ans.* $\frac{27}{73}$.
7. Find the cube root of $\frac{132651}{25672375}$. . . . . . *Ans.* $\frac{51}{295}$.

A more convenient method of forming the Divisors, in Extracting the Cube Root.

§ **308.** For the *first incomplete divisor*, take 3 times the square of the first figure in the root; and with this divisor find the second figure, as before.

Complete the divisor by annexing to it two 0s, and adding the product of the last figure in the root with 3 times the other part of the root prefixed to it, multiplied by the last figure.

Each succeeding incomplete divisor will be found by adding to the last complete divisor, the *product which completed it*, and the *square of the last figure* in the root. The divisors are all completed in the same manner.

EXAMPLE.

To extract the cube root of 95443993.

	95'443'993(457
	64
48	31443
4800+625=5425	27125
5425+625+25=6075	4318993
607500+9499=616999	4318993

The first incomplete divisor is 48, equal to 3 times 16, the square of 4,—which divisor gives the quotient 5, the second figure in the root.

To complete the divisor 48, we annex to it two 0s, and add 625, which is the product of 125, (that is, 5 with three times 4 prefixed to it), multiplied by 5. The divisor thus completed is 5425.

For the next incomplete divisor, we add to the last complete divisor the product 625 which completed it, and 25, the square of 5.

Having found the quotient 7, we complete the divisor 6075 by annexing to it two 0s, and adding 9499, which is the product of 1357, (that is, 7 with three times 45 prefixed to it), multiplied by 7. The divisor thus completed is 616999.

☞ This method of forming the divisors, involves the same principle, (§ 306,) with the Rule before given.

Thus, the 625 added to complete the first divisor is $125\times5=(120+5)\times5=120\times5+5^2=3$ times $40\times5+5^2$.

And the incomplete divisor $6075=4800+625+625+25=3$ times $40^2,+3$ times $40\times5+5^2,+3$ times $40\times5+5^2,+5^2=3$ times 40^2+6 times $40\times5+3$ times $5^2=3$ times (40^2+ *twice* $40\times5+5^2$)$=3$ times 45^2.

The divisors are, therefore, the same as required by the Rule. (§307.) ☜

EXERCISES.

8. Find the cube root of 20796875. . . . . *Ans.* 275.
9. Find the cube root of 28372625. . . . . *Ans.* 305.
10. Find the cube root of 69934528. . . . . *Ans.* 412.
11. Find the cube root of 91125000. . . . . *Ans.* 450.
12. Find the cube root of 125000000. . . . . *Ans.* 500.
13. Find the cube root of 131872229. . . . . *Ans.* 509.
14. Find the cube root of 241804367. . . . . *Ans.* 623.
15. Find the cube root of 997002999. . . . . *Ans.* 999.

Cube Root of Decimals, Imperfect Cubes, &c.

§ **309.** In extracting the cube root, an *integer* is separated into periods from *right* to *left*. A *decimal* must be separated into periods of three figures each, from the *decimal point towards the right;* and one or two 0s must be annexed when necessary to complete the last period.

The number of *decimal figures* in the root, must be the same as the number of *periods* in the given decimal.

In finding the root of an imperfect cube, (§ 292), a period of 0s may be annexed to the last remainder, and the operation continued in this manner to any required exactness; observing that each period thus annexed must be counted as a decima period belonging to the given number.

A fraction will be an *imperfect cube*, if either of its terms is an imperfect cube. Its root in such case will be found, most readily, by extracting the root of its *equivalent decimal.*

A *vulgar fraction* annexed to an integer may be reduced to a decimal. and the root of the mixed number be then extracted.

☞ The product of three decimal fractions, contains just as many decimal figures as all the three factors. The cube of a decimal fraction has, therefore, three times as many decimal figures as the decimal itself Hence each decimal *period* must contain *three figures;* and the number of decimal figures in the root, must equal the number of decimal periods. ☜

EXERCISES.

16. Find the cube root of .389017. . . . . *Ans.* .73.
17. Find the cube root of .202262003. . . *Ans.* .587.
18. Find the cube root of .23456. . . . . *Ans.* .616'.
19. Find the cube root of 734.673. . . . . *Ans.* 9.023'.
20. Find the cube root of 7386. . . . . . *Ans.* 19.474'.
21. Find the cube root of $9873\frac{1}{4}$. . . . . *Ans.* 21.452'.
22. Find the cube root of $7370\frac{2}{3}$. . . . . *Ans.* 19.461'.
23. Find the cube root of 479.2735. . . . *Ans.* 7.825'.
24. Find the cube root of $8377\frac{23}{150}$. . . . *Ans.* 20.309'.

APPLICATION OF THE CUBE ROOT.

§ 310. The *solidity* or *volume* of a body, in cubic measure, is equal to the product of its *length* into its *breadth* into its *height* or thickness.

Since a *cube* has its length, breadth, and thickness, equal to one another, its solidity is equal to the cube of either of its three sides or dimensions.

Hence, the solidity of a cube being given, either of its three dimensions will be found by extracting the *cube root of its solidity.*

EXERCISES.

25. What must be the length, breadth, or height of a cube, that its volume or solidity may be 1728 cubic feet?
Ans. 12 feet.

26. What must be the length, breadth, or depth, of a cubical box, that its capacity may be 2000 cubic feet?
Ans. 12.598' *ft.*

27. What must be the depth of a cubical cistern which shall contain 5000 gallons of water? Of one to contain 500 barrels? *Ans.* 9.344' *ft.*; 14.321' *ft.*

28. What must be the dimensions of a cubical granary which shall contain 3000 bushels of wheat? One to contain 10000 bushels? *Ans.* 15.513' *ft.*; 23.173' *ft.*

29. What must be the dimensions of a cubical cellar whose capacity shall be equal to that of another 30 feet long, 20 feet wide, and $10\frac{1}{2}$ feet deep? *Ans.* 18.469' *ft.*

30. The solidity of a cubical block of marble is 1331 cubic

feet. What is the length of the block? and the area of its surface?

The pupil must consider that the surface of the block consists of *six equal square faces* or sides.

Ans. Length 11 *ft.*; area 726 *sq. ft.*

31. A cubical cistern is to be constructed which shall contain 300 barrels of water; the bottom and walls of which are to be plastered with hydraulic lime. How many square yards of plastering will there be? *Ans.* 81.056' *sq. yd.*

32. A farmer wishes to construct a crib whose capacity shall be 2000 bushels,—its breadth and height to be equal, and each of these one half of its length. What must be the length of the crib?

One half of the given capacity will be the *contents of a cube*, each of whose three dimensions is equal to the breadth or height of the crib.

Ans. 21.512 *ft.*

33. How many square feet will there be in the bottom and four sides of a reservoir, the capacity of which is to be 10000 gallons of water, and the length, breadth, and depth equal to one another? *Ans.* 693.017' *sq. ft.*

34. The capacity of the reservoir being as in the preceding question, how many square feet would be contained in the bottom, the two sides and the two ends, allowing its length to be double each of its other dimensions? *Ans.* 698.482' *sq. ft.*

35. What must be the dimensions of a wine vat which shall contain 5 barrels,—supposing its length to be equal to three times its breadth or depth?

The vat will evidently consist of *three cubes* having their length, breadth, or depth equal to the breadth or depth of the vat.

Ans. Length 5.742'; breadth and depth 1.914' *ft.*

36. A brewer has a cistern which contains 6 barrels of beer, and whose length and height each equal to twice its breadth. What then are the dimensions of the cistern?

Ans. Length and height 4.13'; breadth 2.065' *ft.*

37. What would be the expense of plastering the bottom and walls of a cubical reservoir which shall contain 100 barrels of water, at \$0.37½ per square yard? and what would it cost to plaster in like manner, and at the same rate, another of equal capacity, but having its length 4 times its breadth or height?

Ans. \$14.612'; and \$16.237'.

EXERCISES ON CHAPTER XII.

1. Find the square, and also the square root of 284.
Ans. Square 80656; square root, 16.852'

2. Find the square, and also the square root of 3.26.
Ans. Square, 10.6276; square root, 1.8055'

3. Find the square, and also the square root of 7.52.
Ans. Square, 56.5504; square root, 2.7422

4. Find the square, and also the square root of 83.9.
Ans. Square, 7039.21; square root, 9.159

5. Find the square, and also the square root of 90.8.
Ans. Square, 8244.64; square root, 9.528

6. Find the square, and also the square root of .947.
Ans. Square, .896809; square root, .973'

7. Find the square, and also the square root of $\frac{9}{256}$.
Ans. Square, $\frac{81}{65536}$; square root, $\frac{3}{16}$

8. Find the square, and also the square root of $\frac{25}{231}$.
Ans. Square, $\frac{625}{53361}$; square root, .328.

9. Find the square, and also the square root of $235\frac{1}{4}$.
Ans. Square, $55342\frac{9}{16}$; square root, 15.337'.

10. Find the square, and also the square root of $476\frac{2}{3}$.
Ans. Square, $227211\frac{1}{9}$; square root, 21.832'.

11. Find the cube, and also the cube root of 165.
Ans. Cube, 4492125; cube root, 5.484'.

12. Find the cube, and also the cube root of 27.4.
Ans. Cube, 20570.824; cube root, 3.014'.

13. Find the cube, and also the cube root of 3.28.
Ans. Cube, 35.287552; cube root, 1.485'.

14. Find the cube, and also the cube root of .463.
Ans. Cube, .099252847; cube root, .773'.

15. Find the cube and also the cube root of $125\frac{1}{2}$.
Ans. Cube, $1976656\frac{3}{8}$; cube root, 5.006'.

16. How many rods of fence will be required to enclose 10 acres of ground, in the form of a square? *Ans.* 160 rods.

17. How many rods of fence would be required to enclose a field containing 15 *A.* 2 *R.* 20 *P.*, and having its length equal to twice its breadth? *Ans.* 212.130' rods.

18. A farmer has two tracts of land containing the same number of acres. One of the tracts is a square which is *10000 rods in* compass, and the other an oblong whose breadth

is one-third of its length; required the dimensions of the latter. *Ans.* Length 173.205', breadth 57.735' rods.

19. How many square feet of plank would be required to line the four sides of a cubical cistern which shall contain 500 barrels of water? *Ans.* 820.364' *sq. ft.*

20. A granary whose capacity is 3000 bushels, has its length equal to twice its breadth or height. What are the dimensions of the granary?

Ans. Breadth or height 12.312', length 24.624' *ft.*

21. Another granary whose capacity is 5000 bushels, has its length equal to twice its height, and its height equal to twice its breadth. What are its dimensions?

Ans. Breadth 9.196', height 18.392', length 36.784'.

22. A certain orchard contains 822649 trees, and the number of rows of trees is equal to the number of trees in each row Required the number of rows. *Ans.* 907.

23. A bale of linen contained 1521 yards, and the number of pieces in it was equal to the number of yards in each piece How many pieces were there? *Ans.* 39.

24. A farmer wishes to know what must be the depth of a cubical box which shall contain 100 bushels of grain.

Ans. 4.992' *ft.*

25. A gentleman has an oblong garden 40 poles in length, and 23 *p.* in breadth. Intending to reduce it to the form of a square, of the same area, he wishes to know what must be the length of each side. *Ans.* 30.331' *p.*

26. A certain reservoir for water is 150 *ft.* in length, 100 *ft.* in breadth, and 20 *ft.* in depth, and is lined at the bottom and sides with plank which cost $1.50 per 100 *sq. ft.* Had the reservoir been in the form of a cube of equal capacity, what would have been gained or lost in the cost of the plank for lining it? *Ans.* $38.898' gained.

SUPPLEMENT.

ABBREVIATED OPERATIONS

AND

THE HIGHER PRINCIPLES AND APPLICATIONS

OF

ARITHMETIC.

CONTENTS

OF THE FOLLOWING SUPPLEMENT

ABBREVIATED OPERATIONS

IN

ARITHMETIC.

There are certain abbreviated or contracted methods of calculation in particular cases, in which the pupil may be profitably exercised after he shall have become familiar with the general Rules of Arithmetic. The most useful of these will be given under their appropriate heads.

We shall first explain the methods of *proving* Addition, Subtraction, &c., by the following

Property of the Number 9.

§ **311.** Any number divided by 9 will leave the same remainder as the *sum of its digits* divided by 9.

Take any number, as 345, which is $300+40+5$.

$$300=3\times100=3\times(99+1)=3\times99+3;$$
$$\text{and}\quad 40=4\times\ 10=4\times(\ 9+1\ )=4\times9+4;$$
$$\text{then}\ 345=3\times99+4\times9+3+4+5.$$

Now it is evident that 3 *times* $99+4$ *times* 9 is divisible by 9, *without a remainder;* and therefore the remainder left by $345\div9$ must be that which is left by the sum of the digits $(3+4+5)\div9$.

Similar illustration will apply to any other number consisting of two or more places of figures.

This property of 9, it may be remarked, is a consequence of 9's being one less than 10, the *basis* of numeration. The same property belongs to 3, because 3 is a *measure* of 9.

Addition Proved by Rejecting 9s.

§ **312.** *To prove Addition,*—reject all the 9s in the sum of the digits of each of the numbers which are added together, and set down each *excess* or remainder. Then reject the 9s in the sum of these *excesses:* the remainder must be equal to that found by rejecting the 9s in the *sum of the numbers*.

EXAMPLE.

3 4 7 8 3	7	
4 8 3 5 0	2	
3 7 9 3 8	3	
1 2 1 0 7 1	12,	remainder 3.

Beginning with the upper line, we say, 3 and 4 are 7, and 7 are 14, which is 5 above 9; 5 and 8 are 13, 4 above 9; 4 and 3 are 7. In like manner we find the excesses 2 and 3.

The sum of these excesses is 12, which is 3 above 9; and the excess above the 9s in 121071 is also 3.

The two excesses last found being the same, namely, 3, the addition is *proved to be correct.* This is evident from considering, that in finding each of these two excesses, we find the *excess above all the* 9s in the numbers which are added together, (§ 311.)

Subtraction proved by Rejecting 9s.

§ **313.** *To prove Subtraction,*—reject all the 9s in the sum of the digits of the *remainder*, and also of the *subtrahend.* Then reject the 9s in the sum of the excesses thus found: the remainder last found must be equal to the excess above the 9s in the *minuend.*

The reason of this is evident from the method of proving Addition,—the minuend being the sum of the remainder and subtrahend.

The pupil may be required to give an Example of this method of proving Subtraction.

Multiplication Proved by Rejecting 9s.

§ **314.** *To prove Multiplication,*—reject the 9s in the sum of the digits of *each factor.* Then reject the 9s in the *product* of the excesses thus found: the remainder must be equal to the excess above the 9s in the product of the two numbers.

EXAMPLE.

	37589	5	
	4864	4	
Product	182832896	20	remainder 2.

The excess above the 9s in the multiplicand is 5, and the excess above the 9s in the multiplier is 4. The product of these two excesses is 20, in which the excess above the 9s is 2; and 2 is also the excess above the 9s in the product 182, &c. The multiplication is thus *proved to be correct.*

The reason of this method will appear from considering, that if we reject all the 9s in the multiplicand and multiplier, and all the 9s in the product of the *two remainders*, the effect must be to reject all the 9s contained in the *product of the two numbers.*

Division Proved by Rejecting 9s.

§ **315.** *To prove Division,*—reject the 9s in the divisor, and also in the quotient; and then reject the 9s in the product of the excesses thus found: the remainder must be equal to the excess above the 9s in the dividend.

If there be any *remainder in the division*, its excess above the 9s must be added to the product of the excesses above 9s in the divisor and quotient.

This method of proving Division results from the one just given for Multiplication,—the dividend being equal to the product of the divisor and quotient, *plus* the remainder, if any.

The pupil may be required to give an example of this method of proving Division.

Note. The preceding, as well as all other methods of arithmetical proof, afford but a high *probability*, not an absolute certainty, of the correctness of arithmetical calculations. For an error in the proof may balance an error in the calculation itself, so that the two erroneous results will agree. The only absolute proof consists in the *intuitive truth* of each particular result arising from the operation, performed according to the principles of the science.

Arithmetical Complement—used in Abbreviating the Subtraction of two or more Numbers.

§ **316.** The *arithmetical complement* of a number, is the difference between that number and *a unit with as many* 0*s* annexed as there are figures in the given number.

Thus the Complement of 6 is 10—6 =4;
and the Complement of 64 is 100—64=36.

The Complement of a number will be found by subtracting its right hand figure from 10, and each of its other figures from 9.

Any number may be subtracted by *adding its complement* to the minuend, and subtracting 1 from that figure in the result which is next on the *left of the subtrahend.*

$$\begin{array}{r} 3\,7\,5\,6 \\ 2\,4 \\ \hline 3\,7\,3\,2 \end{array}$$

Thus, instead of subtracting the 24, we may *add its complement* 76, and *subtract* 1 from the resulting 8 on the left.

☞ By adding the complement of 24, we have 3756+(100—24), which exceeds the difference 3756—24 by 100; we must therefore subtract 100, to find the true difference, and this is done by merely subtracting 1 from the *hundreds figure.*

Two or more numbers may be subtracted at once, by mentally *adding their complements* to the minuend, and subtracting 1 for each complement from that figure in the result which is next on the left of each subtrahend.

EXAMPLES.

(I.)		(II.)		(III.)		(IV.)	
	4376		7434		5860		9763
	951		—861		3872		—831
	—764		3703		—2305		—173
	—802		—49		—834		—16
	3761		10227		6593		8743

In these examples the *sign* — is prefixed to those numbers which are to be subtracted from the *sum of the other numbers.*

In the first example we find the *complements* of 2 and 4 by by subtracting these figures from 10, and *add these complements* with 1 and 6. *Carrying* 2, we subtract the 0 and 6 from 9, and add the complements; and in like manner we add the third column. Finally, from the fourth resulting figure 5 we subtract 2, for the *two* complements that were taken in the third column.

Let the pupil be required to perform the operations in the other Examples.

ABBREVIATIONS IN MULTIPLICATION.

One means of abbreviating Multiplication, is, to commit to memory the *products* of numbers beyond the common limit of the *Multiplication Table.*

These products should be recited thus: 13 *times* 2 is 26; 13 *times* 3 is 39, and so on.

The most useful part of the following Table, is from 13×2 to 13×9, and so on to 19×9; though the whole is well worth the labor of learning it.

Supplement to the Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361

This Table having been well committed to memory, we may multiply at once by 13, 14, &c., as readily as by a single figure. Thus

$$\begin{array}{r} 34789 \\ 13 \\ \hline 452257 \end{array}$$

13 times 9 is 117; 13 times 8 is 104, and 11 make 115, &c.

Without using the preceding Table, we may apply the following method:—

§ **317.** *To Multiply by* 13, 14, *&c. to* 19.

1. Multiply by the *units* figure, and add to each partial product (after the first) that figure of the multiplicand which stands next on the right of the one multiplied.

2. Set down only the *right hand figure* of the last product; add the left to the *last figure of the multiplicand,* and set the sum on the left of the other product figures.

EXAMPLE.

To multiply 347045 by 13.

$$\begin{array}{r} 347045 \\ 13 \\ \hline 4511585 \end{array}$$

3 times 5 is 15; 3 times 4 is 12, and 1 are 13, and 5 are 18; 3 times 0 is 0, and 1 and 4 are 5; 3 times 7 is 21; 3 times 4 is 12, and 2 are 14, and 7 are 21; 3 times 3 is 9, and 2 are 11, and 4 are 15; 1 and 3 are 4.

The reason of this method will be understood from considering, that if we multiply by the 3, and then by the 1, according to the Common Rule, the same *additions* of figures will occur in finding the total product.

§ **318.** *Another Method of Multiplying by* 13, 14, *&c.*

Multiply by the *units* figure;—set this product with its *tens* figure under the *units* of the multiplicand, and thus add the product to the multiplicand.

$$\begin{array}{l} 34705 \quad \times 14 \\ 138820 \\ \hline 485870 \end{array}$$

In like manner we multiply by 1 *with* 0s *and a significant figure* annexed—always setting the first figure of the partial product *one more place* to the right of the multiplicand than there are intervening 0s in the multiplier.

$$\begin{array}{l} 34705 \quad \times 104 \\ \;\;138820 \\ \hline 3609320 \end{array} \qquad \begin{array}{l} 34705 \quad \times 1004 \\ \;\;\;138820 \\ \hline 34843820 \end{array}$$

In the first of these examples there is *one* intervening 0 in the multiplier 104, and we set the first figure of the partial product *two places* to the right of the multiplicand.

By this method the several figures of the multiplicand are added to the same figures to which they would be added in finding the product according to the Common Rule.

The pupil may be required to give other Examples of these methods of multiplying.

§ **319.** *To Multiply by* 21, 31, *&c. to* 91.

Multiply by the *tens* figure;—set this product with its units figure under the *tens* figure of the multiplicand, and thus add the product to the multiplicand.

```
   74314×21
  148628
  -------
  1560594
```

In like manner we multiply by any *significant figure with* 0s *and a unit* annexed—always setting the units figure of the partial product *one more place* to the left of the units of the multiplicand than there are intervening 0s in the multiplier.

```
    74314×201        |     74314×2001
 148628              |  148628
 --------            |  ---------
 14937114            |  148702314
```

In the first of these examples there is *one* intervening 0 in the multiplier 201, and we set the units figure of the partial product *two places* to the left of the units of the multiplicand.

This method is evidently equivalent to that by the Common Rule: it merely dispenses with multiplying by the 1 in the right of the multiplier.

The pupil may be required to give other Examples.

§ **320.** *To Multiply by any number of* 9s.

Annex as many 0s to the multiplicand as there are 9s in the multiplier, and from the result *subtract the given multiplicand.*

To multiply 1347 by 999.

```
  1347000
     1347
  -------
  1345653
```

The annexing of *three* 0s to the multiplicand is equivalent to *multiplying it* by 1000; then the multiplicand subtracted from the result leaves 999 *times* the given multiplicand.

Other Examples of this method may be required of the learner.

The following method is applicable to many questions of a practical character, and greatly abridges the calculation in all the cases which are comprehended under it.

§ **321.** *When the Multiplier is an Aliquot Part of a number of Tens, Hundreds, or Thousands.*

Multiply by the *number of tens, hundreds,* or *thousands* of which the given multiplier is an aliquot part, and then take the *same part* of the product thus found.

To multiply 2467 by $16\frac{2}{3}$.

$16\frac{2}{3}$ is $\frac{1}{6}$ of 100, since $16\frac{2}{3}\times6=100$.

Then $2467\times16\frac{2}{3}=2467\times\frac{100}{6}=246700\div6=41116\frac{2}{3}$.

In adopting this method, the pupil has to find some *integer* which multiplied into the given multiplier will produce a number of *tens*, or *hundreds*, or *thousands.* We give a few more examples.

To multiply 157 by $13\frac{1}{3}$.

$13\frac{1}{3}\times3=40$; then $157\times13\frac{1}{3}=157\times\frac{40}{3}=6280\div3$.

To multiply 157 by 25.

$25\times4=100$; then $157\times25=157\times\frac{100}{4}=15700\div4$.

To multiply 157 by $166\frac{2}{3}$.

$166\frac{2}{3}\times6=1000$; then $157\times166\frac{2}{3}=157\times\frac{1000}{6}=157000\div6$.

The products thus obtained are correct, because the improper *fractions* taken as multipliers are equal to the given multipliers.

Thus in the last example the multiplier $166\frac{2}{3}=\frac{1000}{6}$.

Exercises in the application of this method will be found in connexion with the corresponding method of Division, which immediately follows.

Other methods of abbreviating Multiplication, in particular cases, might be given. Some of them will occur to the ingenious student, on the occasions demanding them, and others are of little practical utility. We have explained those which are most likely to be adopted in practice.

ABBREVIATIONS IN DIVISION.

The most useful method of abbreviating Division, results from the one just given for Multiplication.

§ 322. *When the Divisor is an Aliquot Part of a number of Tens, Hundreds, or Thousands.*

Multiply the dividend by the number by which the *divisor must be multiplied* to produce a number of *tens*, *hundreds*, or *thousands*, and divide the product by those *tens*, *hundreds*, or *thousands*.

To divide 2467 by $16\frac{2}{3}$.

Since $16\frac{2}{3}\times6=100$, the divisor $16\frac{2}{3}=\frac{100}{6}$;
then $2467\div\frac{100}{6}=2467\times\frac{6}{100}=2467\times6\div100$.

To divide 2467 by 125.

Since $125\times4=1000$, the divisor $125=\frac{1000}{4}$;
then $2467\div\frac{1000}{4}=2467\times\frac{4}{1000}=2467\times4\div1000$.

For the divisor 15 we may take its equivalent $\frac{60}{4}$; for 35 we may take $\frac{70}{2}$; and in like manner whenever the divisor *multiplied by an integer* produces a number of *tens*, hundreds, or thousands, &c.

This method, it will be seen, is but the converse of the one last given for Multiplication, (§ 321); and admits of equally extensive application.

A multiplier or divisor which bears any simple ratio to 100, or 1000, &c., as $37\frac{1}{2}$, $62\frac{1}{2}$, $87\frac{1}{2}$, 375, 625, &c., may be brought within the application of the same methods.

Thus $37\frac{1}{2}$, being $\frac{3}{8}$ of 100, is $\frac{1}{8}$ of 300; then for $37\frac{1}{2}$ we may take its equivalent $\frac{300}{8}$. For $62\frac{1}{2}$ we may take $\frac{500}{8}$; and for $87\frac{1}{2}$ we may take $\frac{700}{8}$.

Again; 375, being $\frac{3}{8}$ of 1000, is $\frac{1}{8}$ of $3000,=\frac{3000}{8}$.
625, being $\frac{5}{8}$ of 1000, is $\frac{1}{8}$ of $5000,=\frac{5000}{8}$.
875, being $\frac{7}{8}$ of 1000, is $\frac{1}{8}$ of $7000,=\frac{7000}{8}$.

When *decimal fractions* are involved in these abbreviated multiplications and divisions, the number of decimal figures in the *product* or *quotient* must be adjusted according to the Common Rules.

MISCELLANEOUS EXERCISES.

The pupil should *explain the operations* in these exercises, and repeat them until the subject is familiar to him.

By abbreviated methods find the Product

1. Of 3725×17,—of 57039×19,—and of 73165×109.
Ans. 63325; 1083741; and 7974985.
2. Of 2709×41,—of 38740×71,—and of 48365×801.
Ans. 111069; 2750540; and 38740365.
3. Of 6347×99,—of 54398×$12\frac{1}{2}$,—and of 68371×$133\frac{1}{3}$.
Ans. 628353; 679975; and $9116133\frac{1}{3}$.
4. Of 1450×$11\frac{1}{9}$,—of 20732×$16\frac{2}{3}$,—and of 56814×125.
Ans. $16111\frac{1}{9}$; $345533\frac{1}{3}$; and 7101750.
5. Of 2135×25,—of 41640×103,—and of 13472×$111\frac{1}{9}$.
Ans. 53375; 4288920; and $1496888\frac{8}{9}$.
6. Of 7318×$37\frac{1}{2}$,—of 38431×375,—and of 51031×401.
Ans. 274425; 14411625; and 20463431.
7. Of 1693×45,—of 37304×$87\frac{1}{2}$,—and of 43721×$66\frac{2}{3}$.
Ans. 76185; 3264100; and $2914733\frac{1}{3}$.
8. Of 5478×$62\frac{1}{2}$,—of 43821×625,—and of 37951×$333\frac{1}{3}$.
Ans. 342375; 27388125; and $12650333\frac{1}{3}$.
9. Of 37.36×$1.3\frac{1}{3}$,—of 738.30×3.50,—and of 47300×$1.66\frac{2}{3}$.
Ans. $49.813\frac{1}{3}$; 2584.05; and $78833.33\frac{1}{3}$.
10. Of 5.461×.13,—of 9348.3×.225,—and of 73047×.9999.
Ans. .70993; 2103.3675; and 73039.6953.

By abbreviated methods find the Quotient

11. Of 1375÷$3\frac{1}{3}$,—of 76834÷25,—and of 84783÷175.
Ans. $412\frac{1}{2}$; $3073\frac{9}{25}$; and $484\frac{83}{175}$.
12. Of 8364÷35,—of 38639÷$11\frac{1}{9}$,—and of 74950÷$16\frac{2}{3}$.
Ans. $238\frac{34}{35}$; $3477\frac{51}{100}$; and 4497.
13. Of 7460÷$12\frac{1}{2}$,—of 86348÷75,—and of 83764÷$111\frac{1}{9}$.
Ans. $596\frac{4}{5}$; $1151\frac{23}{75}$; and $753\frac{219}{250}$.
14. Of 3701÷45,—of 95406÷$37\frac{1}{2}$,—and of 73841÷$66\frac{2}{3}$.
Ans. $82\frac{11}{45}$; $2544\frac{4}{25}$; and $1107\frac{123}{200}$.
15. Of 4632÷$62\frac{1}{2}$,—of 14783÷55,—and of 83760÷$133\frac{1}{3}$.
Ans. $74\frac{14}{125}$; $268\frac{43}{55}$; and $628\frac{1}{5}$.
16. Of 3761÷25,—of 30304÷$87\frac{1}{2}$,—and of 73641÷875.
Ans. $150\frac{11}{25}$; $346\frac{58}{175}$; and $841\frac{107}{175}$.
17. Of 83.46÷$12\frac{1}{2}$,—of 40.763÷125,—and of 94378÷$3.33\frac{1}{3}$.
Ans. 6.6768; .326104; and 28313.4.
18. Of 7.600÷$16\frac{2}{3}$,—of 13721÷$.37\frac{1}{2}$,—and of 73491÷$1.66\frac{2}{3}$.
Ans. .456; $36589\frac{1}{3}$; and 44094.6.

ABBREVIATIONS IN MULTIPLICATION OF DECIMALS.

By the common Rule for the Multiplication of Decimals, the product will often contain more decimal figures than it is necessary to find. In such cases we may adopt the following method of *contracting* the operation.

§ **323.** *To Contract the Multiplication of Decimals to the finding of a given number of decimal figures in the product.*

The method of doing this will be exhibited by the following

EXAMPLE.

To multiply 37.14586 by 92.83, so far as is necessary to find *three decimal figures* in the product.

$$\begin{array}{r}
37.14586 \\
38.29 \\
\hline
3343127 \\
74292 \\
29716 \\
1114 \\
\hline
3448.249
\end{array}$$

We set the *units figure* of the multiplier under that decimal place of the multiplicand which is the *last to be retained* in the product. In this example the units 2 in the multiplier is set under the *third* decimal figure in the multiplicand, since we are to find *three* decimal figures in the product.

The remaining figures of the multiplier are set in *reverse order* —the *integer* 92 on the right of the decimal point, with its digits reversed, and the decimal 83 on the left, with its digits reversed.

In multiplying we begin with that figure of the multiplicand which stands *directly over the multiplying figure;* thus in multiplying by 9 we begin with the 8; in multiplying by 2 we begin with the 5; and so on. But to secure an *average correctness* in the first figures of the several products, we add to each the nearest number of *tens* that would arise from multiplying the *rejected right hand figure* of the multiplicand. And here observe, that

When the rejected right hand figure would give a product *midway between two numbers of tens*, as 15, 25, 35, and so on, we take the *greater* number as the *nearest value*, because such product would generally be increased from multiplying the other rejected figures.

The several products thus formed will all begin with the *same order of decimals* as the last decimal figure to be retained in the product required. In this example, the .0008 multiplied by the 9, which is 9 *tens* or 90, produces 72 *thousandths*; the .005×2 produces 10 *thousandths*, and so on; and *thousandths* is the last order to be found in the product of the given numbers.

The first figures of the several products must therefore be set *one under another;* and in this order the products must be added together.

Note. The last decimal figure in the product found as above, will often be the same that would be found in that place by the common Rule, and will very seldom be wrong by more than a *unit*.

By that Rule the product in the preceding Example would be

3448.2501838.

When the multiplicand has not as many decimal figures as are required in the product, 0s may be annexed to supply the deficiency.

EXERCISES.

1. Multiply 73.1285 by 4.1316, for two decimal figures in the product. *Ans.* 302.12'.

2. Multiply 130.375 by .47348, for three decimal figures in the product. *Ans.* 61.728'.

3. Multiply 8714.38 by 37.505, for three decimal figures in the product. *Ans.* 326832.822'.

4. Multiply 570.794 by 1.7383, for four decimal figures in the product. *Ans.* 992.2111'.

5. Multiply 97375.8 by .37649, for four decimal figures in the product. *Ans.* 36661.0149'.

6. Multiply 30.7307 by 21.378, for five decimal figures in the product. *Ans.* 656.96090'.

7. Multiply .873805 by .75074, for five decimal figures in the product. *Ans.* .65599'.

ABBREVIATIONS IN DIVISION OF DECIMALS.

When there are many figures in the divisor, and it is necessary to find only a few decimal figures in the quotient, the division may be abbreviated by omitting a part of the customary operation.

§ **324.** *To Contract the Division of Decimals when only a given number of decimal figures are required in the quotient.*

The method of doing this will be illustrated by the following

EXAMPLE.

To divide 2508.92806 by 92.4135, so far as is necessary to find *two decimal* figures in the quotient.

```
9 2.4,1,3 5 ) 2 5 0 8.9 2 8 0 6 ( 2 7.1 4
              1 8 4 8 3
              ---------
                6 6 0 6
                6 4 6 9
                -------
                  1 3 7
                    9 2
                  -----
                    4 5
                    3 7
```

We first consider how many figures the quotient will contain. In this example the quotient will contain *four figures*, two integral figures from dividing 92 into 2508, and the two required decimals.

We divide at first by as many figures in the left of the given divisor as there are to be *figures in the quotient*:—in this example we divide by 9241; and instead of affixing the next figure of the dividend to the remainder, for a new dividend, we *reject another figure* from the divisor, and divide 924 into 6606; and so on.

This divisor and dividend have each *one-tenth* of the value that would accrue to them from an additional figure on the right of each, (§ 16); hence the quotient figure remains the same, (§ 57).

In multiplying the partial divisors by the quotient figures, it is proper to carry the *tens* that would arise from multiplying the rejected figure on the right, as in contracted multiplication of decimals, (§ 323).

Note. When the divisor has not as many figures as are required to be found in the quotient, the division must proceed according to the common Rule, until the figures in the divisor are one more than those *remaining to be found in the quotient.*

EXERCISES.

1. Divide 2857.35 by 743.672, so that the operation shall terminate with two decimal figures in the quotient.
Ans. 3.84'.

2. Divide 738.973 by 205.864, so that the operation shall terminate with three decimal figures in the quotient.
Ans. 3.589'.

3. Divide 5037.64 by 7.38713, so that the operation shall terminate with three decimal figures in the quotient.
Ans. 681.948.

4. Divide 1384.47 by 237.416, so that the operation shall terminate with four decimal figures in the quotient.
Ans. 5.8314'.

5. Divide 4743.86 by 12.3075, so that the operation shall terminate with four decimal figures in the quotient.
Ans. 385.4447'.

6. Divide 743.640 by 247.381, so that the operation shall terminate with five decimal figures in the quotient.
Ans. 3.00605'.

7. Divide 708.349 by 46.8413, so that the operation shall terminate with five decimal figures in the quotient.
Ans. 15.12232'.

8. Divide 3.783943 by .785, so that the operation shall terminate with five decimal figures in the quotient.
Ans. 4.82030'.

9. Divide 7348.3646 by 3.7, so that the operation shall terminate with four decimal figures in the quotient.
Ans. 1986.0444'.

10. Divide 1037.3987 by 276.03, so that the operation shall terminate with six decimal figures in the quotient.
Ans. 3.758283'.

ABBREVIATIONS IN EVOLUTION.

§ 325. In extracting roots, the successive divisors are soon found to be alike in the first two or three *left hand figures*. If the root is to contain decimals, and the figures remaining to be found are less in number than those in the *last complete divisor*, the operation may then be continued as in Contracted Division of Decimals, (§ 324.)

EXAMPLE.

1. To extract the square root of 2 to *six* decimal figures.

```
               2   (1.414214'
               1
              ---
24     )100
         96
        -----
281    )  400
          281
          -----
282,4)   11900
         11296
         -----
282    )   604
           565
           ---
28     )    39
            28
            --
2      )    11
```

We proceed according to the Common Rule to find the first four figures, 1.414, in the root. The divisors, 281 and 2824, have become the same in their two left hand figures; and the three figures remaining to be found in the root do not exceed in number those in the divisor 2824.

The operation is continued by rejecting the right hand figure of the divisor, and using its other figures and the remainder in the same manner as in Contracted Division of Decimals.

The correctness of this method will be evident from considering, that the value of each succeeding figure in the root, depends

on the first two or three figures in the left of the corresponding divisor.

Instead of annexing 00 to the remainder, we reject two figures in the right of the divisor, namely, the right hand one of the preceding divisor, and the one obtained from dividing. This does not affect the value of the resulting root or quotient, since the divisor and dividend thus taken are each reduced to *one hundredth* of the value that would accrue to them from the additional figures on the right of each, (§ 57.)

The same method of contraction is applicable to the *cube root*, and for similar reasons.

Note. The number of figures that may be found in the root after the contraction commences, will evidently be one less than the number in the last complete divisor that was used; but the last figure thus found may be incorrect. In the preceding example, the root carried to the same extent by the Common Rule, is

1.414213'.

This uncertainty in regard to the last figure, is of little importance when the root contains several decimal figures.

The preceding method of contraction is to be applied to the following

EXERCISES.

1. Extract the square root of 3 to five decimal figures. *Ans.* 1.73205'.
2. Extract the square root of 5.13 to five decimal figures. *Ans.* 2.26495'.
3. Extract the square root of 7.35 to six decimal figures. *Ans.* 2.711088'.
4. Extract the square root of 21.345 to six decimal figures. *Ans.* 4.620065'.
5. Extract the square root of 7342.5 to six decimal figures. *Ans.* 85.688393.
6. Extract the *cube root* of 4 to five decimal figures. *Ans.* 1.58740'.
7. Extract the cube root of 5.18 to five decimal figures. *Ans.* 1.73025'.
8. Extract the cube root of 285.75 to five decimal figures. *Ans.* 6.58669'.

THE HIGHER ROOTS.

§ **326.** The *fourth root* of a number is that number whose fourth *power* is equal to the given number; the *fifth root* of a number is that number whose fifth power is equal to the given number; and so on.

Thus 3 is the 4th root of 81, since the fourth power of 3 is 81; and 2 is the 5th root of 32, since the fifth power of 2 is 32.

§ **327.** The fourth root may be obtained, most readily, by extracting the *square root of the square root* of the given number

Thus the square root of 625 is 25, and the square root of 25 is 5; then 5 is the 4th root of 625, since, from the manner in which the 5 has been obtained, its 4th power must be equal to 625.

The square root of 6561 is 81, and the square root of 81 is 9; then 9 is the *fourth root* of 6561.

In general terms,

§ **328.** Any root whose fractional exponent is *resolvable into two factors*, may be found by extracting such root of the given number as is denoted by one of those factors, and then such root of *that root* as is denoted by the *other factor*.

The 6th root is equal to the *cube root of the square root*, or the square root of the cube root,—the fraction $\frac{1}{6}$, which is the *exponent* of the 6th root, being equal to $\frac{1}{2}\times\frac{1}{3}$.

The 5th root cannot be extracted in this way, since the exponent $\frac{1}{5}$ cannot be resolved into two factors. A similar remark is applicable to the 7th root, &c.

Note. General Rules might be given for extracting the higher roots of numbers. These rules, however, are very tedious in their application; and these higher roots are not required in any calculations which properly come within the sphere of Arithmetic. At a more advanced stage in his mathematical studies, the pupil will meet with the proper applications of these roots, and the *best methods of extracting them.*

REMARK.

The preceding, it is believed, comprise most, if not all, of the methods of abbreviating the operations under the General Rules of Arithmetic, that can be considered as practically useful.

By pursuing these methods in connection, as has been done in the present work, they may more conveniently be made a subject of distinct study by the pupil; while, by the same arrangement, the most important directions relating to them, have been given in brief, but comprehensive terms.

REPEATING DECIMALS.

§ **329.** A *repeating decimal*, also called a *Repetend*, is one in which the same figure or figures recur in immediate and continual succession.

In reducing $\frac{1}{3}$ to a decimal, we obtain the *repetend* .333 and so on, which is denoted by $.\dot{3}$, a point over the repeating figure.

In reducing $\frac{2}{11}$ to a decimal, we obtain the *repetend* .181818 &c., which is denoted by $.\dot{1}\dot{8}$, a point over the *first* and the *last* repeating figure.

§ **330.** A *mixed repetend* is a decimal in which other figures precede a repetend or repeating decimal. These precedent figures are called *finite* figures,—the number of figures in the repetend itself being *infinite*.

Thus if we reduce $\frac{5}{12}$ to a decimal, we shall obtain the *mixed repetend* $.41\dot{6}$, in which .41 are the *finite* figures.

In what cases Repetends occur.

§ **331.** Decimal division will always produce a *repetend* when the divisor and dividend are *prime to each other*, and the divisor contains any other prime factors than 2 or 5.

Suppose that 13 is to be divided, decimally, by 15; these numbers having no common measure greater than a unit.

We annex decimal 0s to 13, and divide the result as an integer. The 0s annexed, multiply the 13 successively by 10, or 2×5; and this introduces the factors 2 and 5 into the *dividend*.

But the factors of 15 are 3 and 5, the first of which is not a factor of the dividend. Hence in dividing 13 with 0s annexed by 15 there will always be a remainder; that is, the division will never terminate.

Every case of interminable division will result in a *repeating decimal*. For as the remainders, to which 0s are annexed, are always less than the divisor, some one remainder must occur a second time before the *number of divisions* is equal to the divisor; and then the same figures will recur in the quotient that succeeded the first occurrence of that remainder; that is, the quotient will become a *repetend*.

Repeating Decimals Reduced to Equivalent Vulgar Fractions.

§ 332. A Repetend is always equal to a vulgar fraction whose numerator is the *repeating figure* or *figures*, and denominator as many 9s as there are repeating figures.

For, by reducing $\frac{1}{9}$ to a decimal, we obtain the repetend $.\dot{1}$; then $.\dot{1}=\frac{1}{9}$; and consequently $.\dot{2}=\frac{2}{9}$; $.\dot{3}=\frac{3}{9}$, &c.

By reducing $\frac{1}{99}$ to a decimal, we obtain the repetend $.\dot{0}\dot{1}$; then $.\dot{0}\dot{1}=\frac{1}{99}$; $.\dot{0}\dot{2}=\frac{2}{99}$; $.\dot{0}\dot{3}=\frac{3}{99}$; $.\dot{0}\dot{4}=\frac{4}{99}$, &c.

By reducing $\frac{1}{999}$ to a decimal, we obtain the repetend $.\dot{0}0\dot{1}$; then $.\dot{0}0\dot{1}=\frac{1}{999}$; $.\dot{0}0\dot{2}=\frac{2}{999}$; $.\dot{0}0\dot{3}=\frac{3}{999}$, and so on.

The same method of illustration may be applied to a repetend consisting of four, five, or more figures.

Thus we should find $.\dot{0}00\dot{1}=\frac{1}{9999}$; $.\dot{0}00\dot{2}=\frac{2}{9999}$, &c.

§ 333. The value of a *mixed repetend* may be expressed by a complex or mixed decimal, (§ 132); and this decimal may then be reduced to a vulgar fraction.

Thus in $.41\dot{6}$ the repeating figure 6 annexed to .41, is equivalent to $\frac{6}{9}$ or $\frac{2}{3}$ annexed. (§ 332.)

$$\text{Then } .41\dot{6}=.41\tfrac{2}{3}=\frac{41\frac{2}{3}}{100}=\tfrac{125}{3}\div100=\tfrac{125}{300}=\tfrac{5}{12}.$$

These principles are to be applied in the following

EXERCISES.

1. Reduce $.\dot{5}$ to an equivalent vulgar fraction. *Ans.* $\frac{5}{9}$.
2. Reduce $.\dot{1}\dot{5}$ to an equivalent vulgar fraction. *Ans.* $\frac{5}{33}$.
3. Reduce $.\dot{2}\dot{4}$ to an equivalent vulgar fraction. *Ans.* $\frac{8}{33}$.
4. Reduce $.\dot{5}1\dot{3}$ to an equivalent vulgar fraction. *Ans.* $\frac{19}{37}$.
5. Reduce $.41\dot{2}$ to an equivalent vulgar fraction. *Ans.* $\frac{371}{900}$.
6. Reduce $.5\dot{0}\dot{3}$ to an equivalent vulgar fraction. *Ans.* $\frac{83}{165}$.
7. Reduce $.\dot{2}6\dot{4}$ to an equivalent vulgar fraction. *Ans.* $\frac{88}{333}$.
8. Reduce $.\dot{3}03\dot{6}$ to an equivalent vulgar fraction. *Ans.* $\frac{92}{303}$.
9. Reduce $.24\dot{1}\dot{2}$ to an equivalent vulgar fraction. *Ans.* $\frac{199}{825}$.
10. Reduce $.33\dot{3}\dot{0}$ to an equivalent vulgar fraction. *Ans.* $\frac{1099}{3300}$.

CONTINUED FRACTIONS.

§ **334.** A *continued fraction* is one whose numerator is *unity*, and denominator the *sum of an integer and a fraction* whose numerator is *unity*, and so on, as before.

The following is the usual method of expressing a continued fraction:

$$\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{5+\frac{1}{6}}}}.$$

The *partial fractions* $\frac{1}{2}$, $\frac{1}{3}$, &c.,—each succeeding one of which is to be added to the *denominator* of the preceding one,—may be called the *terms* of the continued fraction.

A continued fraction may also be expressed by placing its terms one directly after another, with the sign $+$ between *the denominators;* thus

$$\frac{1}{2}{+}\frac{1}{3}{+}\frac{1}{5}{+}\frac{1}{6}$$

§ **335.** Continued fractions are employed to find, in lower terms, successive *approximations* to the value of a fraction or *ratio* whose terms are large, and *prime to each other*.

For example, suppose we wish to find, in lower terms, *approximate values* of $\frac{131}{418}$, whose exact value cannot be expressed in lower terms.

Dividing both the given terms by the less, we find

$$\frac{131}{418}=\frac{1}{3\frac{25}{131}}$$

Disregarding the fraction in the denominator, we have $\frac{1}{3}$ for the *first approximate value* of the given fraction.

This first approximation is *greater than the true value*, because the denominator 3 is less than the true denominator $3\frac{25}{131}$. But since this denominator is between 3 and 4, the true value is between $\frac{1}{3}$ and $\frac{1}{4}$.

To find the *second approximate value*, we reduce the $\frac{25}{131}$ in the same manner as the given fraction; thus

$$\frac{25}{131}=\frac{1}{5\frac{6}{25}}.$$

$$\text{Then}\quad \frac{131}{418}=\frac{1}{3+}\frac{1}{5\frac{6}{25}}.$$

Disregarding the last fraction in the denominator, we have $\frac{1}{3+}\frac{1}{5}=1\div\frac{16}{5}=\frac{5}{16}$, for the *second approximation.*

This second approximation is *less than the true value*, because, by rejecting the $\frac{6}{25}$, the term $\frac{1}{5}$ becomes greater than the true value to be added to the denominator 3.

To find the *third approximate value*, we reduce the $\frac{6}{25}$ in the same manner as before; thus

$$\frac{6}{25}=\frac{1}{4\frac{1}{6}}$$

$$\text{Then}\quad \frac{131}{418}=\frac{1}{3+}\frac{1}{5+}\frac{1}{4\frac{1}{6}}$$

Disregarding the last fraction in the denominator, we have

$\frac{1}{3+}\frac{1}{5+}\frac{1}{4}=\frac{1}{3+1\div\frac{21}{4}}=\frac{21}{67}$, for the *third approximation.*

This third approximation is *greater than the true value;* because by rejecting the $\frac{1}{6}$ the $\frac{1}{4}$ becomes too much to add to the denominator 5; and thence results too small a value to add to the denominator 3.

We have thus obtained the three approximating fractions, $\frac{1}{3}$, $\frac{5}{16}$, and $\frac{21}{67}$, which are alternately greater and less than the given fraction. If the remaining partial fraction $\frac{1}{6}$ be included in the valuation, the result will be the given fraction.

From the preceding example we may derive the following principles in relation to

Approximating Fractions.

§ **336.** Any given proper fraction may be reduced to a *continued fraction* by dividing both its terms by the numerator,—proceeding in like manner with the fraction formed of the remainder and divisor,—and so on; and connecting each succeeding *partial fraction*, by the sign $+$, to the preceding *denominator* Then,

1. The *first partial fraction* will be the *first approximation* to the value of the given fraction.

Thus $\frac{1}{3}$ in the preceding example (§ 335) is the first approximation to the value of $\frac{131}{418}$.

2. The *second approximation* will have for its numerator the denominator of the *second partial fraction;* and for its denominator the product of the denominators of the first and second partial fractions, *plus* 1.

Thus in $\frac{5}{16}$, in the preceding example, the numerator is the second partial denominator; and the denominator 16 was found by multiplying 5 into 3, and adding 1.

3. The two terms of each *succeeding approximation*, will be found by multiplying the corresponding partial *denominator* into the two terms, respectively, of the preceding approximation, and adding the terms of the next preceding one.

Thus in the *third approximation*, $\frac{21}{67}$, the numerator 21 was found by multiplying 4 into 5, and adding 1; and the denominator 67 by multiplying 3 into 21, and adding 4, which is equivalent to $4\times16+3$.

If we proceed in like manner to find the *fourth* approximation, we have $6\times21+5=131$, for the numerator; and $6\times67+16=418$, for the denominator; that is, we reproduce the given fraction $\frac{131}{418}$.

We may also conclude from the same example, that

§ 337. Every *odd* aproximating fraction, as the *first, third*, and so on, will be *greater* than the given fraction; while every *even* one will be less than the given fraction.

Thus we found $\frac{1}{3}$ to be *greater*, $\frac{5}{16}$ to be *less*, and $\frac{21}{67}$ to be greater, than $\frac{131}{418}$.

The value, therefore, of the given fraction *lies between any two consecutive approximating fractions*. But it is desirable to ascertain more definitely the accuracy of each successive approximation.

§ 338. Any one of the approximating fractions differs in value from the given fraction by less than a *unit* divided by the denominator of that approximation multiplied into the denominator of the next approximation.

To prove this we first observe that if two consecutive approximating fractions be reduced to a *common denominator*, the resulting numerators will always differ by a *unit*.

Thus taking the $\frac{5}{16}$ and $\frac{21}{67}$, found in the example before given, and reducing them to a common denominator, the numerators will become

$$5\times67 \text{ and } 21\times16.$$

Recollecting the composition of the 67 and 21, according to § 336, we have $5\times67=5\times(4\times16+3)=5\times4\times16+5\times3$;
and $21\times16=(4\times5+1)\times16=4\times5\times16+1\times16$.

From the manner in which these numerators are formed, as just shown, they necessarily differ from each other by a *unit;* and this method of illustration will apply to all like cases.

The difference between the $\frac{5}{16}$ and $\frac{21}{67}$ will then be $\frac{1}{16\times67}$. But we have already seen that the value of the given fraction lies between $\frac{5}{16}$ and $\frac{21}{67}$, (§ 335,) and therefore differs from either of these less than they differ from each other.

Hence $\frac{5}{16}$ differs in value from $\frac{131}{418}$ by less than $1\div(16\times67)$.

From the preceding principle it also follows, that

§ **339.** Any particular approximation differs in value from the given fraction by less than a *unit* divided by the *square of the denominator* of that approximation.

Thus $\frac{5}{6}$ differs in value from $\frac{131}{418}$ by less than $1\div16^2$, since $1\div16^2$ is less than $1\div(16\times67)$.

The principles of § 336 may now be applied to the following

EXERCISES.

1. Find approximate values of $\frac{149}{1217}$. *Ans.* $\frac{1}{8}$, $\frac{5}{41}$, $\frac{6}{49}$.
2. Find approximate values of $\frac{487}{2381}$. *Ans.* $\frac{1}{4}$, $\frac{1}{5}$, $\frac{9}{44}$.
3. Find approximate values of $\frac{349}{1787}$. *Ans.* $\frac{1}{5}$, $\frac{8}{41}$, $\frac{25}{128}$, $\frac{108}{553}$.
4. Find approximate values of $\frac{163}{1493}$. *Ans.* $\frac{1}{9}$, $\frac{6}{55}$, $\frac{19}{174}$, $\frac{25}{229}$, $\frac{69}{632}$.

To obtain approximate values of a *decimal fraction*, the denominator of the fraction must be supplied.

Thus for .83 we would take $\frac{83}{100}$.

5. Find approximate values of .329. *Ans.* $\frac{1}{3}$, $\frac{25}{76}$, $\frac{76}{231}$.
6. Find approximate values of 2.17. *Ans.* $2\frac{1}{5}$, $2\frac{1}{6}$, $2\frac{8}{47}$.
7. Find three approximate values of 3.14159', which is the ratio of the circumference to the diameter of a circle.
Ans. $3\frac{1}{7}$, $3\frac{15}{106}$, $3\frac{16}{113}$.

ARITHMETICAL PROGRESSION.

§ **340.** An ARITHMETICAL PROGRESSION is a series of quantities which continually increase or decrease by a *common difference.*

Thus 1, 3, 5, 7, 9, is a progression which increases by the continual addition of the *common difference* 2;

And 15, 12, 9, 6, 3, is a progression which decreases by the continual subtraction of the common difference 3.

The *first* and *last terms* of a progression are called the two *extremes*, and all the intermediate terms the *means.*

The following are the most useful principles relating to Arithmetical Progression.

The Last Term.

§ **341.** The *last term* of an increasing arithmetical progression, is equal to the first term *plus* the product of the common difference multiplied into the number of terms *less one.*

This is evident from considering that the 2d term is formed by adding the common difference to the 1st term, the 3d by adding the common difference to the 2d, and so on,—the number of these *additions* being always one less than the *number of terms* in the progression.

When the progression *decreases,* each succeeding term will be found by *subtracting* the common difference from the preceding one. Hence,

§ **342.** In a decreasing arithmetical progression the *last term* is equal to the first term *minus* the product of the common difference multiplied into the number of terms *less one.*

Or, more generally,

§ **343.** The *greater* of the two extremes of an arithmetical progression, is equal to the *less* plus the product of the common difference multiplied into the number of terms *less one;* or the less extreme is equal to the greater *minus* the same product.

The Common Difference.

§ **344.** The *common difference* of the terms in an arithmetical progression, is equal to the difference between the two extremes divided by the number of terms *less one.*

This follows from the manner in which either one of the *two extremes* is derived from the other extreme, common difference, and number of terms, (§ 343.)

The Sum of the Two Extremes.

§ **345.** The *sum of the two extremes* in an arithmetical progression, is equal to the sum of any two terms *equidistant from them*, or to twice the middle term when the number of terms is *odd.*

Taking the first term 1, and the common difference 2, we have the progression,

1, (1+2), (1+*twice* 2), (1+3 *times* 2), (1+4 *times* 2).

Of these five terms the sum of the *first* and *last* is

1+1+4 *times* 2, equal to 2+4 *times* 2;

and the sum of the *second* and *fourth*, equidistant from the two extremes, is 1+2+1+3 *times* 2, also equal to 2+4 *times* 2.

We also see that the sum of the two extremes is equal to *twice the middle term* (1+*twice* 2).

Similar illustration will apply to any number of terms.

§ **346.** An *arithmetical mean* between two given terms, is equal to *half the sum* of those terms.

For the sum of the two given terms, considered as the two extremes of a progression, is equal to twice the mean or middle term.

The Sum of all the Terms.

§ **347.** The *sum of all the terms* of an arithmetical progression, is equal to half the sum of the two extremes multiplied into the number of terms.

To prove this we add the several terms of an arithmetical progression to those of the same progression *reversed;* thus

$$\begin{array}{ccccc} 1, & 3, & 5, & 7, & 9 \\ 9, & 7, & 5, & 3, & 1 \\ \hline 10, & 10, & 10, & 10, & 10 \end{array}$$

The sum 10+10+10 &c. of the *two series* is the sum 10 of the two extremes, 1 and 9, in either series, multiplied into the number of terms; hence the sum of either series is equal to half the sum of the two extremes multiplied into the number of terms.

The preceding principles are applicable to the following

EXERCISES.

1. The first term of an arithmetical progression is $2\frac{1}{2}$, the common difference $1\frac{3}{4}$, and the number of terms 13: what is the last term? *Ans.* $23\frac{1}{2}$.

2. The first term of a decreasing progression is 150, the common difference $3\frac{1}{2}$, and the number of terms 16: it is required to find the last term. *Ans.* $97\frac{1}{2}$.

3. If the first term of an arithmetical progression is 7, the last term $68\frac{3}{4}$, and the number of terms 21, what is the common difference? *Ans.* $3\frac{7}{80}$.

4. What is the first term of an arithmetical progression in which the common difference is 5, the number of terms 17, and the last and greatest term $137\frac{1}{2}$? (§ 343.) *Ans.* $57\frac{1}{2}$.

5. If the first term of an arithmetical progression is $5\frac{1}{2}$, the last term $187\frac{1}{2}$, and the number of terms 25, what is the 13th or middle term? *Ans.* $96\frac{1}{2}$.

6. What number inserted between the numbers 150 and 582, will form with those numbers an arithmetical progression? that is, what is the *arithmetical mean* between 150 and 582? *Ans.* 366.

7. What is the sum of all the terms in an arithmetical progression, in which the first and last terms are 31 and $487\frac{1}{2}$ respectively, and the number of terms 100? *Ans.* 25925.

8. What is the sum of all the numbers in the natural series 1, 2, 3, 4, and so on to 1000 inclusive? *Ans.* 500500.

9. What is the sum of all the terms in an arithmetical progression, in which the first term is 1, the common difference 5 and the number of terms 1728? *Ans.* 7462368.

10. Allowing 450 yards of cloth to be sold at 5 cents for the 1st yard, 10 for the 2d, 15 for the 3d, and so on, what would be the price of the last yard? *Ans.* $ 22.5.

11. If a person having a journey of 575 miles to perform in 15 days, travel the first day 30 miles,—what must be the average increase of his daily progress that 50 miles may remain for the last day? *Ans.* $1\frac{3}{7}$ miles.

12. A has $225\frac{1}{2}$ acres of land, B has $368\frac{3}{4}$ acres, and C has as much less than B as A has less than C; how many acres then has C? *Ans.* $297\frac{1}{8}$.

13. Four persons on comparing their ages find that the first is as much younger than the second as the second is younger, and the fourth older, than the third. The ages of the first and third are $21\frac{2}{3}$ and $56\frac{1}{4}$ years respectively; how old is the fourth?

Ans. $73\frac{13}{24}$ years.

14. Suppose a person to build 320 rods of fence on the conditions of receiving 3 cents for the 1st rod, 5 for the 2d, 7 for the 3d, and so on; what would be the price of the last rod? and what would the whole work amount to?

Ans. \$6.41, and \$1030.4.

15. If $20\frac{1}{2}$ and $35\frac{1}{2}$ be taken for the two extremes of an arithmetical progression consisting of six terms, what numbers will form the four *intermediate* terms?

The difference between the *two extremes* $\div$ the number of terms *less one*, is equal to the *common difference* of the terms, (§ 343.)

Hence $(35\frac{1}{2}-20\frac{1}{2})\div 5=3$ is the common difference in the present case; and the four mean terms may now be readily found.

Ans. $23\frac{1}{2}$; $86\frac{1}{2}$; $29\frac{1}{2}$; $32\frac{1}{2}$.

16. A person having a journey to accomplish in 7 days, proposes to travel on the first and last days 30 and 40 miles, respectively, and to have his daily advances throughout in arithmetical progression. What distances must he go on the intermediate days? *Ans.* $31\frac{2}{3}$, $33\frac{1}{3}$, 35, $36\frac{2}{3}$, and $38\frac{1}{3}$ miles.

17. If the sum of five terms of an arithmetical progression, is 300, and the first of them is 20; what are the remaining terms?

The sum of the five terms $\div$ 5, is equal to *half the sum* of the two extremes, (§ 347.)

Hence $300\div 5=60$ is half the sum, and, consequently, 120 is the sum, of the two extremes. Then $120-20=100$ is the *last term.*

Ans. 40; 60; 80; 100.

18. A debt of \$3000 is to be paid in 6 equal installments, the first of which is to be \$600, and the others are to decrease regularly by a common difference. What will the remaining installments be? *Ans.* \$560; 520; 480; 440; 400.

GEOMETRICAL PROGRESSION.

§ **348.** A GEOMETRICAL PROGRESSION is a series of quantities in which each succeeding term has the *same ratio* to the term which immediately precedes it.

Thus 1, 2, 4, 8, 16, is an increasing progression in which each succeeding term is *double* the one which immediately precedes it; that is, the *ratio of the progression* is 2, (§ 212).

And 27, 9, 3, 1, $\frac{1}{3}$, is a decreasing progression in which each succeeding term is *one-third* of the one which immediately precedes it; and the ratio of the progression is consequently $\frac{1}{3}$.

Hence the *successive terms* of a geometrical progression consist of the *first term multiplied* continually into the *ratio*, that is, multiplied into the successive *powers of the ratio*.

The *first* and *last terms* of the progression are called the *two extremes*, and the intermediate terms the *means*.

The following are the most useful principles relating to Geometrical Progression.

The Last Term.

§ **349.** The *last term* of a geometrical progression is equal to the first term multiplied into that *power of the ratio* which is expressed by the number of terms *less one*.

If the first term be 1, and the ratio of the progression 3, the series will be

$$1,\ 1\times3,\ 1\times3\times3,\ 1\times3\times3\times3, \text{ and so on.}$$

The *second term*, 1×3, is the first term multiplied into the *first power* of the ratio; the third term, $1\times3\times3$, is the first term multiplied into the *second power* of the ratio; and so on, to the last term, in which the power of the ratio will evidently be expressed by the number of terms *less one*.

From the preceding, it follows, that

§ **350.** The *first term* of a geometrical progression, is equal to the last term divided by that power of the ratio which is expressed by the number of terms *less one*.

Product of the two Extremes.

§ **351.** The *product of the two extremes* in a geometrical progression, is equal to the product of any two terms *equidistant from them*, or to the square of the middle term when the number of terms is *odd*.

In the progression

$$2,\ 2\times3,\ 2\times3^2,\ 2\times3^3,\ 2\times3^4,$$

the product $2\times2\times3^4$, of the first and last terms is equal to that of the *second* and *fourth* terms, which are equidistant from them; and is also equal to the *square* of the middle term 2×3^2.

This principle will be found to be true whatever be the number of terms in the progression.

§ **352.** A *geometrical mean*, or a *mean proportional*, between two given terms, is equal to the *square root of the product* of those terms.

For the product of the two given terms, considered as the *two extremes* of a geometrical progression, is equal to the *square* of the mean or middle term.

The Sum of all the Terms.

§ **353.** The *sum of all the terms* of a geometrical progression, is equal to the difference between the *first term* and the *product of the last term* multiplied into the ratio, divided by the difference between the ratio and a *unit*.

Take the progression 1, 3, 9, 27, in which the *ratio* is 3.
Multiplying each term by the *ratio*, we obtain
the series 3, 9, 27, 81.

The sum of the terms in this second series is 3 *times* the sum of the given series. If then the first series be subtracted from the second, the remainder will be equal to *twice the given series*.

In subtracting, the 3, 9, and 27 in the first series, will cancel the same terms in the second series, and the remainder will be 81—1, or

$$27\times3-1.$$

This remainder, as already shown, is *twice* the sum of the given series. Hence $(27\times3-1)\div2=$the *sum of the series*.

The dividend in this expression is the difference between the first term 1 and the product of the last term 27 multiplied into the ratio 3; and the divisor 2 is the difference between the ratio and a *unit*.

A Decreasing Progression with an Infinite Number of Terms.

§ **354.** When the ratio of a geometrical progression is a *proper fraction*, the succeeding terms will *continually diminish;* and if the number of terms be supposed *infinite*, the last term will be 0.

In the series 3, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, and so on, in which the ratio of progression is $\frac{1}{3}$, the terms continually diminish, and the series would terminate in 0 if the *number of terms were infinite.*

§ **355.** The sum of an *infinite number of terms* in a decreasing geometrical progression, is equal to the first term divided by the difference between the ratio and a *unit.*

The sum of the series

3, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, and so on, to an infinite number of terms, according to § 353, is

(3—the *last term* × the *ratio*)$\div(1-\frac{1}{3})$.

But the last term would be 0 (§ 354); hence the sum of the *infinite series* is $3\div(1-\frac{1}{3})=3\div\frac{2}{3}=4\frac{1}{2}$.

On the principle just demonstrated we may also compute the value of a *repeating decimal.*

For example, take the repetend $.\dot{4}$, or .4444 &c.

This decimal is equal to

$\frac{4}{10}+\frac{4}{100}+\frac{4}{1000}+\frac{4}{10000}$, and so on, without end.

We have then $.\dot{4}$ equal to the sum of an *infinite geometrical series* of which the first term is $\frac{4}{10}$ and the ratio $\frac{1}{10}$.

Hence $.\dot{4}=\frac{4}{10}\div(1-\frac{1}{10})=\frac{4}{10}\div\frac{9}{10}=\frac{4}{9}=\frac{2}{3}$.

This value, it will be observed, is the same that would be found by the method before given, (§ 328).

EXERCISES.

1. What is the last term in a geometrical progression of which the first term is $2\frac{1}{2}$, the ratio 3, and the number of terms 5?
Ans. $202\frac{1}{2}$.

2. What is the sixth term in a geometrical progression of which the first term is $7\frac{1}{4}$ and the ratio $\frac{1}{2}$? *Ans.* $\frac{29}{128}$.

3. What will be the first term of a geometrical progression, if the ratio be 2, the number of terms 9, and the last term 1280?
Ans. 5.

4. What is the second or mean term in a geometrical progression of which the first and third terms are $6\frac{1}{4}$ and $30\frac{1}{4}$ respectively? *Ans.* $13\frac{3}{4}$.

5. What is the last term, and the sum of all the terms, in a geometrical progression whose first term is 23, ratio 5, and number of terms 4? *Ans.* 2875, and 3588.

6. What is the sum of the first six terms in a geometrical progression whose first term is 100, and ratio $\frac{2}{3}$?
Ans. $273\frac{161}{243}$.

7. What is the sum of an infinite number of terms in a geometrical progression whose first term is 1000, and ratio $\frac{1}{2}$?
Ans. 2000.

8. What is the sum of the series 5, 1, $\frac{1}{5}$, and so on to an infinite number of terms? *Ans.* $6\frac{1}{4}$.

9. What is the sum of the infinite series 100, 50, 25, &c.?
Ans. 200.

10. What is the difference between the *arithmetical* and the *geometrical* mean between the numbers 100 and 1000?
Ans. 233.773'.

11. If 9 acres of land be sold at \$1 for the first acre, \$2 for the 2d, \$4 for the 3d, and so on, to the last, what would the last acre amount to? *Ans.* \$256.

12. A has \$400, B \$900, and C a sum which bears the same ratio to A's that B's does to C's: how many dollars then has C?
Ans. \$600.

13. If 11 yards of cloth were sold at 1 cent for the first yard, 3 for the 2d, 9 for the 3d, and so on to the last, what would be the price of the last yard? and what would the whole amount to? *Ans.* \$590.49; and \$885.73.

14. How far would a person travel in 6 days, allowing him to go 40 miles the first day, and to diminish his rate of travelling in such a manner that each succeeding day's journey shall be $\frac{3}{4}$ of the one immediately preceding. *Ans.* $131\frac{67}{128}$ miles.

15. Allowing a person to commence trading on a capital of \$1000, and to increase it by $\frac{1}{4}$ of itself each year for 10 years, what would then be the amount of his capital?
Ans. \$7450.580'.

16. If 50 acres of land were to be sold at 1 *cent* for the first acre, 2 for the second, 4 for the third, and so on, what would the 50 acres amount to? *Ans.* \$11258999068426.23.

PROGRESSIONS APPLIED TO INTEREST.

I. *Simple Interest.*

§ **356.** A Principal at *simple interest* forms the *first term* of an Arithmetical Progression—in which the *common difference* is the Interest for one year—the number of *terms* one more than the number of years for which interest is computed—and the *last term* the Amount of principal and interest.

For example, the Interest of \$100, at 6 per cent, being \$6 for 1 year, \$12 for 2 years, \$18 for three years, and so on,—

We have the Principal, and the Amounts for *one*, *two*, *three*, &c. years, in the terms of the Arithmetical Progression,

\$100, \$106, \$112, \$118, &c.

The *common difference* of these terms is \$6, the Interest for one year.

II. *Compound Interest.*

§ **357.** A Principal at *compound interest* forms the *first term* of a Geometrical Progression—in which the *ratio* is the Amount of \$1 for one year or *period* of interest—the number of *terms* one more than the number of years or periods of interest—and the *last term* the Amount of principal and interest.

The *amount* of \$100, at 6 per cent, for the 1st year, would be \$106, and this would form the *principal* for the 2d year, (§ 282).

But 106 is equal to 100×1.06; and \$1.06 is the Amount of \$1 for one year; that is, the *principal* for the 1st year $\times$ 1.06 produces the Amount for the 1st or the *principal* for the 2d year.

In like manner, $\$106\times1.06$ or $\$100\times1.06\times1.06$ is the Amount for the 2d or the principal for the 3d year; $\$100\times(1.06)^3$ is the Amount for the 3d or the *principal* for the 4th year, &c.

We have then the Principal, and the Amounts for *one*, *two*, *three*, &c. years, in the terms of the Geometrical Progression,

$\$100$, $\$100\times1.06$, $\$100\times(1.06)^2$, $\$100\times(1.06)^3$, &c.

The *ratio* of this progression is 1.06, the Amount of \$1 for one year, at the given rate per cent.

The given Principal subtracted from the Amount for the last year, will leave the *Compound Interest.*

Tabular Method of Calculating Compound Interest.

§ **358.** In finding the Amount from the Principal, or the principal from the amount, in Compound Interest, on the principles of geometrical progression,—the *ratio* must be raised to that power which is expressed by the number of years, or periods.

The TABLE on the next page contains these powers to the 50th order, for various rates of interest; and thus supersedes the necessity of involving the ratio in each particular calculation.

These powers show the Amount of $1 for each corresponding rate and number of years.

Thus the Amount of $1, at 5 per cent, for 1 year, is $1.05; for 2 years it is $\$1 \times (1.05)^2 = (1.05)^2$; for 3 years, $\$1 \times (1.05)^3 = (1.05)^3$; and so on, in the ascending powers of the ratio 1.05. (§ 357).

Hence the following methods—

§ **359.** 1. *To find the Amount at Compound Interest*,—Multiply the Principal by tabular amount of $1 for the given rate and number of years or periods; and

2. *To find the Principal*,—Divide the given Amount by the tabular amount of $1 for the given rate and number of years or periods for which interest is computed.

Thus to find what $100 would amount to in 20 years, at 6 per cent, allowing the interest to be compounded annually,

Opposite to 20 years, and under 6 per cent, in the TABLE, we find that the amount of $1 would be $3.20713'.

Then $\$3.20713 \times 100 =$ *Answer* $320.713'.

When the Number of Years or Periods exceeds 50.

§ **360.** The Amount of $1 at Compound Interest for any number of years or periods, multiplied by the Amount for any other number, produces the Amount of $1 for the *sum of those two numbers*, (§ 295.)

Thus 18.42015, the Amount of $1 for 50 years, at 6 per cent, $\times$ 4.29187', the amount for 25 years, $=$ 79.05688', the Amount of $1, at 6 per cent, for 75 years.

§ 361. *Table of the Amount of* $1.00 *at Compound Interest.*

Years.	3 p. ct.	3½ p. ct.	4 p. ct.	4½ p. ct.	5 p. ct.	6 p. ct.	7 p. ct.
1	1.03000	1.03500	1.04000	1.04500	1.05000	1 06000	1.07000
2	1.06090	1.07122	1.08160	1.09202	1.10250	1.12360	1.14490
3	1 09272	1.10871	1.12486	1.14116	1.15702	1 19101	1.22504
4	1.12550	1.14752	1.16985	1.19251	1.21550	1.26247	1.31079
5	1.15927	1.18768	1.21665	1.24618	1.27628	1.33822	1 40255
6	1.19405	1.22925	1.26531	1.30226	1.34009	1.41851	1.50073
7	1.22987	1.27227	1.31593	1.30086	1.40710	1.50363	1.60578
8	1.26677	1.31680	1.36856	1.42210	1.47745	1.59384	1.71818
9	1.30477	1.36289	1.42331	1.48609	1.55132	1.68947	1.83845
10	1.34391	1.41059	1.48024	1.55296	1.62889	1.79084	1.96715
11	1.38423	1 45997	1.53945	1.62285	1.71033	1.89829	2.10485
12	1.42576	1.51106	1.60103	1.69588	1.79585	2.01219	2.25219
13	1.46853	1.56395	1.66507	1.77219	1.88564	2.13292	2.40984
14	1.51259	1.61869	1.73167	1.85194	1.97993	2.26090	2.57853
15	1.55796	1.67534	1.80094	1.93528	2.07892	2.39655	2.75903
16	1.60470	1.73398	1.87298	2.02237	2.18287	2.54035	2.95216
17	1.65284	1.79467	1.94790	2.11337	2.29201	2.69277	3.15881
18	1.70243	1.85748	2.02581	2.20847	2.40661	2.85433	3.37993
19	1.75350	1.92250	2.10684	2.30786	2.52695	3.02559	3.61652
20	1.80611	1.98978	2.19112	2.41171	2.65329	3.20713	3.86968
21	1.86029	2.05943	2.27876	2.52024	2.78596	3 39956	4 14056
22	1.91610	2.13151	2.36991	2.63365	2.92526	3.60353	4.43040
23	1.97358	2.20611	2.46471	2.75216	3.07152	3 81975	4.74052
24	2.03279	2.28332	2.56330	2.87601	3.22510	4.04893	5.07236
25	2.09377	2.36324	2.66583	3.00543	3.38635	4.29187	5.42743
26	2.15659	2.44595	2.77247	3.14067	3.55567	4.54938	5.80735
27	2.22128	2.53156	2.88336	3.28200	3.73345	4.82234	6.21386
28	2.28792	2.62017	2.99870	3.42970	3.92012	5.11168	6.64883
29	2.35656	2.71187	3.11865	3.58403	4.11613	5.41838	7.11425
30	2.42726	2.80679	3.24339	3.74531	4.32194	5.74349	7.61225
31	2.50008	2.90503	3.37313	3.91385	4.53803	6.08810	8.14511
32	2.57508	3.00670	3.50805	4.08998	4.76494	6.45338	8.71526
33	2.65233	3.11194	3.64838	4.27403	5.00318	6.84059	9.32533
34	2.73190	3.22086	3.79431	4.46636	5 25334	7.25102	9.97811
35	2.81386	3.33359	3.94608	4 66734	5.51601	7.68608	10,67657
36	2.89827	3.45026	4.10393	4.87737	5.79181	8 14725	11.42393
37	2.98522	3.57102	4.26809	5.09686	6.08140	8.63608	12.22361
38	3.07478	3.69601	4.43881	5.32621	6.38547	9.15425	13.07927
39	3.16702	3.82537	4.61636	5.56589	6.70475	9.70350	13 99482
40	3.26203	3.95926	4.80102	5.81636	7.03998	10.28571	14 97446
41	3.35989	4.09783	4.99306	6.07810	7.39198	10.90286	16.02267
42	3.46069	4.24125	5.19278	6.35161	7.76158	11.55703	17.14426
43	3.56451	4.38970	5.40049	6 63743	8.14966	12.25045	18.34436
44	3.67145	4.54334	5.61651	6.93612	8.55715	12.98548	19.62846
45	3.78159	4.70235	5.84117	7.24824	8.98500	13 76461	21.00246
46	3.89504	4.86694	6.07482	7.57442	9.43425	14.59048	22 47263
47	4.01189	5.03728	6.31781	7.91526	9.90597	15 46591	24.04571
48	4.13225	5.21358	6.57052	8.27145	10 40126	16.39387	25.72891
49	4.25621	5.33606	6 83334	8.64367	10.92133	17.37750	27 52994
50	4.38390	5.58492	7.10668	9 03263	11.46740	18.42015	29.45703

EXERCISES.

To be performed on the principles of Arithmetical and Geometrical Progression.

1. What would $100 amount to in 100 years, allowing interest at 6 per cent? *Ans.* $700.

2. What would $500 amount to in 5 years, allowing compound interest at 6 per cent? *Ans.* $669.11'.

3. What principal would amount to $2500 in 20 years at 7 per cent. compound interest? *Ans.* $646.04'.

4. What principal would amount to $3000 in 4 years at 7 per cent compound interest? *Ans.* $2288.696'.

5. What would be the compound interest on $450 for 3 years at 6 per cent? *Ans.* $85.95'.

6. What principal would amount to $10000 in 5 years at 6 per cent compound interest? *Ans.* $7472.612.

7. Which would be the most advantageous to the lender, the simple interest of $500 for 5 years at 6 per cent, or the compound interest at 5 per cent? *Ans.* The simple interest by $11.86'.

8. A debt of $1000 will be due in 5 years, without interest. What is the *present worth* of the debt, allowing the comp. interest to be 6 per cent? (§ 275.) *Ans.* $747.26'.

9. What is the present worth of $1200, due in 4 years without interest, on the supposition that money can be loaned at 6 per cent compound interest? *Ans.* $950.517'.

10. A owes B $3250 to be paid in 5 years, without interest. What sum in hand would be an equivalent for the debt, if the present worth could be put at interest at 7 per cent, and the interest compounded annually? *Ans.* $2317.20'.

11. What would $375.5 amount to in 15 years at 7 per cent, compound interest? *Ans.* $1036.015'.

12. What principal must be put at compound interest, at 5 per cent, to amount to $20000 in 25 years? *Ans.* $5906.06'.

13. If $500 were put at interest at 6 per cent, and the interest collected annually, and put out at the same rate, and so with all the interest annually due, what would be the amount in 20 years? *Ans.* $1603.56'.

14. A is indebted to B $4750, to be paid in two equal installments in 3 and 6 years, without interest. What is the present worth of the debt, on the supposition that money will produce 6 per cent at compound interest? *Ans.* $3668.397'.

ANNUITIES.

§ **362.** An ANNUITY is properly a sum of money payable *annually;* but the term annuity is also applied to a sum which is to be paid semi-annually, quarterly, or at any regular intervals.
Pensions, salaries, rents, &c., are of the nature of annuities.

An *annuity certain* is one which is to continue for a given number of years; as the annual rent of a house for five years.

A *contingent annuity* is one whose continuance depends on some uncertainty. When limited by the duration of a given life or lives, it is called a *life annuity.*

A *perpetual annuity*, or *perpetuity*, is an annuity which is to continue forever. Of this kind may be the interest which a Government pays on borrowed money.

An *annuity in reversion* is one of which the periods of payment are to be reckoned from a specified future time, or the occurrence of a specified future event.

§ **363.** An annuity *forborne*, or in *arrears*, is one on which the periodical payments have not been made as they became due; and the *amount* of the annuity is the *sum of the payments due*, with the interest on each for the time it has remained unpaid.

Amount of Annuities—how found.

§ **364.** The Amount of an annuity *in arrears*, at simple interest, is equal to the *sum of the terms* of an arithmetical progression whose *first term* is the annuity—common difference, the interest of the annuity for *one year*, or period of payment—and number of terms the *number of years* or periods for which the annuity is due.

For example, suppose an annuity of $100 a year to be due for 4 years, and the Amount to be required at 6 per cent., simple interest.

For the last of the 4 years, that is, for the 4th year, the annuity $100 is due, without interest.

The 1st term is then the annuity $100.

For the 3d year the annuity is due, with interest for 1 year.

The 2d term is therefore $100+6=$106.

For the 2d year the annuity is due, with interest for 2 years.

The 3d term is therefore $\$100+12=\$112$.

For the 1st year the annuity is due, with interest for 3 years.

The 4th term is therefore $\$100+18=\$118$.

The sum of all these terms, that is, of the *progression*,

\$100, \$106, \$112, \$118, is the Amt. of the annuity.

§ **365.** The Amount of an annuity *in arrears*, at compound interest, is equal to the *sum of the terms* of a geometrical progression whose *first term* is the annuity—*ratio*, the amount of \$1 for *one year*, or period of payment—and number of terms, the number of years or periods for which the annuity is due.

For example, suppose that an annuity of \$100 a year has remained unpaid for 4 years, and that the amount is to be computed at 6 per cent, compound interest.

For the last of the 4 years, that is, for the 4th year, the annuity \$100 is due, without interest.

The 1st term is therefore the annuity \$100.

For the 3d year the annuity is due, with interest for 1 year.

The 2d term is then $\$100+6=\$100\times1.06$.

For the 2d year the annuity is due, with comp. int. for 2 years.

The 3d term is therefore $\$100\times(1.06)^2$.

In like manner the 4th term will be $\$100\times(1.06)^3$.

And the sum of all terms,

$\$100$, $\$100\times1.06$, $\$600\times(1.06)^2$, $\$100\times(1.06)^3$,

is the Amount of the annuity.

The series of terms thus found form a geometrical progression whose first term is the annuity,—the ratio \$1.06 is the amount of \$1 for 1 year, and the number of terms 4 is the number of years the annuity has remained unpaid.

EXERCISES.

1. A laborer's wages were \$125 a year, and remained unpaid for 5 years. What amount was then due him, allowing interest to be computed at 7 per cent? (§ 364.) *Ans.* \$712.5.

2. The rent of a house, which is \$250 a year, has been forborne for 10 years. What amount should now be paid, if interest be charged at the rate of 10 per cent? *Ans.* \$3625.

3. An annuity of \$300 per annum has been forborne for 25 years. What is the present amount of the annuity, allowing 6 per cent, compound interest?

Find the last term of the progression, (§ 365), and thence the

sum of the terms. The power of the *ratio* 1.06 which enters into the last term, will be found in the Table (§ 361).

Ans. \$16459.32'.

4. What is the amount due on a pension of \$500 a year, which has remained unpaid for 13 years, allowing interest to be compounded at the rate of 5 per cent? *Ans.* \$8856.42'.

5. What difference is there between the amount at simple, and at compound interest, at 6 per cent, of an annuity of \$375 per annum, on which the payments have been forborne for 20 years?

Ans. The amt. at comp. int. is the greater by \$2019.53'.

6. A person at the age of 22 put \$100 at interest, at 6 per cent, and \$100 each year afterwards, until he was 40 years old. He also collected the interest annually, and converted the same into *principal:* what amount was, by these means, accumulated? *Ans.* \$3090.56'.

Present Worth of Annuities.

§ 366. The Present Worth of an *annuity certain* is equal to that *principal* which, at interest till the termination of the annuity, would increase to the *amount* of the *annuity in arrears* to the same time.

For example, suppose we wish to find the Present Worth of an annuity of \$100 a year for 3 years.

The first payment on the annuity will be due in 1 year, without interest. The *present worth* of this is that principal which would amount to \$100 in 1 year. This *principal*, therefore, and the *first payment* on the annuity would, at *cmp. int.* increase to the same *amount*, by the termination of the 3 years.

In like manner the present worth of the *second payment*, and that payment itself, would increase to the same amount, by the termination of the 3 years; and so on.

The *sum of the present values* of all the payments on the annuity, is the present value of the annuity; and therefore the present worth of the annuity is equal to the present worth of the *amount of the annuity in arrears* to its termination.

If the present worth be computed on the principles of compound interest, we shall have for the *amount of the annuity* for 3 years,—allowing the rate of interest to be 6 per cent,—

\$318.36, (§ 365).

The present worth of the annuity is the *principal* which would amount to this sum in 3 years; that is,

\$318.36÷1.19101=\$266.121, (§ 359–2).

§ **367.** The Present Worth of a *perpetuity*, or *perpetual annuity*, is that sum of money whose interest is equal to the annuity.

Thus the present worth of an annuity of \$100 a year, forever, allowing interest at 5 per cent, is

$100÷.05=$2000, (§ 250).

§ **368.** The Present Worth of an *annuity in reversion*, is the present worth of what *would be the value of the annuity* at the time from which its periods of payment are to be reckoned.

For example,—to find the present worth of an annuity of \$100 a year, for 3 years,—to commence at the expiration of 4 years from the present time.

If the computation be made at 6 per cent, compound interest, the value of the annuity 4 years hence, as already found, (§ 366,) will be

\$266.121.

The present worth of the annuity is therefore the principal which would amount to this sum in 4 years; that is,

$266.121÷1.26247=$210.792,' (§ 359–2).

In like manner we compute the present worth of a *perpetuity in reversion*.

Thus, to find the present worth of a perpetuity of \$100 a year, —to commence at the expiration of 5 years from the present time.

At 5 per cent, compound interest, the value of the perpetuity 5 years hence, as already seen, (§ 367,) will be

\$2000.

The present worth of the perpetuity is then the principal which would amount to this sum in 5 years; that is,

$2000÷1.27628=$1567.051', (§ 359–2).

EXERCISES.

7. What would be the present worth of an annuity of \$200 a year, for 12 years, to be computed at 6 per cent, simple interest? *Ans.* \$1855.813'.

8. The annual rent of an estate is \$500. What would be the present worth of the rents for 7 years, to be computed at 6 per cent, compound interest? *Ans.* \$2791.17'.

9. What is the present value of a perpetual annuity of \$500 per annum, allowing the rate of interest to be 5 per cent? *Ans.* \$10000.

10. An annuity of $1000 a year is to continue for 15 years, but is not to commence till the expiration of 10 years from the present time. What sum in hand would be an equivalent for the annuity, on the principles of compound interest at 6 per cent? *Ans.* $5423.294'.

11. A perpetuity of $300 per annum is in reversion for 20 years. What is its present value, computed at 5 per cent, compound interest? *Ans.* $2261.34'.

12. A gentleman bequeathed to his son, for 10 years, an estate worth $500 a year, to his daughter the reversion of the same estate for the next 20 years, and to his widow the reversion of the estate forever afterwards. What was the value in ready money of each bequest, allowing compound interest at 6 per cent? *Ans.* $3680.015; $3202.37: $1450.918'.

PERMUTATIONS AND COMBINATIONS.

§ 369. PERMUTATIONS are the different *orders of succession* in which a given number of things may be taken—either the *whole number together*, or the whole number taken *two* and *two*, or *three* and *three*, &c.

Thus the different permutations of the letters *a*, *b*, and *c*, when *all are taken together*, are

abc, acb, bac, cab, bca, cba,

And the different permutations of the same letters when taken *two* and *two*, are *ab, ba, ac, ca, bc, cb.*

§ 370. COMBINATIONS are the different *collections* which may be formed out of a given number of things, by taking *less than the whole*, but the same number in each collection—without regard to the order of succession.

Thus the different combinations which may be formed out of the three letters *a*, *b*, and *c*, by taking two at a time, are

ab, bc, ac.

Observe that *ab* and *ba* would not be different *combinations*, but different *permutations*, of the letters *a* and *b*.

In Permutations we have regard to the *order of succession*, and may therefore have two permutations of *two things*. In Combinations we do not consider the order of succession, so that the combination of two or more things is the same, in whatever order they are taken.

Number of Permutations.

§ **371.** The number of Permutations which may be formed of a given number of things, *all different from each other*, is equal to the *given number* × the given number *minus* 1, × the given number *minus* 2, × the given number *minus* 3, and so on, until the number of *factors multiplied together* is equal to the number of things taken in each permutation.

For example, suppose we have *four* letters, a, b, c, d, to be subjected to *permutations.*

If we write one of the letters, as a, before each of the other *three letters*, we shall find 3 *permutations* of the 4 letters taken *two* and *two*, in which a stands first.

In like manner we should find 3 *permutations* of the 4 letters taken *two* and *two*, in which b stands first; and so on for each of the 4 letters. Hence we have

4×3 *permutations* of 4 letters taken *two* and *two.*

Suppose now that the 4 letters are to be taken *three* and *three* in forming the permutations. By taking b, c, and d, and proceeding after the same manner as before, we should find

3×2 *permutations* of these 3 letters taken *two* and *two;*

and by writing a before each of these permutations, we have

3×2 *permutations* of the 4 letters taken *three* and *three*, in which a stands first.

In like manner we should find 3×2 *permutations* of the 4 letters taken *three* and *three*, in which b stands first; and so on for each of the four letters. Hence we have

$4 \times 3 \times 2$ *permutations* of 4 letters taken *three* and *three.*

The results thus obtained are in accordance with the principle which has been stated, and similar demonstration may be employed in all like cases.

Number of Combinations.

§ **372.** The number of Combinations which can be formed out of a given number of things, is equal to the number of *permutations* that could be formed in the like case, divided by $1 \times 2 \times 3$ and so on, until the number of *factors composing the divisor* is equal to the number of things in each combination

For example, suppose we wish to find the number of *combinations* that can be formed out of 4 *letters*, by taking 3 *letters* in each combination.

The number of *permutations* that could be formed of 4 *letters* taken *three* and *three*, is

$$4\times3\times2=24, \text{ (§ 371)}.$$

On the same principle there are $3\times2\times1$ *permutations* for each *combination* of 3 *letters* taken *all together*. Hence the number of combinations that can be formed in this case, is

24, the number of permutations, $\div1\times2\times3, =24\div6=4$.

EXERCISES

In Permutations and Combinations.

1. How long would it require for 5 persons to seat themselves in a different order each day at dinner?

Here the *whole number is to be taken in each permutation*: hence we have $5\times4\times3\times2\times1$, (§ 371).

Ans. 120 days.

2. In how many different ways may a class containing 10 pupils be arranged? *Ans.* 3628800.

3. How many different numbers may be expressed by the 10 digits, allowing five figures to each number? *Ans.* 30240.

4. In how many different ways may the names of the 12 months be placed one after another? *Ans.* 479001600.

5. How many different arrangements of 5 men could be formed out of a company consisting of 15 men?

Ans. 360360.

6. How many different *collections* of 3 persons in each could be formed out of a company of 13 persons?

$$13\times12\times11\div1\times2\times3, \text{ (§ 372)}.$$

Ans. 286.

7. How many different combinations of 4 letters may be made from the first 12 letters of the alphabet? *Ans.* 495.

8. How many different compositions of 2 kinds of metal could be formed of 5 different metals? *Ans.* 10.

8. In how many different ways might a company of 20 men be arranged in a procession? *Ans.* 2432902008176640000.

10. A farmer wishes to select a team of 6 horses out of a drove containing 17 horses. How many different choices for the team will he be able to make? *Ans.* 12376.

Permutations of Things among which there are Several of the Same Kind.

§ **373.** When of a given number of things there are *two or more of the same kind*, the number of Permutations that may be formed of *all together*, will be found by

Dividing the number of permutations that could be formed if the things were *all different*, (§ 371), by the number that could be formed of *as many of them as are alike.*

When there are *two or more sets of like things*, the divisor to be used will be the *product* of the numbers of *permutations that could be formed of each set*, supposing the things to be different.

For example, to find the number of Permutations that may be formed of the letters *aaabc*, taken all together.

If the letters were all different, the number of permutations that could be formed would be

$$5\times4\times3\times2\times1=120.$$

The *three* letters corresponding to the three *a's*, would admit of $3\times2\times1=6$ permutations, and each of these *six* would combine with the different positions of *b* and *c*,—whereas *aaa* admits of but *one order* of succession to be combined with *b* and *c*.

Hence the number of permutations will be

$$\frac{5\times4\times3\times2\times1}{3\times2\times1}=\frac{120}{6}=20.$$

To find the number of Permutations that may be formed of the letters *aababcd*, taken all together.

If the letters were *all different*, the number of permutations that might be formed, would be $7\times6\times5\times4\times3\times2\times1=5040$.

The number of permutations of the three letters corresponding to the three *a's*, would be $3\times2\times1$, and of those corresponding to the two *b's* would be 2×1. Hence the number of Permutations required is $5040\div(3\times2\times1\times2\times1)=420$.

11. In how many different orders of succession may the letters *aabcd* be placed? *Ans.* 60.

12. How many variations may occur in the succession of the figures in the number 32233? *Ans.* 10.

13. In how many different orders of succession may the letters in the word *Virginia* be arranged? *Ans.* 6720.

14. How many variations may occur in the succession of the letters in the word *Constitution*? *Ans.* 9979200.

Combinations formed out of two or more Sets of Different Things.

§ **374.** The number of Combinations that may be formed by taking *one* from each of two or more *sets of different things*, will be found by multiplying together the numbers of things in the different sets respectively.

For example, to find how many different collections of 3 men could be chosen out of 3 companies containing 4, 5, and 6 men, respectively, by taking one from each.

Each of the 4 men in the first company could be combined, separately, with every one of the 5 men in the second company. This would give $4\times5=20$ different combinations of 2 men in the first two companies.

Again; each of these 20 combinations of 2 men, could be combined, separately, with every one of the 6 men in the third company; we thus find

$$4\times5\times6=120 \text{ combinations of 3 men,}$$

by taking one man from each of the three companies in each combination.

15. How many different combinations of 4 persons may be formed, by taking one out of each of 4 companies containing 5, 7, 8, and 9 men, respectively? *Ans.* 2520.

16. How many variations might occur in forming a class of 5 pupils, by taking one from each of 5 other classes consisting of 6, 9, 13, 10, and 12 pupils, respectively? *Ans.* 84240.

17. How many different choices of 6 horses together could be made out of 6 droves containing 5, 8, 10, 13, 15, and 20 horses, respectively, if one be selected from each drove? *Ans.* 1560000.

18. Five companies are composed of 20 soldiers in each. How many different parties of 5 men could be selected out of them, by taking one man from each company? *Ans.* 3200000.

19. How many different combinations of 6 letters might be obtained by taking one from each of the sets, *abc...defg...hijkl... mno...pqrst...uvwxyz*? *Ans.* 5400.

20. A *die* is a small cube, having its six faces marked with the numbers from 1 to 6, respectively. How many changes or different combinations of these numbers might be presented on their superior faces, in throwing 5 dice? *Ans.* 7776.

REMARKS.

The GENERAL EXERCISES which follow, will be found suitable for an examination on the whole subject of Arithmetic, as it has been treated in the present work.

These Exercises are arranged in progressive order, and divided into sections embracing kindred topics; so the whole science, as it has been exhibited, may be reviewed in continuation, or any desirable portion of it may be readily selected for the like purpose.

The student should be required not merely to perform the operations requisite for finding the *Answers;* but to give *definitions*, state *principles*, assign *reasons*,—in fine, to *discuss* the whole matter of the Exercises prescribed to him.

GENERAL EXERCISES

IN

ARITHMETIC.

I.

FUNDAMENTAL OPERATIONS.—COMMON MEASURE.—COMMON MULTIPLE.—VULGAR AND DECIMAL FRACTIONS.—FEDERAL MONEY.

1. What is the Sum of 3 millions 94 thousand and sixty, 105 thousand and ten, 45 millions 2 hundred and five, and 19 thousand and thirteen? *Ans.* 48218288.

2. What is the Sum of 4 billions 24 millions 30 thousand, 350 millions and forty, 135 thousand four hundred, and 500 millions 35 thousand? *Ans.* 4874200440.

3. What is the Sum of 460, 18 thousand and four, 13 millions five hundred and nine, and 75 billions 7 millions 304 thousand 2 hundred and one? *Ans.* 75020323174.

4. What is the Difference between 9 trillions 31 millions 360 thousand 5 hundred and three, and 10 billions 5 millions 273 thousand 804? *Ans.* 8990026086699.

5. What is the Difference between 360 billions 204 millions 34 thousand 3 hundred, and 2 trillions 375 millions 183 thousand 7 hundred and 16? *Ans.* 1640171149416.

6. What is the Difference between 190 trillions 3 billions 236 millions 2 thousand and seventy-five, and 16 billions 104 thousand 5 hundred and one? *Ans.* 189987235897574.

7. What is the Product of 350 thousand and nineteen, multiplied by the sum of 5 thousand 4 hundred, and 100 thousand 2 hundred and ten? *Ans.* 36965506590.

8. What is the Product of 70 thousand 5 hundred, multiplied by the sum of 30 thousand 1 hundred, and 4 thousand five hundred and sixty-three? *Ans.* 2443741500.

9. What is the Product of 4 millions 3 thousand 7 hundred, multiplied by the sum of 3 hundred, and 21 thousand nine hundred and seventy-seven? *Ans.* 89190424900.

10. What is the Quotient of 95 trillions 200 millions, divided by the difference between 275 millions 100 thousand, and 75 millions 100 thousand? *Ans.* 475001.

11. What is the Quotient of 495 millions 66 thousand 570, divided by the difference 150 thousand 100, and 55 thousand four hundred and 41? *Ans.* 5230.

12. What is the Quotient of 10 trillions 300 millions, divided by the difference between 120 millions 240 thousand, and 20 millions 240 thousand? *Ans.* 100003.

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13. A sold 125 cords of wood at $3 per cord, and laid out the *proceeds* for flour at $6 per barrel: how many barrels of flour did he purchase?

The pupil should be required to give oral, *elementary* solutions of these Exercises. In this one, he would state the *operations* to be performed by saying, Multiply 125 by 3, and divide the product by 6; but, for an *elementary solution*, he should say,

The wood amounted to 125 *times* $3, and the *number of barrels* bought is the number of times that $6 *is contained in that amount.*

Ans. 62½ barrels.

14. How many tons of coal, at $8 per ton, could be bought with the proceeds of 340 barrels of corn, at $2 per barrel?

Ans. 85 tons.

15. A merchant invested the proceeds of 524 *cwt.* of hemp, at $5 per *cwt.*, in sugar at $7 per 100*lb.*: how many pounds of sugar did he purchase? *Ans.* $37428\frac{4}{7}$ *lbs.*

16. What is the entire quantity of hay that could be bought for $3, at 5 dollars per ton; for $4, at 7 dollars per ton; and for $5, at $9 per ton? *Ans.* $1\frac{229}{315}$ tons.

17. A bought of B 50 head of cattle, for $1750, and sold the same to C at $37 a head. What did he gain or lose by the speculation? *Ans.* Gained $100.

18. What would be the entire amount of land that could be obtained for 13 dollars, at $20 per acre; for 17 dollars, at $20 per acre; and for 17 dollars, at $40 per acre?

Ans. $1\frac{37}{40}$ acres.

19. If 34 *cwt.* of iron be hauled 43 miles for a certain sum of money, how far ought 53 *cwt.* to be hauled for the same sum?

Ans. $27\frac{31}{53}$ miles.

20. Allowing 149 men to accomplish a certain amount of work, in 41 days, in how many days ought 200 men to perform 3 times as much work? *Ans.* $91\frac{127}{200}$ days.

21. A and B together have $3768¼, and A has $923¾ more than B: what sum has each? *Ans.* A $2346; B $1422¼.

22. A farmer wishes to purchase land at $20, $35, and $40 per acre, and the same quantity at each rate: how many acres at each rate can he buy for $15850? *Ans.* $166\frac{16}{19}$ acres.

23. C and D together have 1780 head of cattle, and D has 524 less than C: how many has each?

Ans. C 1152; D 628.

24. A person performed a journey of 1520 miles by traveling half the time at the rate of 35 miles per day, and the other half at the rate of 45 miles per day. How many days was he in completing the journey? *Ans.* 38 days.

25. An agriculturist has 2145 bushels of wheat, with which he wishes to fill a number of sacks of equal capacity. What are the several smallest capacities, either of which would answer his purpose? *Ans.* 1 *bu.*; 3 *bu.*; 5 *bu.*; 11 *bu.*; or 13 *bu.*

26. A and B purchased horses at the same price per head. A's horses amounted to $623, and B's to $1068: what was the number purchased by each? *Ans.* A 7, and B 12.

27. What are the three smallest numbers, greater than a *unit*, each of which will divide 10465, without a remainder?

Ans. 5, 7, and 13.

28. How many acres of land would admit of being divided into a number of farms containing 150 acres, or 200, or 250 acres, each? *Ans.* 3000 acres, 6000 acres, &c.

29. Three hundred and eighty-five Englishmen, 455 Frenchmen, and 700 Germans are to be ferried in a small boat over a river. Into what equal companies may they be divided, so that only those of the same nation shall cross together?

Ans. Into companies of 5, 7, or 35.

30. A, B, C, and D start from different points, and travel around an island which is 600 miles in circumference. A goes 20 miles per day, B 30, C 25, and D 40: in how many days would they all arrive, at once, at the same points from which they started? *Ans.* 120 days.

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31. What should be paid for $\frac{7}{8}$ of a yard of cloth, at the rate of $\$6\frac{1}{2}$ per yard?

$\frac{7}{8}$ of a yard will cost $\frac{7}{8}$ of the price of *one yard;* that is, $\$6\frac{1}{2}$ *multiplied* by $\frac{7}{8}$, (§ 116). *Ans.* $\$5\frac{11}{16}$.

32. A farmer sold $\frac{3}{4}$ of a tract of land containing 375 acres, at $\$33\frac{1}{3}$ per acre. What did the land sell for? *Ans.* \$9375.

33. A merchant bought 3 pieces of cotton, each containing $31\frac{3}{4}$ *yd.*, at $\$\frac{1}{8}$ per yard. What did the cotton amount to? $\$$*Ans.* $11\frac{29}{32}$.

34. A person bought $\frac{4}{5}$ of a piece of ground for $\$73\frac{1}{2}$, and sold $\frac{2}{3}$ of his purchase for what it cost him. What part of the whole piece did he sell? and for what sum? *Ans.* $\frac{8}{15}$; and \$49.

35. What is the price of a ton of hay when $\frac{4}{5}$ of a ton cost $7\frac{1}{2}$ dollars? and what should be paid for 13 tons at the same rate?

The *price* of a ton $\times\frac{4}{5}$ produces $\$7\frac{1}{2}$; the *price* of a ton is therefore equal to $\$7\frac{1}{2}\div\frac{4}{5}$, (§ 122).

Ans. $\$9\frac{3}{8}$; and $\$121\frac{7}{8}$.

36. Allowing a person to walk $3\frac{1}{4}$ miles in $\frac{5}{6}$ of an hour, at what rate does he walk per hour? and what distance would he go in $9\frac{1}{2}$ hours? *Ans.* $3\frac{9}{10}$ miles; and $37\frac{1}{20}$ *m.*

37. A merchant laid out $\$62\frac{1}{2}$ for calico, at $\$\frac{1}{4}$ per yard, and sold the same at a profit of $\$\frac{1}{8}$ per yard. What was the entire profit made? *Ans.* $\$31\frac{1}{4}$.

38. An upholsterer bought a quantity of carpeting for \$150, at $\$1\frac{1}{4}$ per yard, and sold $\frac{3}{4}$ of it at a profit of $\frac{1}{3}$ of a dollar per yard. What amount of profit did he make on the quantity sold? *Ans.* \$30.

39. If a railroad car runs 35 miles in $3\frac{1}{2}$ hours, and 41 miles in 4 hours, and $62\frac{1}{2}$ miles in 5 hours, what is its average rate per hour? *Ans.* $11\frac{2}{25}$ miles.

40. If 3 men can mow $2\frac{1}{4}$ acres of grass in one day, in what time ought 17 men to mow $25\frac{3}{4}$ acres? *Ans.* $2\frac{1}{51}$ days.

41. A bought of B $783\frac{1}{2}$ bushels of wheat for $\$587\frac{5}{8}$, and sold 500 bushels of it to C for $\$562\frac{1}{2}$. How much did he gain or lose per bushel on what he sold? *Ans.* Gained $\$\frac{3}{8}$.

42. Bought 175 cords of wood for $\$437\frac{1}{2}$, and sold 93 cords of it at a profit of $\$\frac{1}{2}$ per cord. At what rate must the remainder be sold to make $\$97\frac{3}{4}$ on the whole? *Ans.* $\$3\frac{1}{8}$ per cord.

43. A person who has a journey of 570 miles to perform, proceeds for 9 days at the rate of $33\frac{1}{3}$ miles per day. How much must his daily rate be increased or diminished, to complete the journey in 9 more days? *Ans.* Diminished $3\frac{1}{3}$ *m.*

44. A bought of B 34 yards, and of C 46 yards, of cloth, at $\$5\frac{1}{2}$ per yard. Having sold $\frac{1}{4}$ of these purchases to D, at a profit of $\$1\frac{1}{4}$ per yard, at what rate must the remainder be sold, that his profit may be $150 on the whole? *Ans.* $\$7\frac{7}{12}$ per yard.

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45. What is the Sum of 230 and 3 *tenths*, 29 and 13 *hundredths*, 173 and 5 *hundredths*, 75 thousandths, and 1350 and 3 ten-thousandths? *Ans.* 1782.5553.

46. What is the Sum of 625 *thousandths*, 162 and 5 *hundredths*, 346 and 9 tenths, 375 thousandths, and 8374 and 15 ten-thousandths? *Ans.* 8883.9515.

47. What is the Sum of $18\frac{3}{4}$ *hundredths*, 470 and 7 tenths, $95\frac{1}{2}$ thousandths, 436 and $3\frac{1}{4}$ tenths, and 4730 and 73 hundred-thousandths? (§ 144.) *Ans.* 5637.30873.

48. What is the Product of 375 and 125 *thousandths*, multiplied by the difference between 75 hundredths and 31 thousandths? *Ans.* 269.714875.

49. What is the Product of 43 and 74 *ten-thousandths*; multiplied by the difference between 5 hundred-thousandths and $3\frac{1}{2}$ hundredths? *Ans.* 1.503108630.

50. What is the Product of 25 *millionths* multiplied by the sum of 300 and 5 tenths, 17 thousandths, and 10 and $6\frac{1}{4}$ tenths? *Ans.* .00777855.

51. What is the Quotient of 384 and 19 *hundredths*, divided by the sum of 9 thousandths, 6 hundredths, 2 tenths, and 3 and 5 tenths? *Ans.* 101.934'.

52. What is the Quotient of 7340 and 16 *hundred-thousandths*, divided by the difference between 6357 and 8 tenths, and 736 and $3\frac{1}{4}$ hundredths? *Ans.* 1.305'.

53. What is the Quotient of 803 and 154 *hundred-thousandths*, divided by the difference between 25 hundred-thousandths, and 24564 and 625 ten-thousandths? *Ans.* .0326'.

54. Find the sum that should be paid for $3\frac{1}{2}$ cords of wood at \$2.18$\frac{3}{4}$ per cord, 10 bushels of apples at $87\frac{1}{2}$ cents a bushel, and 125 pounds of beef at $6\frac{1}{4}$ cents a pound.

In these Exercises, reduce Vulgar Fractions to *Decimals*.

Ans. \$24.21875.

55. A gives B $117\frac{1}{2}$ yards of silk at $93\frac{3}{4}$ cents per yard, for 20 yards of cloth at \$4.37$\frac{1}{2}$ per yard, and enough of calico at $12\frac{1}{2}$ cents per yard to pay the balance. How much calico must A receive? *Ans.* 181.25 *yd.*

56. How much rice at $4\frac{1}{2}$ cents a pound, would be an equivalent for 100 pounds of coffee at $12\frac{1}{2}$ cents a pound, 300 pounds of sugar at 7 cents a pound, and $25\frac{1}{4}$ pounds of tea at \$1 a pound? *Ans.* 1305.555' *lb.*

57. A miller sold 75 barrels of flour at \$4.87$\frac{1}{2}$ a barrel, and with the proceeds intends to purchase, in equal quantities, wheat at 75 cents a bushel, rye at $31\frac{1}{4}$ cents a bushel, and corn at $37\frac{1}{2}$ cents a bushel. What quantity of each can he purchase?

Ans. 254.347' bushels.

58. A gives B 37 *cwt.* of hemp at \$4.33$\frac{1}{3}$ per *cwt.*, for 1000 pounds of bacon at $6\frac{1}{4}$ per pound, \$20 in cash, and as much sugar at $7\frac{1}{4}$ cents per pound as will pay the balance. What quantity of sugar must A receive? *Ans.* 1073.393' *lb.*

59. A merchant bought 100 yards of cloth at \$3.93$\frac{3}{4}$ per yard, and $82\frac{1}{2}$ yards at \$4.12$\frac{1}{2}$ per yard. At what average price per yard should he sell the whole, to realize a profit which shall be equal to $\frac{1}{4}$ of the cost? \$*Ans.* 5.027'.

60. A speculator bought 210 barrels of flour at \$4 a barrel, $965\frac{1}{2}$ bushels of oats at $33\frac{1}{3}$ cents a bushel, and 50 barrels of pork at \$5.06$\frac{1}{4}$ a barrel. He sold the flour and pork at an advance of \$2.5 a barrel, and the oats at a loss of 3 cents a bushel: what was the result of the speculation?

Ans. He gained \$621.03'.

II.

OPERATIONS ON MONOMIALS AND POLYNOMIALS.—CURRENCIES.—DUODECIMALS.—ALIQUOT PARTS.

61. A jeweler sold 3 *lb.* 9 *oz.* 15 *dwt.* 18 *gr.* of silver plate, at \$2.25 per *oz.* What did the whole amount to?

The pupil should be required to explain the different methods of solving questions of this kind. (§ 178, § 209.)

Ans. \$103.02'.

62. A farmer sold 3 T. 16 *cwt.* 3 *qr.* 21 *lb.* of hemp, at \$5 per *cwt.*, and invested the proceeds in land at \$37$\frac{1}{2}$ per acre. How much land did he purchase? *Ans.* 10 A. 1 R. 1.28' P.

63. A grocer exchanged 29 *gal.* 3 *qt.* 1 *pt.* of brandy, at 43$\frac{3}{4}$ cents per gallon, for rye at 31$\frac{1}{4}$ cents per bushel. What quantity of rye did he thus obtain? *Ans.* 41 *bu.* 3 *pk.* 2 *qt.* .8 *pt.*

64. A brewer bartered ale at 37$\frac{1}{2}$ cents per gallon, for 25 *bu.* 2 *pk.* 1 *qt.* of barley at 40 cents per bushel. What quantity of ale was thus disposed of? *Ans.* 27 *gal.* 1.864' *pt.*

65. A laborer dug 130 *rd.* 4 *yd.* 2$\frac{1}{2}$ *ft.* of ditching, at \$2.5 per rod, for which he took \$100 in cash, and wheat at 87$\frac{1}{2}$ cents a bushel. How much wheat did he get?

Ans. 259 *bu.* 2 *pk.* 4 *qt.* 1.66' *pt.*

66. Find the sum that should be paid for 3 T. 2$\frac{1}{2}$ *cwt.* and 3$\frac{1}{4}$ *qr.* of hemp, at \$4$\frac{1}{2}$ per *cwt.* *Ans.* \$284.904'.

67. A townsman who had a lot of ground containing 5$\frac{1}{4}$ A., sold to each of two persons 3$\frac{1}{2}$ *R.*, at the rate of \$100 per acre. What is the remainder of it worth at the same rate?

Ans. \$350.

68. A merchant sold some remnants of cloth, containing 3$\frac{1}{2}$ *yd.*, 2$\frac{1}{8}$ *yd.*, 3$\frac{1}{4}$ *qr.*, and 1$\frac{1}{2}$ *qr.*, at the rate of \$3 per yard. What did the whole amount to? *Ans.* 20.436'.

69. A person who had a journey of 735 miles 5 *fur.* to perform, went 13 days at the rate of 40 *m.* 3 *fur.* 20 *p.*, and 2 days at the rate of 39$\frac{1}{2}$ miles per day. What distance then remained to be traveled? *Ans.* 130 *m.* 7 *fur.* 20 *p*

70. A bought a tract of land containing 570 *A.* 3 *R.*; of which he sold to one person 90 A. 1 *R.* 20 P., and three times that quantity to another, at \$27$\frac{1}{2}$ per acre. What is $\frac{3}{8}$ of the remainder of the tract worth, at the same rate? *Ans.* \$3813.

71. At the rate of 1 *bu.* 3 *pk.* per acre, how much ground could be sowed with 35 *bu.* 2 *pk.* 5 *qt.* of wheat?

Ans. 20 A. 1 *R.* 20 P.

72. If 45 *yd.* 2 *qr.* of carpeting be required to cover the floor of a room, what part of the floor may be covered with 13 *yd.* 3$\frac{1}{2}$ *qr.* of the same? *Ans.* .304'.

73. A bought of B 42 T. 13 *cwt.* 2 *qr.* of iron, of which he sold 18 T. 14 *cwt.* to C, and the remainder to D. What part of the whole quantity did he sell to each?

Ans. .438' to C; .561' to D.

74. What should be paid for plastering one side of a wall which is 30 *ft.* 10 *in.* long, and 8 *ft.* 9 *in.* high, at 18¾ cents per square yard? *Ans.* \$5.620'.

75. How many square yards of paper would be required to cover the walls of a room which is 10 *ft.* 6 *in.* high, and 56 *ft.* in compass,—deducting $\frac{1}{12}$ for doors, windows, &c.?
Ans. 59 *s. yd.* 8 *s. ft.*

76. The invoice of a lot of broadcloth imported from Liverpool, amounted to 125£ 16*s.* 10*d.* sterling. Required the amount in Federal Money, according to the legal value of the pound sterling in the United States. *Ans.* \$609.070'.

77. What would be the cost of excavating a cellar 36 *ft.* long, 25 *ft.* 8 *in.* wide, and 8 *ft.* 10 *in.* deep, at \$1.06¼ per cubic yard?
Ans. \$321.189'.

78. A Southern merchant purchased in New-York, 95 *yd.* 3 *qr.* of calico at 1*s.* 6*d.* per yard, 39½ *yd.* of cloth at 20*s.* per yard, and 45 *yd.* 2½ *qr.* of silk at 8*s.* 6*d.* per yard. What was the amount of his bill in Federal Money? *Ans.* \$165.179'.

79. A reservoir for water is 10 *ft.* 3 *in.* in length, 8 *ft.* in breadth, and 4 *ft.* 11 *in.* in depth. How many barrels of water will it contain? *Ans.* 68 *bar.* 22 *gal.* 1 *qt.* 1.712' *pt.*

80. A farmer wishes to construct a crib which shall contain 1000 bushels. What must be the length of the crib, allowing its height to be 7 *ft.*, and its breadth 9 *ft.* 6 *in.*?
Ans. 18.713' *ft.*

III.

RATIO.—PROPORTION.—ANALYSIS.

81. Find the *fourth proportional* to 3, 5, and 7 *cwt.* 3 *qr.*, that is, the quantity to which 7 *cwt.* 3 *qr.* has the same ratio that 3 has to 5. *Ans.* 12 *cwt.* 3 *qr.* 18.592' *lb.*

82. Find the *inverse* fourth proportional to 5, 13, and 2 *T.* 10 *cwt.* 20 *lb.*, that is, the quantity to which 2 *T.* 10 *cwt.* 20 *lb.* has the *inverse ratio* 5 to 13. *Ans.* 19 *cwt.* 1 *qr.* 3.36' *lb.*

83. Find the *direct* and also the *inverse* fourth proportional to 15 *bu.* 3 *pk.*, 25 *bu.* 2 *pk.* 5 *qt.*, and 3£ 12*s.* 8*d.*
Ans. 5£ 18*s.* 4.08'*d.*; and 2£ 4*s.* 7.2'*d.*

84. If 12 could accomplish a certain work in 9½ days, in what time ought 17 men to do the same work?

In these Exercises the pupil should be required to state, in general terms, the *kind of proportion* involved in the question.

Thus, The time required *is inversely as the number of men*, (§ 220).

Ans. $6\frac{12}{17}$ days.

85. If .75 *T.* of iron cost $\$37\frac{1}{2}$, what should be paid for 18 *cwt.* 3 *qr.* 10 *lb.* of iron at the same rate?

The pupil should also be required to give *verbal, analytical solutions* of these questions. The Analysis of the present one might be as follows:

$\$37\frac{1}{2}$ divided by .75 is the value of 1 *ton;* and the value of *one* ton multiplied by 18 *cwt.* 3 *qr.* 10 *lb.*, reduced to the denomination of *ton*, is the value of 18 *cwt.* 3 *qr.* 10 *lb.*

Ans. $47.05'.

86. If $5\frac{1}{2}$ acres of ground produce 170 *bu.* 6 *qt.* of wheat, how many bushels would 37 *A.* 2 *R.* 16 P. 20 *s. yd.* produce at the same rate? *Ans.* 1163 *bu.* 2 *pk.* 2.656' *qt.*

87. Allowing a mechanic to earn $\$62.87\frac{1}{2}$ in a month, by working $9\frac{1}{4}$ hours per day, what ought he to earn in a month by working $11\frac{1}{2}$ *hr.* per day? *Ans.* $78.168'.

88. If $20 will supply a family with flour at $\$5\frac{1}{2}$ a barrel, for $3\frac{1}{3}$ months, how long would the same sum supply them with flour at $\$4\frac{3}{4}$ a barrel? *Ans.* $3\frac{49}{57}$ months.

89. What should be paid for 17 *yd.* 2 *qr.* of cloth, if 4 *yd.* $3\frac{1}{2}$ *qr.* of the same sells for $\$27.87\frac{1}{2}$? *Ans.* $100.064'.

90. A company of emigrants has a supply of bread for 25 days, at an allowance of $1\frac{1}{4}$ *lb.* to each per day. How long would the supply last them at an allowance of 12 *oz.* per day? *Ans.* $41\frac{2}{3}$ days.

91. A lot of ground which is 40 *p.* 2 *yd.* long, and 27 *p.* wide, is equivalent in area to another which is $53\frac{1}{2}$ *p.* long. What is the breadth of the latter? *Ans.* 20.37' *p.*

92. Three pipes of equal size will fill a cistern with water in 13 *hr.* 40 *m.* In what time would 5 such pipes fill a cistern whose capacity is $2\frac{1}{2}$ times that of the first one?

Ans. $20\frac{1}{2}$ hours.

93. If A could build a wall in 3 days, B in 5 days, and C in 6 days, in what time could the three together build the wall?

In this question, as well as those which immediately follow, the *proportion* involved is not very apparent, so that the *analytical* will probably be found the most obvious method of solution.

Ans. $1\frac{3}{7}$ days.

94. A young man squandered $\frac{1}{3}$ of his fortune in one month: $\frac{3}{5}$ of the remainder went in the next two months, when he had but \$3000 left. What had he at first? *Ans.* \$11250.

95. A and B together can mow a meadow in 5 days, and B could do it himself in 8 days. In what time could A mow the meadow? *Ans.* $13\frac{1}{3}$ days.

96. A cistern receives water through 2 pipes, one of which would fill it in 8 hours and the other in 5 hours; but by leakage the cistern loses at the rate of $\frac{1}{10}$ of its whole capacity per hour. In what time will the two pipes running together fill the cistern? *Ans.* $4\frac{4}{9}$ hours.

97. A and B start at the same time from two places which are 75 miles apart, and travel toward each other. A goes $4\frac{1}{2}$ miles per hour, and B $5\frac{1}{3}$ miles per hour: in what time will they meet each other? *Ans.* $7\frac{37}{59}$ hours.

98. The sum of \$1000 is to be divided among four persons in the proportion of 1, $1\frac{1}{2}$, 2, and $2\frac{1}{2}$: what are the several shares?
Ans. \$142.857, \$214.285', \$285.714', \$357.142'.

99. A bankrupt is indebted to A \$300, to B \$500, and to C \$600. He is able to pay \$800: what is the share of this sum that each creditor ought to receive?
Ans. A \$171.428', B \$285.714', C \$342.857'.

100. Three men contracted to grade a turnpike road for \$5000. In accomplishing the work one of the men furnished 30 laborers for 45 days, another 42 laborers for 34 days, and the other 50 laborers for 30 days. How should the \$5000 be divided among the contractors?
Ans. The 1st should receive \$1577.84', the 2d \$1669', the 3d \$1753.15'.

101. The sum of \$2000 is to be divided between A, B, and C, in such a manner that A's share shall be to B's as 2 to 3, and B's to C's as 4 to 5. What are the shares?
Ans. A's $\$457\frac{1}{7}$; B's $\$685\frac{5}{7}$; C's $\$857\frac{1}{7}$.

102. Four persons in partnership gain \$1800. One *third* of the capital employed belonged to the first, $\frac{1}{4}$ of it to the second, and the remainder equally to the third and fourth: what share of the \$1800 should be assigned to each?
Ans. To the 1st \$600; 2d \$450; 3d and 4th each \$375.

103. A gentleman wishes to divide \$1500 among his three sons, so that the first shall have $1\frac{1}{2}$ times as much as the second, and the second $2\frac{1}{4}$ times as much as the third. What sum must each receive? *Ans.* The 1st $\$764\frac{8}{53}$; 2d $\$509\frac{23}{53}$; 3d $\$226\frac{22}{53}$.

104. A, B, and C formed a partnership,—A advancing $5000 of the joint capital, B $4000, and C $3500. At the end of 6 months A withdrew $1500 from the business, B withdrew $500, and C increased his stock by $\frac{1}{2}$ of its original amount. At the end of 12 months a dissolution occurred, when their profits amounted to 3765.12\frac{1}{2}$: how should this sum have been divided between them?

Ans. A should have had $1293.07'; B $1140.94'; and C $1331.10'.

105. A grocer wishes to mix three different kinds of liquor which rate at 20 cents, 22 cents, and 31 cents, a gallon, in such proportions that the mixture shall be worth 25 *cts.* a gallon. In what proportions must the three kinds be taken?

Ans. 6 *gal.* at 20 *cts.*, 8 at 31, and 6 at 22.

106. A gentleman bequeathed the sum of $5000, to be divided between his widow, son, and daughter, in the proportion of $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. The widow dying soon after, the whole sum was divided in due proportion between the two children: how much did each receive?

Ans. The son 2777\frac{7}{9}$; the daughter 2222\frac{2}{9}$.

107. How many ounces of gold which is 15 carats fine, must be mixed with 3 *oz.* 18 carats fine, and 5 *oz.* 23 carats fine, that the compound may be 20 carats fine? *Ans.* 1$\frac{4}{5}$ *oz.*

108. If 50 *bu.* of wheat be exchanged for 80$\frac{1}{2}$ *bu.* of rye, and 3 *bu.* of rye for 4$\frac{1}{2}$ *bu.* of corn, and 10 *bu.* of corn for 12 *bu.* 3 *pk.* of oats, and 3$\frac{1}{3}$ *bu.* of oats be worth $1,—what is the value of 100 *bu.* of wheat? *Ans.* $92.374'.

109. What quantities of brandy at 25 *cts.*, 30 *cts.*, and 33$\frac{1}{3}$ *cts.*, per gallon, and of water, should be taken to form a mixture of 36 gallons which shall cost 20 *cts.* per gallon?

Ans. 8$\frac{8}{53}$ *gal.* of each kind of brandy, and 11$\frac{27}{53}$ of water.

110. If 12 oxen eat 3$\frac{1}{3}$ acres of grass in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen would eat 24 acres in 18 weeks,—the grass being at first equal on every acre, and *growing uniformly*?

This question may be readily solved after having determined the *ratio of the weekly growth* of grass to the quantity at first on the ground.

If 12 oxen eat 3$\frac{1}{3}$ acres of grass *and its growth*, in 4 weeks, it would require $10 \div 3\frac{1}{3} = 3$ *times* 12 oxen, that is, 36 oxen to eat 10 acres of grass *and its growth*, in 4 weeks.

Hence $\frac{4}{9}$ of 36, that is, 16 oxen would eat 10 acres of grass *with* 4 *weeks' growth* of grass, in 9 weeks. But 21 oxen were required to consume 10 acres *and their growth*, in 9 weeks; then 5 oxen were fed for, the 9 weeks on 5 *weeks' growth of grass*, and, consequently, 9 oxen were fed on the 9 weeks' growth.

The remaining 12 oxen consumed what was on the ground at first, and 1 ox consumed $\frac{1}{12}$ of that quantity. But as 9 oxen eat 9 weeks' growth, 1 ox must have eaten 1 weeks' growth; and, therefore, 1 *weeks' growth was equal to* $\frac{1}{12}$ *of the quantity on the ground* at first.

We may therefore disregard the constant growing of the grass, if we *increase the number* of acres by $\frac{1}{12}$ of itself for *each week* of the respective periods of time.

Ans. 36 oxen.

IV.

PERCENTAGE,—INTEREST,—DISCOUNT,—EQUATION OF PAYMENTS.

☞ Questions in Percentage may be solved by the practical Rules which have been given on this subject; but the pupil should also be exercised in solving them *analytically*, and by Proportion.

111. An invoice of imported merchandise amounted to \$3763.5; what would be the amount of duty to be paid at the Custom House, at 12 per cent?

12 per cent would be $\frac{12}{100}$ of the cost, or *basis of percentage*.
Ans. \$451.62'.

112. A College edifice which is valued at \$25000, is insured on $\frac{2}{3}$ of that sum, at $\frac{7}{8}$ per cent. What is the amount of the premium paid? *Ans*. \$145.833'.

113. A merchant sold cloth at \$10.50 per yard, and in so doing realized a profit of 25 per cent. What did the cloth cost him per yard? *Ans*. \$8.40.

114. What would be the percentum of profit or loss on a quantity of spirits, if purchased at 43$\frac{3}{4}$ cents a gallon, and sold at 12$\frac{1}{2}$ cents a pint? *Ans*. 128.571' per cent. profit.

115. A tax of \$1133.33$\frac{1}{3}$ was raised on property amounting to \$340000. At what percentum was the tax levied?
Ans. $\frac{1}{3}$ per cent

116. An agent receives $3275 to invest in merchandise, at a commission of $1\frac{1}{2}$ per cent on the amount of purchase that can be made after this percentum is deducted. What will be the amount of his commission? *Ans.* $48.4'.

117. What amount of stock in the capital of a canal Company, at a discount of $3\frac{1}{2}$ per cent, could be purchased for $3860? and what amount, at an advance of 4 per cent, could be purchased for $6240? *Ans.* $4000; and $6000.

118. A merchant bought 37 *yd.* 3 *qr.* of cloth, at 4.87\frac{1}{2}$ per yard, and 49 *yd.* $2\frac{1}{2}$ *qr.* of silk, at $93\frac{3}{4}$ cents per yard. For what sum must the whole be sold to make a profit of $33\frac{1}{3}$ per cent? *Ans.* $307.328'.

119. A shipment of goods from New York to Mobile, amounting to $5362.36, is insured, at $2\frac{1}{8}$ per cent., on such a sum, that, in case of a total loss at sea, the insurance Company shall be liable for $\frac{3}{4}$ of the value of the goods and the premium of insurance. For what amount was the policy taken? *Ans.* 4109.088'.

120. By selling a lot of carpeting at $1.25 per yard, an upholsterer realized a profit of 20 per cent. If the same had been sold at 0.87\frac{1}{2}$ per yard, what would have been the percentum of profit or loss? *Ans.* 15.946' per cent. loss.

121. A broker in Lexington receives $5000 in Southern funds, with instructions to purchase stock in the Northern Bank of Kentucky. These funds are at a discount of $2\frac{1}{2}$ per cent, while the Bank stock is at an advance of 10 per cent; and the broker charges a commission of $\frac{3}{4}$ per cent on the stock purchased. What amount of purchase can he make? *Ans.* $4398.826'.

$1723.50. *St. Louis, May 10th,* 1848.

122. Twelve months after date, I promise to pay to John Smith, one thousand seven hundred and twenty-three dollars and fifty cents, with interest, for value received.

Edward Earnest.

The following payments were endorsed on this note:

May 10th, 1849, received $1000.
June 25th, 1850, received $ 25.50.
Dec. 30th, 1850, received $ 49.75.

What is the balance due on the 1st of March, 1851? *Ans.* $841.517'.

123. A loaned B $320 for 2 *yr.* 9 *m.* 25 *da.*, at the end of which time the Amount was found to be $383.155'. At what rate per cent was the interest computed? *Ans.* 7 per cent.

$2485.75. *New Orleans, June 3d,* 1847.

124. On or before the 1st of January next, I promise to pay to Enos Goodfellow, Two thousand four hundred and eighty-five dollars and seventy-five cents, with interest, for value received.

Timothy Trustworthy.

Endorsements:—March 4th, 1848, received $100.
May 20th, 1848, received $340.50.

What balance was due on 25th of December, 1849?
Ans. $2337.87'.

125. A debt of $375.37½, in the District of Columbia, remained unpaid until it had legally amounted to $500. How long was it on interest? *Ans.* 5.533' years.

126. A gentleman in New York proposes to loan a sum of money which, at lawful interest, shall amount to $5000, in 2 years and 6 months. What must be the amount of the loan?
Ans. $4255.319'.

127. For what sum must a note be drawn, at 4 months, that the proceeds of it when discounted in Bank, at 6 per cent, shall be $1000? *Ans.* $1020.929'.

128. A merchant bought a stock of goods amounting to $7381.25 on a credit of 6 months, but paid the debt in 3 months and 21 days from the time of purchase. What discount should have been allowed him,—rating interest at 6 per cent?
Ans. $83.919'.

129. A speculator buys 320 mules at $62½ each, and pays for them with the proceeds of a note which is discounted in Bank, for 4 months, at 6 per cent. At the end of one month he sells his mules at $87½ each, and puts the proceeds at interest, at 10 per cent, until his note is to be paid in the Bank; what profit does he make by these transactions? *Ans.* $7653.75'.

130. A farmer bought 230 A. 2 R. 30 P. of land, at $32½ per acre, on the conditions of paying $1000 in hand, $1500 in 12 months, and the remainder in 1 *yr.* 6 *m.* If the whole were converted into one payment, in what time should that payment be made? *Ans.* 1 *yr.* 2*m.* 11 *da.*

131. A person owes a note of $800 in a Bank, and has $500 with which he proposes to pay a *part of the note*, and the discount at 6 per cent, on a note at 4 months, to be given for the *remainder?* How much is he able to pay on the note? and what will be the discount on the remainder?

If $500 were paid on the note, the remainder would be $300; the remainder is the *avails* or present worth of the *note to be given at* 4 *months*—and the Bank discount on this note subtracted from $500, will leave the *payment* that can be made on the first note.

Ans. $493.73'; and $6.27'.

132. A merchant took a farmer's note for $325.50, due, without interest, on the 1st of June, 1848, and some time afterwards the farmer got possession of a note against the merchant for $500, due, without interest, on the 20th of January, 1850. Settlement was had on the 15th of August, 1849: how stood the matter of debt between them, allowing money to be at 7 per cent? *Ans.* $132.51' in favor of the farmer.

133. What would a person gain or lose by borrowing $1000 for 2 years and 6 months, at 7 per cent, and lending the same sum at 6 per cent, and on such conditions as will enable him to *compound* the interest every 6 months?

Ans. He would lose $15.727'.

134. A person owes a debt of $1500, towards the payment of which he is able to raise but $900. He offers this sum, to be applied in part to the payment of the debt, and in part to paying the interest, at 8 per cent, in advance, on his note at 12 months for the remainder. His creditor accepting, for what sum must the note be drawn? *Ans.* $652.17'

If $900 were paid on the debt, the remainder would be $600. This remainder is the present worth of the note to be given at 12 months. *Ans.* $652.17'

135. A is indebted to B $400, of which $100 will be due on the 1st of July, $150 on the 3d of September, and the remainder on the 16th of December,—without interest. On what day might the whole be equitably paid at once?

Consider the two last payments as having credit from the time at which the first payment will be due.

Ans. September 25th.

V.

SQUARE AND CUBE ROOTS.

136. What must be the length of each side of a square tract of land which shall contain 100 acres? *Ans.* 126.491' *p.*

The number of *poles* in each side is the square root of the number of square poles in the area, (§ 303).

137. A field containing 35 A. 3 R. 25 P. has its length equal to twice its breadth. What is the length of the field?
Ans. 107.19' *p.*

138. What must be the length of each edge of a cubical block of marble which shall contain 1000 cubic feet? *Ans.* 10 *ft.*

139. A reservoir containing 325 barrels of water, has its length equal to twice its breadth or depth. What is the depth of the reservoir? *Ans.* 9.846' *ft.*

140. What must be the dimensions of a granary which shall contain 2000 bushels of wheat,—its length to be equal to twice its breadth, and its breadth equal to twice its height?
Ans. Length 27.1', breadth 13.55', height 6.775 *ft.*

141. How many square feet are contained in the bottom and the four sides of a cubical cistern, the capacity of which is 175 barrels of water? *Ans.* 509.34' *sq. ft.*

142. What would be the difference in *area* between two fields of the same compass or perimeter,—one of them to be in the form of a square,—the other to be 75 *r.* in length, and 30 *r.* 3 *yd.* in breadth? *Ans.* 3 A. 13.92' P.

143. A person enclosed a garden 16 *r.* 2 *yd.* in length, and 10 *r.* in breadth, at $3.75 per rod for paling, and his neighbor enclosed the same area at the same rate for paling, but chose to have his in the form of a square. What did the latter gain or lose by such expedient? *Ans.* Gained $5.842'.

144. What would be the difference in the number of square feet contained in the four sides of a wine vat of 10 barrels capacity, whether it be of a cubical form, or of a length equal to twice its breadth or depth? *Ans.* 27.817' *sq. ft.*

Other exercises in the application of the Square and Cube Roots will be found in the Mensuration of Surfaces and Solids.

VI.

REPEATING DECIMALS.—CONTINUED AND APPROXIMATING FRACTIONS.—PROGRESSIONS.—ANNUITIES.—PERMUTATIONS AND COMBINATIONS.

145. What would be the percentum of profit on a quantity of spirits, if purchased for \$195, and sold for \$325?

$$325-195=130;\text{ and }130\div195=.\dot{6}=\tfrac{6}{9}=\tfrac{2}{3}.$$

We thus find the *ratio* of the profit to the cost, and from this ratio is readily found the *percentum* of profit.

The ratio of profit is also expressed by $\frac{130}{195}=\frac{2}{3}$. But calculations in Percentage are most usually performed by *decimal* operations, and it will sometimes be expedient to reduce a repeating decimal to an equivalent vulgar fraction.

Ans. $66\frac{2}{3}$ per cent.

146. A tax of \$400 is to be assessed on property which is valued at \$120000. At what rate per cent must the tax be assessed? *Ans.* $\frac{1}{3}$ per cent.

147. A merchant bought a quantity of cloth for \$720. At what percentum must he set his profit in selling to gain \$240 on the cloth? *Ans.* $33\frac{1}{3}$ per cent.

148. The pyramid of Cheops in Egypt, which is the loftiest structure ever raised by man, is 543 feet in height, and the altitude of the loftiest mountain is 29000 feet. What are the *approximate ratios* of these altitudes?

Ans. $\frac{1}{53}$; $\frac{2}{107}$; $\frac{5}{267}$; &c.

149. The altitude of the loftiest mountain on the Earth being, as already stated, 29000 feet, and the Earth's diameter being, in round numbers, 8000 miles, what are the approximate ratios of the altitude of this mountain to the diameter of the Earth?

Ans. $\frac{1}{1456}$; $\frac{1}{1457}$; $\frac{2}{2913}$; &c.

150. A man had 12 children, whose ages were in arithmetical progression; the youngest being 1 year old, and the eldest 34. What was the common difference of their ages?

Ans. 3 years.

151. A gentleman bequeathed to the eldest of his four sons \$4000, and to the youngest \$9000, while his second and third sons were to have the geometrical and arithmetical *means*, respectively, between the portions of the other two. What did the second and the third receive? *Ans.* \$6000; and \$6500.

152. A person completed a journey in 15 days, and his daily advances were in arithmetical progression. On the eighth day he traveled $26\frac{1}{2}$ miles: what was the length of his journey? *Ans.* $397\frac{1}{2}$ miles.

153. A lady who was married on a New-year's day, received from her father $1 towards her fortune, and the sum was tripled on the first day of each month to the end of the year. What was the amount of her fortune? *Ans.* $265720.

154. If a body were to move at the rate of 60 miles the first hour, 40 miles the second hour, and so on in decreasing geometrical progression for ever, what would be the whole distance that it would describe? *Ans.* 180 miles.

155. What would $1 amount to in 100 years, at 6 per cent, compound interest? *Ans.* $339.301'.

156. What would $100 amount to in $5\frac{1}{2}$ years, at 7 per cent, if the interest were compounded *semi-annually?*

7 per cent *per annum* is at the rate of $3\frac{1}{2}$ per cent for *half a year;* and the amount required is evidently the same that would arise from $3\frac{1}{2}$ per cent per annum in double the given time, that is, in 11 *years.*

Ans. 145.997'.

157. If $250.25 were loaned at 6 per cent, and the interest were collected and put at interest every 6 months, what would be the amount in 20 years? *Ans.* $816.323'.

158. On the birth of his first son a gentleman put at interest, at 8 per cent, a sum of money which, by adding the interest annually due to the principal, he augmented to the amount of $5000 by the time his son was 21 years old. What was the sum originally put at interest? *Ans.* $962.875'.

159. The rents of an estate, at $500 per annum, have remained unpaid for 7 years. What amount is now due, allowing *simple* interest, and what, allowing *compound* interest, at 6 per cent? *Ans.* $4130; and $4196.838'.

160. A pension of $200 a year, payable semi-annually, has been forborne for 13 years. What amount is due, if compound interest be allowed, at 6 per cent? *Ans.* $3855.27'.

161. The rent of a house is $300, and the owner wishes to convert this rent, for 5 years to come, into an equivalent sum in ready money. Required the sum that should be paid him, if compound interest be charged, at 6 per cent.

Ans. $1263.68'.

162. What is the difference in present value between a term of 15 years in an estate of $800 per annum, and the reversion of the same estate for ever, after the expiration of the 15 years, reckoning compound interest at 5 per cent? *Ans.* $493.15'.

163. In an exhibition of a public school, 7 speakers are to be taken from a class of 12 students. How many different selections of the seven might be made? and in how many different ways might they succeed each other in the delivery of their speeches? *Ans.*

164. A *die* is a small cube whose six faces are marked with the numbers from 1 to 6 inclusive. How many different combinations of five numbers might be exhibited on their superior faces, in throwing five dice together? *Ans.*

165. A company of 10 persons engaged to remain together so long as they might be able to combine in different couples in an evening walk. How long will it require them to fulfil their engagement? *Ans.*

166. In how many different ways might the seven prismatic colors, *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, and *violet*, have been arranged in the solar spectrum? and how many different combinations of three colors might be formed out of these seven? *Ans.* 5040; and 840.

167. A person who enjoyed a perpetuity of $1000 per annum, provided in his will that, after his decease, it should descend to his only son for 10 years, to his only daughter for the next 20 years, and to a benevolent Institution for ever afterwards. What was the value of each bequest, at the time of his decease, allowing compound interest at 6 per cent?

Ans. 7360.03'; $6405.75'; $5196.75.

MENSURATION.

§ 375. MENSURATION consists in applying the principles of Geometry (§ 4) to the determination of the dimensions and *contents* of surfaces and solids.

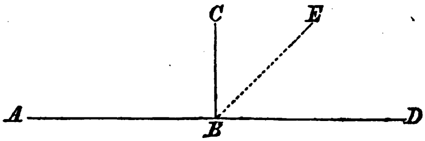
GEOMETRICAL DEFINITIONS.

§ 376. A *point* is mere *position*, without length, breadth, or thickness.—A *line* is an extension in *length*, without breadth, or thickness.—A *straight line* is one which has the *same direction* throughout its whole extent.—A *curved line* is one which continually changes its direction.

§ 377. An *Angle* is the *divergence from each other* of two straight lines which have a common point. This point is called the *vertex* of the angle, and the lines themselves are the *sides* of the angle.

A *right angle* is one of the *two equal angles* which one straight line may make at a point in another straight line.

Thus if the two angles A B C and C B D are *equal to each other*, each of them is a *right* angle.



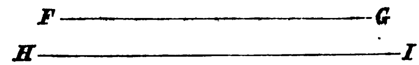
An *acute angle* is one which is less than a right angle, as the angle C B E; and an *obtuse angle* is one which is greater than a right angle, as the angle A B E.

§ 378. One straight line is *perpendicular* to another when they form a *right angle* with each other.

Thus the straight line B C is *perpendicular* to A D.

§ **379.** Two straight lines are *parallel* to each other when they are everywhere *equidistant from each other.*

F G and H I may represent two straight lines which are *parallel* to each other.



Two parallel straight lines, it is evident, would never meet, how far soever they might be produced.

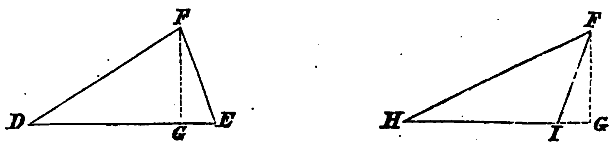
§ **380.** A *surface* is an extension in *length* and *breadth*, without thickness; and a *plane* is a surface in which if any two points be taken, the straight line which joins them *lies wholly in the surface.*

Polygons.

§ **381.** A *plane figure* is a bounded plane surface.—A *rectilineal* figure, or *polygon*, is a plane bounded by three or more straight lines which are called the *sides* of the polygon, and all the sides together make up the *perimeter* of the polygon.

§ **382.** A *triangle* is a polygon of *three sides.* A triangle is *equilateral* when its three sides are equal—*isosceles* when two of its sides are equal—and *scalene* when no two of its sides are equal.

§ **383.** A *right angled* triangle is one which has a *right angle.* The side opposite to the right angle is called the *hypothenuse:* the other two are the *perpendicular sides.*



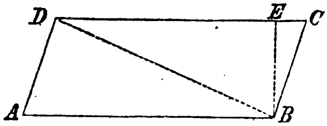
Thus D G F is a right angled triangle, having the right angle at G. The side D F is the *hypothenuse.*

§ **384.** Any side of a triangle may be considered as its *base*, and the *perpendicular distance* from the base, or the base produced, to the vertex of the opposite angle, is the *altitude* of the triangle.

Thus G F is the altitude of the triangle D E F or H I F.

§ **385.** A *quadrilateral* is a polygon of *four sides.* A *parallelogram* is a quadrilateral whose opposite sides are *parallel;* a *rectangle* is a parallelogram whose four angles are *right angles;* and a *square* is a rectangle whose sides are all *equal.*

A *trapezoid* is a quadrilateral which has only two of its sides parallel.



A B C D is a *parallelogram;* and A B E D is a *trapezoid,* having the two sides A B and D E parallel to each other.

§ **386.** Any side of a parallelogram may be considered as its *base*, and the perpendicular distance from the base to the opposite side, is the *altitude* of the parallelogram.

The *altitude of a trapezoid* is the perpendicular distance between its *two parallel sides.*

Thus B E is the altitude of the parallelogram A C, or of the trapezoid A E.

§ **387.** A *diagonal* is a straight line which joins the vertices of two angles of a polygon that are not adjacent to each other.

The diagonal of a *parallelogram* divides the parallelogram into two equal triangles.

Thus D B is a diagonal, and it divides the parallelogram A C into the two equal triangles A B D and C D B.

§ **388.** A *pentagon* is a polygon of *five sides;* a hexagon has six sides,—a heptagon seven,—an octagon eight,—a nonagon nine,—a decagon ten,—an undecagon eleven,—a dodecagon twelve, &c.

§ **389.** A *regular polygon* is one which has all its sides equal, and all its angles equal,—being equilateral and equiangular.

§ **390.** Two polygons are *similar* when they have the angles of the one equal to the angles of the other, each to each, and the sides which contain equal angles *proportional.*

Circle and Ellipse.

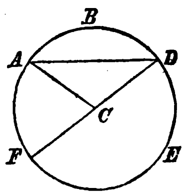
§ **391.** A *circle* is a plane figure bounded by a curve line called the *circumference*, all points of which are equidistant from a point within called the *centre*.

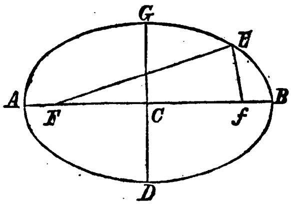
The *radius* of a circle is a straight line drawn from the *centre* to the circumference; and the *diameter* is a straight line drawn through the centre, and terminated both ways by the circumference.

§ **392.** An *arc* of a circle is any part of the circumference; and the *chord* of an arc is a straight line joining the two extremities of the arc.

A *segment* of a circle is the space enclosed by an *arc and its chord;* and a *sector* of a circle is the space enclosed by an *arc and two radii* drawn to its extremities.

A *semicircumference* is half the circumference—a *semicircle* is a segment equal to half the circle—a *quadrant* is a fourth part of the circumference, or a sector equal to a fourth part of the circle.





A B D E F A is the *circumference* of the circle, C the *centre*, C A a radius, D F a diameter, A B D an *arc*, the straight line A D is the *chord* of the arc, A B D A is a segment, A C D B A a sector.

§ **393.** An *ellipse* is a plane figure bounded by a curve line, from any point of which if two straight lines be drawn to *two fixed points*, their sum will be equal to a straight line drawn through the same two points, and terminated both ways by the curve.

Thus A D B E G A may represent an ellipse, in which the two lines E F and E *f*, drawn from any point of the curve, as E, to the two points F and *f*, are together equal to A B.

The two fixed points, F and f, are the two *foci* of the ellipse, —the middle point C of the straight line which joins the two foci, is the *centre*,—and the distance, C F or C f, from the centre to either *focus* is the *eccentricity*.

The diameter A B which passes through the two foci, is called the *transverse axis;* and the diameter D G which is drawn through the centre, perpendicular to A B, is the *conjugate axis*.

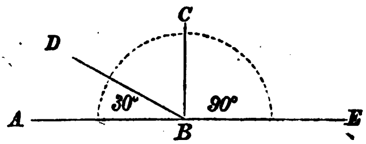
Division of the Circumference, and the Measure of Angles.

§ **394.** The Circumference of every circle is conceived to be divided into 360 equal parts called *degrees*, each degree into 60 equal parts called *minutes*, and each minute into 60 equal parts called *seconds*.

Degrees, minutes, and seconds, are denoted thus

$25° \ 30' \ 45''$, 25 *deg.* 30 *min.* 45 *sec.*

§ **395.** An Angle is measured by the number of *degrees*, &c., intercepted between its sides, when a circle is described from its *vertex* as a centre.



Thus, if a circle be described from the point B as a centre, the angle A B D for example, will be measured by the number of degrees intercepted between the sides A B and B D.

Two *right angles*, as A B C and C B D, will together contain *one half* of the entire circumference, or 180°. Hence a right angle is an angle of 90°.

All the angles that can be formed at a point in a straight line, on one side of the line, are together equal to *two right angles*, or 180°.

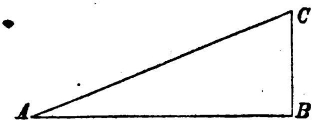
The angles A B D, D B C and C B E together contain 180°.

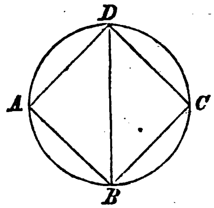
The three angles of every *triangle* together contain 180°; and the four angles of every *quadrilateral* together contain 360°.

Relation between the Sides of a Right Angled Triangle.

§ **396.** The square on the *hypothenuse* or longest side of a right angled triangle, is equal to the *sum of the squares* on the other two sides.

Thus, in the right angled triangle A B C, the square whose side is the hypothenuse A C is equal to the sum of the two squares whose sides are A B and B C respectively.





The property above stated may be expressed thus:

$$AC^2 = AB^2 + BC^2.$$

Hence also the square on either of the two perpendicular sides, is equal to the square of the hypothenuse diminished by the square of the other side; that is,

$$AB^2 = AC^2 - BC^2;\ \text{and}\ BC^2 = AC^2 - AB^2.$$

On these principles, any one side of a right angled triangle may be found when the other two sides are given.

For example, suppose the side $AB = 4$ *ft.*, and $BC = 3$ *ft.*

Then $4^2 + 3^2 = 16 + 9 = 25 = AC^2$; and since 25 is the *square* of A C, the length of this side is the *square root* of 25.

Square Inscribed in a Circle.

§ **397.** The side of a square *inscribed in a circle*, is equal to the square root of *half the square* of the diameter of the circle.

Thus, let A B C D be a square inscribed in a circle whose diameter is B D. Then $BD^2 = BC^2 + CD^2$, or *twice* BC^2.

Hence $BC^2 = BD^2 \div 2$, and $BC =$ the square root of $(BD^2 \div 2)$.

Any polygon is said to be *inscribed* in a circle when all the vertices of the polygon are in the circumference; and the circle is then said to *circumscribe* the polygon.

Ratio between the Diameter and Circumference.

§ **398.** When the Diameter of a Circle is 1, the Circumference is 3.14159'; and the circumferences of circles are proportional to the diameters; hence

any *diameter* × 3.14159' = the *circumference;*
any circumference ÷ 3.14159' = the diameter.

Instead of the decimal part of this ratio, we may use its approximate value $\frac{1}{7}$ when great accuracy is not required; thus the circumference of a circle is $3\frac{1}{7}$ times its diameter, *nearly.*

The decimal .14159' is nearly equal to .1416, and this last is often used in calculation. The exact ratio between the diameter and circumference cannot be determined.

Length of an Arc of a Circle—how found.

§ **399.** The length of 1° on the circumference of a circle whose diameter is 1, is 3.14159 ÷ 360 = .0087266'; and the lengths of arcs containing the *same number of degrees,* in different circles, are proportional to the diameters; hence

The *length of an arc* = the number of *degrees* in the arc, × .0087266' × the *diameter* of the circle. Or,

360° : to the *number of degrees* in the arc : : the length of the *circumference* : the length of the *arc.*

§ **400.** *Areas of Regular Polygons whose sides are Unity.*

NAMES.	AREAS.	NAMES.	AREAS.
Triangle	0.4330127'	Octagon	4.8284271'
Square	1.0000000'	Nonagon	6.1818242'
Pentagon	1.7204774'	Decagon	7.6942088'
Hexagon	2.5980762'	Undecagon..........	9 3656399'
Heptagon...........	3.6339124'	Dodecagon	11.1961524'

This table gives the area of a regular Polygon when each side of the polygon is 1.

Thus if each side of a regular Pentagon be 1, the area will be 1.7204774'; if each side of a regular Octagon be 1, the area will be 4.8284271'.

AREAS OF PLANE FIGURES.

§ **401.** The *Area* of any figure is the ratio of its surface to some assumed unit of surface, as a *square inch*, square foot, &c

The dimensions of figures being expressed in linear inches, or feet, &c., their *areas* in *square inches*, or square feet, &c., will be as follows:—

I. The Area of a Parallelogram is equal to its *base* × its *altitude*.

II. The Area of a Triangle is equal to its *base* × its *altitude* ÷ 2; or it is equal to the *square root* of the Product formed by multiplying *half the sum* of the three sides by the three *remainders*, continually, after each side is subtracted from the same half sum.

III. The Area of a Trapezoid is equal to the *sum of its two parallel sides* × its *altitude* ÷ 2.

IV. The Area of any Polygon of more than three sides is equal to the *sum of the areas* of the Triangles or Trapezoids into which it may be divided by diagonal lines.

V. The Area of a Regular Polygon is equal to the *perpendicular* from the *centre* to one of the sides × the *perimeter* ÷ 2; or it is equal to the *square* of one side × the *area of a similar polygon* whose side is a *unit*. (§ 400.)

VI. The Area of a Circle is equal to its *radius* × its *circumference* ÷ 2; or it is equal to the *square* of its radius, × 3.14159.

VII. The Area of a Sector of a circle is equal to the *radius* × the *length of the arc* of the sector ÷ 2; or

360° : the number of *degrees* in the arc of the sector : : the area of the *circle* : the *area of the sector*.

VIII. The Area of a Segment of a circle is equal to the area of the *sector* having the same arc, + or — the *triangle* formed by the *chord* of the segment and the *radii* of the sector, according as the segment is *greater or less than a semicircle*.

IX. The Area of an Ellipse is a geometrical mean between the areas of the two circles whose diameters are the two *axes* of the ellipse; and is equal to the *product of the semi-axes*, × 3.14159.

☞ The *demonstration* of the preceding propositions depends on principles of Geometry which cannot be given in this work.

EXERCISES

In the Mensuration of Plane Figures.

1. The two perpendicular sides of a right angled triangle are 30 and 40 feet, respectively. What is the length of the hypothenuse? *Ans.* 50 *ft.*

2. If the hypothenuse of a right angled triangle be 75 feet in length, and the base 60 feet, what is the altitude of the triangle? *Ans.* 45 *ft.*

3. What is the length of the diagonal of a rectangle whose base is 50 feet, and altitude 12 feet 9 inches? *Ans.* 51.6' *ft.*

4. What is the length of the side of a square inscribed in a circle whose diameter is 100 feet? and of one inscribed in a circle whose radius is 25 feet?
Ans. 70 *ft.* 8.52' *in.*; and 35 *ft.* 4.26' *in.*

5. What is the circumference of a circle whose diameter is 37½ feet? and the diameter of one whose circumference is 300 *ft.*? *Ans.* 117.809' *ft.*; and 95.493' *ft.*

6. What is the length of an arc containing 75 degrees on the circumference of a circle whose radius is 5 feet?
Ans. 6.544' *ft.*

7. How many *degrees* are there in an arc of 3 feet in length on the circumference of a circle of radius 10 feet?
Ans. $17^\circ\ 11'\ 16.8''$.

8. Find the area of a parallelogram, and also of a triangle, whose base is 9½ feet, and altitude 8 feet.
Ans. 76 *sq. ft.*; and 38 *sq. ft.*

9. Find the area of a trapezoid whose parallel sides are 15 and 20 feet, respectively, and altitude 6 feet 8 inches.
Ans. 116.655' *sq. ft.*

10. Find the area of a regular pentagon, and also of a regular octagon, supposing each side to be 10 feet.
Ans. 172.047' *sq. ft.*; and 482.842' *sq. ft.*

11. Find the area of a circle whose radius is 25 feet, and also of an ellipse whose axes are 20 and 30 feet.
Ans. 1963.493' *sq. ft.*; and 471.238' *sq. ft.*

12. Find the area of a circular sector whose arc is $60°$ on a circumference of radius 3 feet, and also of the segment (less than a semicircle) having the same arc as the sector.

The chord of an arc containing $60°$ is always equal to the radius of the circle. Hence the triangle which is formed of this chord and the two radii of the sector, is *equilateral.*

Ans. 4.712' *sq. ft.*; and .815' *sq. ft.*

13. What must be the altitude of a triangle whose base is 16 feet, that the area may be 100 square feet?

Since the area of a triangle is equal to its *base* $\times$ its *altitude* $\div 2$; *conversely*, the *area* $\div$ the *base* will be *half the altitude*; or *twice the area* $\div$ the base, will be the altitude.

Ans. $12\frac{1}{2}$ *ft.*

14. What must be the altitude of a parallelogram whose base is 25 feet 3 inches, that the area may be 250 *sq. ft.*?

Ans. 9.9' *ft.*

15. What must be the side of an equilateral triangle that its area may be 500 *sq. ft.*?—The given area $\div$ by the tabular area of a triangle whose side is 1, will be the *square* of the required side. *Ans.* 33.98' *ft.*

16. What must be the side of a regular decagon, and also the radius of a circle, that the area of each figure may be 1000 square feet? *Ans.* 11.4' *ft.*; and 17.841' *ft.*

17. Allowing the *perimeter* of a regular hexagon to be 120 feet, and the circumference of a circle also 120 feet, what would be the area of each figure?

Ans. 1039.23' *sq. ft.*; and 1145.91' *sq. ft.*

18. The diagonal of a quadrilateral field is 45 rods, and the two perpendiculars to the diagonal from the opposite angles are 20 and 30 rods, respectively. What is the area of the field?

Ans. 7 A. 4.96' P.

19. A circle of 33 feet in diameter is enclosed by another circle 50 feet in diameter. What is the area of the space included between their circumferences?

The space between the circumferences is the difference of the areas of the two circles. *Ans.* 1108.195' *sq. ft.*

20. An irregular pentagon has been divided, by drawing perpendiculars from three of the angles to the opposite side, into two triangles whose bases are $12\frac{1}{2}$ and 14 *ft.*, with corresponding altitudes of 10 and $15\frac{1}{4}$ *ft.*, and two trapezoids, adjacent to each other, whose parallel sides are the same two altitudes, with an intermediate one of 20 *ft.*, and the distance between the parallel sides of each is 18 *ft.* What is the area of the pentagon?

Ans. $756\frac{1}{2}$ *sq. ft.*

21. The base of an isosceles triangle is 20 feet, and each of the two equal sides $15\frac{1}{2}$ feet. What is the altitude of the triangle?

The perpendicular from the vertex to the base, will *bisect* the base, that is, divide it into two equal parts.

Ans. 11.842' *ft.*

22. Allowing the area of a circle to be 375 square feet, what would be the area of the inscribed square?

Ans. 238.711' *sq. ft.*

23. By drawing a diagonal through a certain tract of land, it will be divided into a parallelogram and a triangle; the base and altitude of the former will be 320 and 193.25 rods, and the sides of the latter 203, $150\frac{1}{2}$, and 175 rods. What is the area?

Ans. 466 A. 2 R. 18.56' P.

24. Find the area of the space included between the curve of an ellipse whose axes are 40 and 50 feet, respectively, and the circumference of a circle whose diameter is the conjugate or minor axis of the ellipse. *Ans.* 314.159' *sq. ft.*

25. What must be the perimeter of a regular dodecagon which shall contain the same area as a circle whose circumference is 1000 feet? *Ans.* 1011.672' *ft.*

26. A gentleman has a garden 100 feet in length, and 80 feet in breadth. What must be the breadth of a walk extending half round the garden, which shall occupy one half of the ground? *Ans.* 25.968' *ft.*

27. The perambulator, or surveying wheel, is so contrived as to turn just twice in the length of one pole, or $16\frac{1}{2}$ feet; what is its diameter? *Ans.* 2.626' *ft.*

28. From a mahogany plank 16 inches in breadth, 6 square feet are to be sawn off. At what distance from the end must the line be struck? *Ans.* $4\frac{1}{2}$ *ft.*

29. A and B departed from an inn at the hour of eight in the morning; A went north-west, at the rate of 6 miles an hour, and B north-east, at the rate of 8 miles an hour; how far were they distant from each other at 12 o'clock of the same day?

Ans. 40 *miles.*

SOLIDS.

§ **402.** A SOLID, in Geometry, is an extension in *length, breadth*, and *depth*, height, or thickness; and is to be regarded as mere *space*, unconnected with any of the properties of matter.

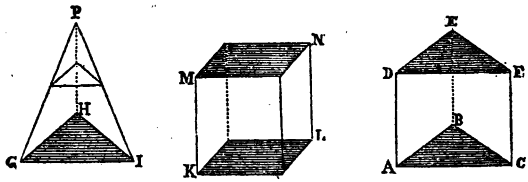
Polyedrons.

§ **403.** A *polyedron* is a solid included between several *plane faces* which meet each other in straight lines, called the *edges* of the polyedron.

Polyedrons, among solids, correspond to *polygons* among plane figures.

§ **404.** A *prism* is a polyedron included between two equal and parallel polygons, called its *bases*, and three or more parallelograms which are its *lateral faces*, and which together make up the *convex surface* of the prism.

A prism is *triangular*, quadrangular, or pentagonal, &c., according as its base is a *triangle*, quadrilateral, or pentagon, &c.



ABC-FED is a *triangular* prism, whose *bases* are the triangles ABC and FED, and lateral faces the parallelograms AE, EC, and CD.

A *right* prism is one whose lateral faces are *perpendicular to its bases;* any other is an *oblique* prism.

The *altitude* of a prism is the *perpendicular distance* between its two bases.

§ **405.** A *parallelopipedon* is a prism included between six parallelograms. When its six faces are all *rectangles*, it is a *rectangular* parallelopipedon; and when they are all *squares*, it is a *cube*.

Thus KL-MN may represent a *cube*, whose faces are six equal squares.

§ **406.** A ***pyramid*** is a polyedron included between a polygon called its *base*, and three or more triangles which form its *convex surface*, and which meet in a point called the *vertex* of the pyramid.

A pyramid is *triangular*, quadrangular, or pentagonal, &c., according as its base is a *triangle*, quadrilateral, or pentagon, &c.

GHI-P is a triangular pyramid, whose base is the triangle GHI, and *vertex* P.

The *altitude* of a pyramid is the *perpendicular* distance from the base, or the plane of the base produced, to the *vertex*.

A *right* pyramid is one whose base is a *regular polygon*, and lateral faces *equal triangles*. Its *slant height* is the perpendicular from the vertex to *any side of the base*. This is sometimes called a *regular* pyramid.

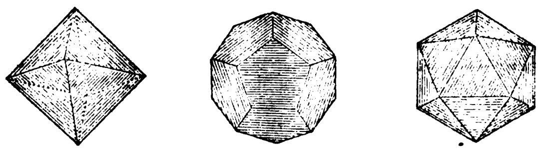
§ **407.** A *frustum* of a pyramid is a portion of it included between the base and a plane *parallel to the base*. The frustum will thus have *two bases*, and its *altitude* will be the perpendicular distance between them. The *slant height* is the perpendicular distance between two corresponding *sides of the two bases*.

§ **408.** *Similar polyedrons* are such as are included between the same number of *similar polygons*, similarly situated.

Five Regular Polyedrons.

§ **409.** A *regular polyedron* is one whose faces are all equal *regular* polygons. Thus a pyramid whose base and three lateral faces are *equilateral* triangles is a regular *tetraedron;* and the *cube*, whose faces are *squares*, is a regular *hexaedron.*

Besides these there may also be the following regular polyedrons, viz.

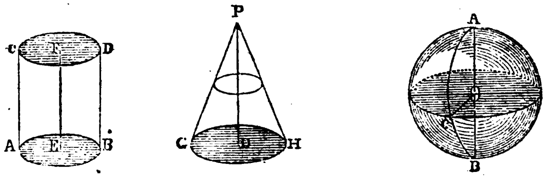


A regular *octaedron* is included between *eight* equilateral *triangles;* a regular *dodecaedron* between *twelve* regular *pentagons;* and a regular *icosaedron* between twenty equilateral *triangles.*—There can be no other regular polyedrons, from the impossibility of arranging the angles of regular polygons, in any other way, around a *solid vertex.*

Cylinder, Cone, and Sphere.

§ **410.** A *cylinder* is a solid included between two equal and parallel circles, called its *bases*, and a *convex surface* in which a straight line may be drawn between any two *corresponding points* in the circumferences of the two bases.

Thus AB-DC may represent a cylinder, whose bases are the circles AB and DC. AC or BD is a straight line lying in the convex surface, between corresponding points in the circumferences of the two bases.



The *axis* of a cylinder, as EF, is the straight line which joins the centres of the two bases. A *right* cylinder is one whose *axis* is perpendicular to its bases; otherwise, the cylinder is *oblique*.

§ **411.** A *cone* is a solid included between a circle, called its *base*, and a *convex surface* in which straight lines may be drawn at pleasure from the circumference of the base to a point called the *vertex* of the cone.

GH-P represents a cone whose base is the circle GH, and vertex P; GP or HP is a straight line lying in the convex surface.

The axis of a cone, as PD, is a straight line which joins the vertex to the centre of the base. A *right* cone is one whose axis is perpendicular to its base; otherwise, the cone is *oblique*.

The *altitude* of a cylinder or a cone—also a *frustrum* of a cone—are to be understood in the same way as for a *prism* and *pyramid*.

§ **412.** A *sphere* is a solid bounded by a *curved surface*, all points of which are equidistant from a point within called the *centre*.

A sphere would be described by *revolving a semicircle* ACB *about its diameter* AB—this diameter remaining unmoved.

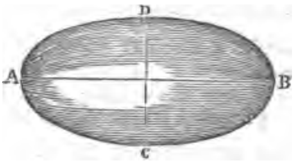
The *radius* of a sphere is a straight line drawn from the centre to any point of the surface; and a *diameter* is a straight line passing through the centre, and terminated both ways by the surface.

The *circumference* of a sphere is the same as that of a circle having for its diameter any *diameter of the sphere*:—such circle is also called a *great circle* of the sphere.

A *hemisphere* is one half of a sphere; that is, one of the two equal parts into which a sphere will be divided by a *plane passing through its centre*.

Spheroids.

§ **413.** A *spheroid* is a solid described by *revolving an ellipse* about one of its *axes*—that axis remaining unmoved or *fixed*.



An *oblate* spheroid is described by revolving an ellipse about its *shorter* axis, CD; and a *prolate* spheroid, by revolving the ellipse about its longer axis, AB.

Mensuration of Solids.

§ **414.** The measure of the *surface* of any solid is expressed by the area of an equivalent *plane* surface; *solidity*, or the *volume* of a solid, is expressed by the ratio of the space occupied by the solid to some assumed *unit* of solidity, as a *cubic inch*, or a cubic foot, &c.

In the following propositions it is to be understood that the *solidities* will be found in *cubic units* corresponding in name to the linear units in which the dimensions are taken.

Thus if the dimensions of a solid be taken in *inches*, the solidity will be found in *cubic inches;* and so for any other denomination.

I. The Convex Surface of a Right Prism or Cylinder, is equal to the *perimeter* or circumference of its *base* × its *altitude*.

II. The Solidity of any Prism or Cylinder is equal to the *area* of its base × its *altitude*.

III. The Convex Surface of a Right Pyramid or Cone, is equal to the *perimeter* or circumference of its *base* × *its slant height* ÷ 2.

IV. The Solidity of any Pyramid or Cone is equal to the *area* of its *base* × its altitude ÷ 3.

V. The Convex Surface of a *Frustum* of a right pyramid or cone, is equal to the *sum of the perimeters* or circumferences of its two *bases* × its *slant height* ÷ 2.

VI. The Solidity of a *Frustum* of any pyramid or cone, is equal to (the *sum* of the areas of its *two bases* + a *mean proportional* between those areas) × its *altitude* ÷ 3.

VII. The Surface of a Regular Polyedron is equal to the area of one of its *equal faces* × the number of its faces; or it is equal to the *square* of one of the *edges* of the polyedron × the *surface* of a similar polyedron whose edge is *unity*; and

VIII. The Solidity of a Regular Polyedron is equal to the *cube* of one of its *edges* × the *solidity* of similar polyedron whose edge is *unity*.

☞ The *edges* of a polyedron are evidently the same as the *sides* of the polygons which form the faces of the polyedron.

IX. The Surface of a Sphere is equal to its *diameter* × its *circumference*; or it is equal to the *square* of its diameter × 3.14159.

X. The Solidity of a Sphere is equal its *surface* × its *radius* ÷3; or it is equal to the *cube* of its diameter × 0.5236.

XI. The Solidity of a Spheroid is $\frac{2}{3}$ of the solidity of a Cylinder the diameter of whose *base* is equal to the *fixed axis* of the *describing ellipse*, and whose *altitude* is equal to the revolving axis of the ellipse.

☞ The *demonstrations* of the preceding propositions depend on principles of Geometry, and cannot be presented in this work.

Surfaces and Solidities of Regular Polyedrons whose Edges are Unity.

NAMES.	SURFACES.	SOLIDITIES.
Tetraedron	1.732051	0.117851
Hexaedron	6.000000	1.000000
Octaedron	3.464102	0.471404
Dodecaedron	20.645729	7.663119
Icosaedron	8.660254	2.181695

EXERCISES

In the Mensuration of Solids.

1. Find the convex surface, and the solidity, of a right triangular prism the sides of whose base are 20, 30, and 40 feet, and whose altitude is 12 feet. *Ans.* 1080 *sq. ft.* and 3485.676' *cu. ft.*

2. Find the solidity of a prism whose altitude is 9 feet, and base a regular pentagon of 20 feet in perimeter, and also of a cylinder of the same altitude, and whose base is 4 feet in diameter. *Ans.* 247.748' *cu. ft.*; and 113.097' *cu. ft.*

3. Find the convex surface of a right pyramid whose slant height is 5 feet, and base a square whose side is 3 feet; and also of a right cone of the same slant altitude, and whose base has a radius of 3 feet. *Ans.* 30 *sq. ft.*; and 47.123' *sq. ft.*

4. Find the solidity of a pyramid whose base is a regular hexagon 30 feet in perimeter, and also of cone whose base is 30 in circumference—allowing each of the two solids to be 10 feet in altitude. *Ans.* 216.506' *cu. ft.*; and 238.7' *cu. ft.*

5. Find the convex surface of a frustum of a right pyramid whose bases are equilateral triangles of 12 and 9 feet, respectively, in perimeters, and its slant height 10 feet:—also the solidity of a frustum of a cone of altitude 4 feet, and bases 5 and 2 feet, respectively, in radius.
Ans. 105 *sq. ft.*; and 163.362' *cu. ft.*

6. Find the surface, and also the solidity, of a regular octaedron whose *edges* are each 1.5 feet.
Ans. 7.794' *sq. ft.*; and 1.59' *cu. ft.*

7 Find the surface, and also the solidity, of a sphere whose circumference is 13 feet and 10 inches.
Ans. 60.906' *sq. ft.*; and 44.684' *cu. ft.*

8. Find the solidity of the *oblate*, and also of the *prolate*, spheroid which would be described by the revolutions of an ellipse whose axes are 3 and 4 feet, respectively.

Ans. 18.849' *cu. ft.*; and 25.132' *cu. ft.*

9. If the radius of a right cylinder's base be $1\frac{1}{2}$ *ft.*, what must the altitude be, to make the convex surface 25 square feet? and what will be the solidity?

The convex surface ÷ the circumference of the base, will be the *altitude*. *Ans.* 2.652 *ft.*; and 18.745' *cu. ft.*

10. If the radius of the base of a right cone be 2 feet, and the convex surface 20 square feet, what is the slant height of the cone? and also its solidity?

The slant height of the cone will be the *hypothenuse* of a right angled triangle, — the radius of the base one of the *perpendicular sides*,—and the *altitude* of the cone the remaining side.

Ans. 3.182' *ft.*; and 10.36' *cu. ft.*

11. How many gallons of wine could be put into a vat, which is in form a frustum of a square pyramid,—each side of the larger end being 5 *ft.*, each one of the smaller $3\frac{1}{2}$ *ft.*, and the depth 4 feet? (§ 179) *Ans.* 546.077' *gal.*

12. A cistern whose form is a frustum of a right cone, has its diameter at the top 12 *ft.*, at the bottom 9 *ft.*, and its depth is 10 *ft.* How many barrels of water will the cistern contain?

Ans. 112.29' *bar.*

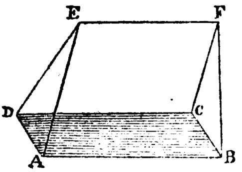
13. The diameter of a legal Winchester bushel is $18\frac{1}{2}$ inches, and its depth is 8 inches. What is the diameter of that bushel whose depth is 7 inches? *Ans.* 19.777' *in.*

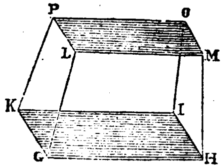
14. A cubic foot of brass is to be drawn into wire $\frac{1}{30}$ of an inch in diameter. What will be the length of the wire, allowing no waste in the metal? *Ans.* 31.252' *miles.*

15. A gentleman has a bowling-green, 500 feet in length, and 300 feet in breadth, which he wishes to raise 1 foot higher, by means of the earth to be dug out of a ditch with which he intends to surround it. What must be the depth of the ditch, if its breadth be everywhere 9 feet? *Ans.* 10.187' *ft.*

Solidity of a Wedge and Prismoid.

§ **415.** A *wedge* is a solid included between a *rectangular base*, two triangular ends, and two parellelograms or trapezoids which meet in a *vertical edge* parallel to the base.





ABCD is the *base* of the wedge; ADE and BCF are the triangular ends; and ABFE and CDEF two parallelograms or trapezoids meeting in the vertical *edge* EF parallel to the base. When the two faces AF and CE are parallelograms, the wedge becomes a triangular *prism*, having the two triangular ends for its *bases*.

The *altitude* of a Wedge is the perpendicular distance from its vertical edge to the base, or the plane of the base produced.

To find the Solidity *of a Wedge*,—Add the length of the *edge* to twice that side of the base which is *parallel to the edge;* multiply the sum by the product of that side of the base which is not parallel to the edge into $\frac{1}{6}$ of the *altitude* of the Wedge.

§ **416.** A rectangular *prismoid* is a solid included between two parallel *rectangular bases*, and four *trapezoids* which are its lateral faces.

GHIK and LMOP are the two rectangular *bases;*—the lateral faces KL, GM, HO, and OK are *trapezoids.*

To find the Solidity *of a Rectangular Prismoid*,—Multiply the sum of the two parallel sides of one of the *trapezoidal faces* by the sum of the two parallel sides of one of the *contiguous* trapezoidal faces; to the product add the areas of the two *rectangular* faces; and multiply the sum thus found by $\frac{1}{6}$ of the *altitude* of the Prismoid.

For example, suppose that in the Prismoid above represented, GH=10; HI=6; LM=8; MO=5; and the *altitude* =9.

The Solidity is

$((10+8)\times(6+4)+10\times6+8\times5)\times\frac{9}{6}=(180+100)\times\frac{3}{2}=420.$

16. What is the solidity of a wedge whose base is 2 feet in length, and 11 inches in breadth, the altitude being 1*ft.* 9*in.*, and its vertical edge, which is parallel to the longer side of the base, 17 inches.

These dimensions must all be used in the *same denomination.*

Ans. 1.448' *cu. ft.*

17. What is the solidity of a rectangular prismoid which measures 13 inches by 10 inches at one end,—the parallel dimensions at the other end being 12 and 15 inches, and the length 14 feet? *Ans.* 15.15' *cu. ft.*

18. A solid is bounded by a rectangular base which measures 19 by 37 inches, two triangular, and two trapezoidal faces meeting in an edge 15 inches long, 3½ feet from the base, and parallel to the shorter side of the base. How many cubic feet does it contain? *Ans.* 7.943' *cu. ft.*

19. An embankment of earth is 13 feet in height; 100 feet long, and 40 wide at the base; and 105 feet long, and 30 wide at the top. How many cubic yards does it contain?

Ans. 1724.77' *cu. yd.*

20. An abutment of masonry is 53½ feet long, and 31 feet wide at the base; 40 long, and 15.6 wide at the top; and its altitude is 23 feet. How many *perches* does it contain,—24¾ cubic feet making a *perch*, in stone-work?

Ans. 1028.26' *perches.*

21. Find the entire surface, and also the solidity, of a rectangular parallelopipedon whose base is 3½ by 2 feet, and its altitude 2 *ft.* 9*in.* *Ans.* 44.25 *sq. ft.*; and 19.25 *cu. ft.*

22. What must be the length, breadth, or height of a cube, that its entire surface may be 1350 square feet? and what will be the solidity of the same cube? *Ans.* 15 *ft.*; and 3375 *cu. ft.*

23. What must be the length of each edge of a regular tetraedron, that its entire surface may be 1200 square feet?

Ans. 26.321' *ft.*

24. Find the number of square miles on the surface of the Earth, on the supposition that the earth is a *sphere* of 7912 miles in diameter. *Ans.* 196662729.75' *sq. m.*

25. A marble pillar, which is in form a frustum of a right cone, is 2 *ft.* 6 *in.* in diameter at one end, 1 *ft.* 9 *in.* at the

other, and 5 *ft.* in length: what is the solidity of the greatest square prism that could be cut out of it?

The *base of the prism* would be a square inscribed in the smaller base of the frustum. The diameter of this base will be a *diagonal of the square*, and *half the square* of the diameter will be the *area* of the inscribed square. *Ans.* 7.65' *cu. ft.*

26. What is the solidity of the greatest square prism that could be sawn out of a log of wood whose diameter at the larger end is 3 *ft.*, at the smaller 1¾ *ft.*, and whose length is 20 feet? *Ans.* 30.625' *cu. ft.*

27. Find the solidity of the largest frustum of a square pyramid that could be hewn out of a frustum of a right cone, whose bases are 30 and 20 inches, respectively, in diameter, and whose altitude is 4½ feet. *Ans.* 9.89' *cu. ft.*

28. A piece of timber is in the form of a square right prism, each side of whose base is 1 *ft.* 6 *in.*, and its altitude or length is 10 feet: what is the solidity of the greatest cylinder that could be cut out of it? *Ans.* 17.671' *cu. ft.*

29. Find the number of cubic feet in a beam whose ends are rectangles; the length and breadth of the greater being 32 and 20 inches; the length and breadth of the less 16 and 8 inches; and the length of the beam 25 feet. *Ans.* 61$\frac{1}{9}$ *cu. ft.*

30. What is the solidity of the largest frustum of a cone that might be cut out of a frustum of a square right pyramid, the side of whose greater base is 25 inches, and of the less 16 inches —the altitude being 3¼ feet? *Ans.* 7.568' *cu. ft.*

31. A wall of stone surrounding a prison is in the form of a hollow square; the perimeter on the outside is 1000 feet, the thickness of the wall 3 feet, and the height 15 feet. How many perches does the wall contain? *Ans.* 1796.36' *pchs.*

32. A conical pile of wood measures 43½ feet in circumference at the base, and the slant height is 12 feet. How many cords does it contain? *Ans.* 3.843' *cords.*

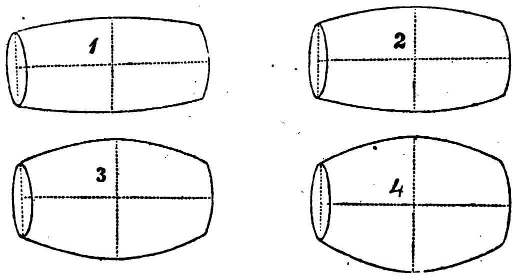
33. The dimensions of a certain receptacle are as follows: at the top, the length is 80½ and the breadth 56 inches; at the bottom, the length is 41½, and the breadth 32 inches; and the perpendicular depth is 47 inches, how many *bushels* will it contain? *Ans.* 60.36' *bu.*

34. The true figure of the Earth is that of an *oblate spheroid*, its diameter at the poles being about 7898.9 miles, and at the equator 7925.3 miles; how many cubic miles does it contain? *Ans.* 259,323,316,083.66' *cu. m.*

Gauging of Casks.

§ **417.** The *Gauging* of a *Cask* consists in determining its capacity in *gallons* from its dimensions in *inches*.

Casks are usually regarded as of *four varieties*, according to their different degrees of *curvature*, from end to end. These different forms, beginning with that which has the least curvature, are represented by the following figures.



§ **418.** The Capacity of a Cask may be computed as that of a *cylinder* of equal length, and having a *base* whose diameter is some quantity intermediate between the *end* and *middle* diameters of the cask. This intermediate diameter is called the *mean diameter* of the cask.

To find the MEAN DIAMETER *of a Cask*,—Add to the *end diameter* the product which arises from multiplying the difference between the *end* and *middle* diameters by .55, .60, .65, or .70, according as the Cask is of the 1st, 2d, 3d, or 4th *degree of curvature*. Then

To find the CAPACITY *of the Cask in* GALLONS,—Multiply the square of the mean diameter in *inches* by the length of the cask in *inches*; multiply the product thus found by .0028, to find the number of gallons in Beer Measure; or multiply the same product by .0034, to find the number of gallons in Wine Measure.

EXAMPLE.

To find the Capacity of a Cask of the 2d variety, whose length is 40 inches, diameter at each *end* 24 inches, and at the *middle* 32 inches.

The *mean diameter* is $24+.6\times(32-24)=24+.6\times8=28.8$;

then $28.8\times28.8\times40\times.0028=92.89$ *gal.* in Beer Measure;

and $28.8\times28.8\times40\times.0034=112.80$ *gal.* in Wine Measure.

☞ The square of the *mean diameter* of the Cask, multiplied by .7854, which is the area of a Circle whose diameter is 1,—would be the *area* of the *base* of the equivalent *cylinder.* This area multiplied by the length of the Cask, would give the Capacity in *cubic inches*,—which might be reduced to *beer* or *wine* gallons by dividing by 282, or 231, (§ 167).

Instead, however, of using the *multiplier* .7854, and the *divisor* 282 or 231, we abbreviate the calculation by using the *multiplier* .0028, which is equal to .7854÷282, or the multiplier .0034, equal to .7854÷231.

☞ It is only a *near approximation* to the true capacity of a Cask that can be obtained by arithmetical computation; inasmuch as from want of conformity in the curvature of the staves to regular geometrical figures, the subject does not admit of being treated in a strictly scientific manner.

Tonnage of Ships.

§ **419.** The *Tonnage* of a Ship is the number of *tons* burden that the ship will carry. This is equal to the weight of water which the Ship displaces in sinking from the "*light water-mark*" to the "*load water-mark;*" and would be found by dividing the number of *cubic feet* contained in so much of the Ship's *hull* as is included between these two *water lines* or marks by 35, the number of cubic feet of sea-water required to weigh *one ton.*

But this method of computing the Tonnage, with entire accuracy, is impracticable, from the want of agreement in figure between the Ship's hull and any of the solids of Geometry.

It is therefore only an approximative measurement that can be obtained; and as Government usually imposes a duty or tax on the capacity or tonnage of Merchant Ships, so the Rule for computing the tonnage, with this end in view, is prescribed by Law.

At the same time, the Carpenter who builds the ship, estimates its cost according to the amount of tonnage found by a different Rule.

Rule *adopted by the Government of the U. S. for computing the Tonnage of Ships.*

1. *When the ship is double-decked.*—From the length between the fore part of the *main stem* and the after part of the *stern-post*, above the upper deck, subtract $\frac{3}{5}$ of the greatest breadth above the *main wales;* multiply the remainder by the breadth,

and the product by $\frac{1}{2}$ of the breadth,—all in *feet*,—and divide the last product by 95.

2. *When the Ship is single-decked.*—Proceed as above with the exception of using the *depth* from the under side of the deck-plank to the ceiling of the hold, as a multiplier, instead of $\frac{1}{2}$ the *breadth.*

Carpenter's RULE *for the Tonnage of Ships.*

Multiply together the length of the *keel*, the breadth at the *main beam*, and, in a double decker, $\frac{1}{2}$ *of the breadth*, but, in a single decker, the *depth of the hold*,—all in feet,—and divide the product by 95.

EXAMPLE.

To find the Government Tonnage of a double-decked ship whose length is 80 feet, and breadth 24 feet.

$$\frac{(80-24\times\frac{3}{5})\times24\times24\times\frac{1}{2}}{95}$$

$$=\frac{(80-14.4)\times24\times12}{95}=\frac{18892.8}{95}=198.9.$$

APPENDIX I.

EXCHANGE.

§ **420.** Exchange, in Commerce, is a transaction by which a debt is paid to a person at a distance, without the transmission of money.

This is effected by means of a

Bill of Exchange.

§ **421.** A *Bill of Exchange* is an open letter from one person, the *drawer*, to another, the *drawee*, requesting the payment of a certain sum to a third person, the *payee*, or to his order.

The payee becomes an *endorser* of the Bill by writing his name across the back of it. This is called a *blank* endorsement, and the bill being put into circulation, becomes payable to the *bearer*. If he endorse that payment shall be made to a particular person, that person is the *endorsee*, and may himself *endorse the bill* in the same manner as the first endorser; and a bill may obtain any number of successive endorsers.

The drawee becomes the *acceptor* of the Bill, and binds himself to pay it as requested, when, upon its being presented to him, he writes the word "*accepted*" and his name across the face of the bill.

A Bill of Exchange is said to be *negotiated* when, being made payable to a person named, or to *his order*, it may by the endorsement of the payee, become payable to any *holder* of the bill, and thus be put into circulation in the same manner as the bills of a Bank.

In this way Bills of Exchange become, to a great extent, the Money of Commerce.

Example in Exchange.

Suppose that John Smith of Louisville is indebted to James Brown of New-York, and Thomas Jones of New-York to William Nelson of Louisville; and that payment is to be made by each debtor to his creditor, to the amount of $500.

John Smith pays this sum to William Nelson for a bill on Thomas Jones, which reads as follows:

$500. LOUISVILLE, AUG. 1, 1851.

Thirty days after date, pay to John Smith, or his order, Five hundred dollars, with or without further advice.

WILLIAM NELSON.

To THOMAS JONES, *Esq.*
Merchant, New-York.

John Smith writes on the back of this bill, "Pay to James Brown, or order," subscribes his name, and sends the bill to James Brown. The latter presents it to Thomas Jones, who, agreeing to pay as requested, writes "Accepted" and his name across the face of the bill.

This Bill, either with or without the acceptance of the drawee, Thomas Jones, might pass by endorsement through different hands, answering the purposes of money to its successive *holders*, until, becoming due, it is presented to the drawee for *payment*.

Inland and Foreign Bills.

§ **422.** An *inland* Bill of Exchange is one of which the drawer and drawee both reside in the same country. Such bills are usually called *drafts*,—or, *checks* when made on a Bank in which the drawer has funds deposited.

§ **423.** A *foreign* Bill of Exchange is one of which the drawer and drawee reside in different countries, or under different State Governments.

According to this definition, a Bill drawn in any one of our States on a person residing in any other one, and thus made payable in the other State, is a *foreign* Bill. (*Jones' Legal Science.*)

§ **424.** A *Set of Exchange* consists usually of three Bills of the *same import*, called respectively the *first*, *second*, and *third* of Exchange, which, to provide against miscarriage, are sent by different conveyances. They virtually form but one bill, and when either of them is received, and *accepted* or *paid*, the others are *void*.

Laws and Customs respecting Bills of Exchange.

§ **425.** The *drawer*, and all the *endorsers* of a Bill of Exchange, (with the exception hereafter made,) are liable for its amount to the *holder* of the Bill, if the drawee *fail to pay it at maturity.* But to bind them to this liability they must receive due notice of such failure. This notice, given in writing by an officer styled a *Notary*, at the instance of the holder, is called a *protest* of the bill, and will be for *non-acceptance* or else for *non-payment.* A protest for *non-acceptance* binds the drawer for the immediate payment of the Bill, even though it should not have reached its maturity.

§ **426.** The exception to the endorsers' being liable for the amount of the Bill, is the case in which an endorsement has been made "*without recourse*," or *at the risk* of the endorsee. A person so qualifying his endorsement is exempt from all liability to pay the bill.

§ **427.** When the time in which a Bill will *mature* is given in *months*, Calendar months are always to be understood, without regard to the number of days; and it is to be observed that the day on which the bill is dated is not included in reckoning the time.

Thus a Bill dated on the 28th of January, payable *one month* after date, will mature on the 28th of February. If dated on the 29th, 30th, or 31st of January, it would mature on the *last day* of February.

§ **428.** *Days of grace* are, in this country and Great Britain, 3 days allowed for payment, beyond the time for which the Bill is drawn.

A Bill payable *on demand*, or without *any time of payment expressed*, should be presented within a reasonable time after receipt, and is payable on *presentment*, without days of grace. Also a Bill which is made payable on a *fixed* day, as the 20th of November, is not entitled to *days of grace.* It is not fully settled, though the weight of authority is in favor of them, that Bills payable *at sight* are entitled to days of grace.

§ **429.** When the last day of grace allowed for the payment of a Bill, is Sunday, or a public *holiday*, such as the 4th of July, the bill must be paid on the preceding day.

Remark. Promissory Notes may be *negotiated*, that is, passed, for value received, from one holder to another in the same manner, and subject to the same Laws and Customs, as Bills of Exchange.

Par of Exchange.

§ **430.** The *Par of Exchange* between two countries is the value of a given amount of the currency of the one, as expressed in the *currency of the other;* and this is either an *intrinsic*, a *legal*, or a *commercial* value.

The *intrinsic* par of exchange depends on the amount of *pure gold* or *silver* in the coins compared: the *legal* par is the value assigned to the foreign currency by *law*; the *commercial* par is the rate at which one currency will, at any time, exchange for the other, with *money-brokers.*

For example, the *intrinsic* value of the English Sovereign, or Pound Sterling, is $4.861, this being the value of the pure gold it contains, as compared with the United States Eagle of $10.

The *legal* value of the Sovereign, or the amount for which it is receivable by *law* in the Custom Houses of this country, is $4.84.

The *commercial* value of the Sovereign varies from $4.86 to $4.83, according as, from any cause, this species of currency is in greater or less demand at the time.

Relative Value of Gold and Silver in Different Countries.

§ **431.** The relative value of gold and silver in any country, is ascertained from the proportional amount of pure gold and silver in its principal coins,—taking these coins of the *value*, *weight*, and *purity*, prescribed by the laws of that country.

Thus in the United States the *Eagle* is required to weigh 258 *grains*,—$\frac{9}{10}$ of it to be pure *gold*, and $\frac{1}{10}$ alloy.

The *Dollar* is required to weigh 412½ *grains*,—$\frac{9}{10}$ of it to be pure *silver*, and $\frac{1}{10}$ alloy.

The Eagle then contains $258 \times .9 = 232.2$ *gr.* of gold;
and the Dollar " $412.5 \times .9 = 371.25$ *gr.* of silver.

And since 1 Eagle is equivalent to 10 Dollars,
232.2 *gr.* of gold are equivalent to 3712.5 *gr.* of silver.

$$3712.5 \div 232.2 = 15.988'.$$

The value of gold in the United States, is therefore 15.988' *times* that of an equal weight of silver. In other words, their *relative value* is as 15.988' to 1.—In the comparison of coins the *alloy* which they contain is considered of no value.

The relative value of gold and silver *is not the same in all countries.* Thus, as found above,

In the United States it is as		15.988'	to 1;
In England	"	14.28'	to 1;
In France	"	15.5'	to 1;
In Spain	"	16.0	to 1;
In China	"	14.25	to 1.

The relative valuation of these metals is sometimes changed in the same country. This occurred in the U. S. in 1834, as will be seen in § 435.

§ **432.** The difference in the relative value of gold and silver in different countries, causes the one or the other of these metals to be employed in the payment of foreign debts—when the circumstances of trade require the transmission of money—according as the one or the other will be *increased in value* in the country to which it is sent.

Thus in England, France, or China, silver is, relatively to gold, more valuable than in the United States. Silver, rather than gold, will therefore be sent from the U. S. to those countries.

Course of Exchange.

§ **433.** The *Course of Exchange* between two countries, is the variable price paid in the one for Bills of exchange *payable in the other.*

The transmission of a Bill of exchange to a distant place, especially to a foreign country, is less expensive and hazardous than of gold or silver. When, therefore, a person has a payment to make at any distant place, he will generally find it advantageous to purchase a Bill upon some one residing there, and send this Bill to the person to whom his payment is to be made.

In this way Bills of Exchange between two places come to be in *greater or less demand* in the one or the other, and their price, like that of any other article, will, to a limited extent, *vary with the demand.*

§ **434.** Exchange between two countries will be *at par* when the *debits* and *credits* between them balance each other—*above par* in the country which owes a *balance of debt* to the other, and *below par* in the country to which a balance of debt is due from the other.

When the *debits* and *credits* arising, in the course of trade, between any two places, as New-York and London, are equal,—that is, when New-York owes London just as much as London owes New-York,—the *debtors* in the two places will *exchange liabilities* with each other; a N. York creditor, for example, draws a Bill on his London debtor, payable to a N.York debtor, which the latter endorses to his London creditor; and by transactions of this kind the indebtedness of each place to the other is discharged by *exchanges at the commercial par*.

If New-York owe a *balance of debt* to London, there will be a competition in N. York for Bills on London, each debtor wishing to avoid the necessity of sending gold or silver to London, and Bills on London will sell *above par*. At the same time, there being a greater amount to be drawn for by London on N. York, than will be needed in the way of exchange, there will be a competition in London in the *sale of Bills* on N. York; and this will cause them to fall *below par*.

The *limit to the premium* which can be obtained on Bills of Exchange, is the expense that would attend the transmission of gold or silver to the place on which the Bills are drawn. For, rather than pay a higher premium, the debtor will transmit the precious metals.

Exchange between the U. S. and England.

§ **435.** To understand the present course of Exchange between the United States and England, we must advert to a change which has been produced in the relative value of the Pound Sterling, by an alteration in the standard U. S. Eagle.

By the standard first adopted the Eagle was to weigh 270 *gr*., $\frac{10}{11}$ of it to be pure *gold*, and $\frac{1}{11}$ alloy; and its value was $10.

By Act of Congress, taking effect August 1st, 1834, the present standard was introduced. By this the weight of this coin was diminished 12 *gr*, and its proportion of alloy increased from $\frac{1}{11}$ to $\frac{1}{10}$, while its value still remained $10.

The relative value of gold, in the United States, *was thus enhanced*; the *old Eagle* came to be worth $10.66½; and the English Pound sterling, *from being worth but* 4.44\frac{4}{9}$ in our currency, was enhanced, intrinsically, to $4.861.

We thus peceive that the legal value, $4.84, of the Pound sterling, according to the new valuation of gold in this country, is about 9 *per cent.* above the *old par* of $4.444';

$$\$4.444' \times 1.09 = \$4.844'.$$

Now it has happened that the *old value of the Pound* has been retained as the basis of Exchange with England, and its present exchangeable value is expressed in the form of a *premium on the former value.*

Hence when it is said that Bills of exchange on England, sell at a *premium of* 9 *per cent.*, it must be understood that the Pound sterling is reckoned at $4.444', and that the exchange is really near the *intrinsic par.* Consequently, English Bills will be above or below par, according as they bring a premium above or below 9 per cent.

United States and France.

§ **436.** In France accounts are kept in *francs* and *centimes.*

100 *centimes*=1 *franc*=18.6 *cents* in the U. S.

This is the legal and current value of the *franc* in this country, and gives 5.37' *francs*, or 5 *francs* 37 *centimes*, nearly, to the *dollar.*

$$100 \div 18.6 = 5.37'.$$

Such is therefore the *par of exchange* between the U. S. and France.

United States and Hamburg.

§ **437.** At Hamburg accounts are kept in *marcs*, *schillings*, *pfenings*, &c.; and money is there distinguished into *banco* or Bank money, and current money. Bank money is transferred from one owner to another, on the books of the Bank, without being drawn out; and bears a premium of 18 to 25 per cent. on current money. This premium is called the *agio.*

12 *pfenings*=1 *schilling*; 16 *schil.*=1 *marc* banco=35 *cts.* in U. S.

Note. For the *par of Exchange* between the United States and other countries, reference must be had to Tables prepared expressly on this subject. Such Tables, should it be found desirable, will be appended to this work; it is supposed, however, that persons needing them will prefer other authorities than a School Arithmetic.

ARBITRATION OF EXCHANGE.

§ 438. ARBITRATION OF EXCHANGE consists in computing the Exchange between two countries through the medium of exchanges between these two and one or more other countries.

This will be illustrated by the following

EXAMPLE.

A merchant in New-York has to pay £100 in London, and Bills on London are at a premium of 10 per cent. The exchange with Paris is 5.4 *francs* to the dollar, and with Hamburg 35 cents per *marc* banco. At the same time the exchange between Paris and London is 25.8 francs per £, and between Hamburg and London 13.5 marcs banco per £. It is required to determine what sum will pay the £100.

With London.—£1 in Exchange is \$4.444'.
Adding \$0.444' for the premium of 10 per cent;
£1=\$4.888; hence £100=\$488.8.

Through Paris.—Arranging the terms in a *Conjoined Proportion,* we have, (§ 238, § 239,)

\$1=5.4 *francs,*
25.8 *fr.*=£1,
£100=*how many* \$?
Ans. (1×25.8×100)÷(5.4×1)=\$477.77'.

Through Hamburg.—Arranging the terms in this case also in a *Conjoined Proportion,* we have

\$0.35=1 *marc,*
13.5 *m.*=£1,
£100=how many \$?
Ans. (.35×13.5×100)÷(1×1)=\$472.5.

We thus find that a direct exchange with London will require \$488.8'; an exchange through Paris \$477.77'; and an exchange through Hamburg \$472.5.

By the same method of CONJOINED PROPORTION an Exchange may be arbitrated between two countries through *two or more intervening* countries; and thus the most advantageous medium through which to make a foreign payment may be ascertained.

These transactions, it will be understood, are effected by Bills of Exchange between the different places; and require that the person so arbitrating exchanges should have, in the intermediate places, *Correspondents* or *Agents* to assist him.

APPENDIX II.

MATHEMATICAL PROBABILITY, AND ITS APPLICATION TO LIFE ANNUITIES AND LIFE INSURANCE.

Contingent Events.

§ **439.** A *contingent event*, in the mathematical sense, is one among a number of similar events, some only of which will *certainly occur*, while it is impossible to assign any reason why any one of them should occur *rather than any other*.

Mathematical Probability.

§ **440.** The *Probability* of a contingent event, is expressed by the ratio of the number of *chances favorable to its occurrence* to the whole number of chances favorable and unfavorable to the same event.

EXAMPLES.

Suppose that *one* person *is to be taken by lot* from among *five* persons, A, B, C, D, E. The Probability that any particular one, as A, will be taken, is *one-fifth*, since each person has *one chance in five*.

If *two* persons were to be taken, by lot, from among the *five*, the Probability that A would be one of those two would be *two-fifths*, since he would then have *two chances in five*.

§ **441.** The Sum of all the Probabilities arising out of the same trial of favorable and unfavorable chances, amounts to UNITY; and unity is therefore the expression for *certainty*.

Thus in the preceding Example, there is a Probability of $\frac{1}{5}$ for each of the five persons from among whom *one is to be taken*. The sum of all the probabilities in the case is therefore

$$\tfrac{1}{5}+\tfrac{1}{5}+\tfrac{1}{5}+\tfrac{1}{5}+\tfrac{1}{5}=1.$$

And since it is *certain* that the lot will fall on some one of the five, the sum of all these probabilities amounts to a *certainty*, which is therefore expressed by *unity*.

§ 442. The Improbability of an event, in other words, the probability of the *non-occurrence* of the event, is expressed by UNITY *minus* the *probability of the same event.*

If 1 ticket is to be drawn from among 7 tickets, of which 3 are *prizes* and 4 are *blanks*, the Probability that a *prize* will be drawn is $\frac{3}{7}$, since there are 3 chances in *seven* in favor of a prize.

The Probability that a *blank* will be drawn is $\frac{4}{7}$, which is equal to the *improbability* that a prize will be drawn; and

$$\tfrac{4}{7}=1-\tfrac{3}{7}.$$

Observe that the probability of a prize + the probability of a blank, is *unity*, since it is certain that either a prize or a blank will be drawn.

The Probability and the Improbability of the same event may be regarded as *opposite probabilities* the sum of which is unity.

Combined Probabilities.

§ 443. The Probability of one, indifferently, of two or more contingent events, is equal to the *sum* of the *separate probabilities* of the same events.

EXAMPLES.

If one person is to be taken by lot out of a company of 9 persons, the Probability that some *one of any three* that might be named, will be taken, is

$$\tfrac{1}{9}+\tfrac{1}{9}+\tfrac{1}{9}=\tfrac{3}{9}=\tfrac{1}{3}.$$

It is plain that, as the *three* together have 3 chances in *nine*, if these three chances be given to one person, that one will have 3 chances in 9, which is a probability of $\frac{3}{9}$ or $\frac{1}{3}$.

§ 444. The Probability of the *concurrence of two or more events*, is equal to the *product* of the separate probabilities of the same events, and in like manner for *improbabilities*.

If 2 tickets are to be drawn from among 9 tickets, of which 4 are *prizes*, and 5 are *blanks*, the Probability that a *prize* will be obtained at the first drawing is $\frac{4}{9}$; and, as 3 prizes would then remain among 8 tickets, the probability that a prize would be obtained at the second drawing is $\frac{3}{8}$.

Then the Probability that 2 prizes would be obtained at two drawings, is

$$\tfrac{4}{9}\times\tfrac{3}{8}=\tfrac{12}{72}=\tfrac{1}{6}.$$

Each of the 9 tickets in the first drawing might be combined with each of the 8 in the second—which produces $9\times8=72$ *chances* or different combinations of 2 tickets. Each of the 4 *prizes* in the *first* might combine with each of the 3 which would remain for the *second* drawing—on the supposition of a *prize* having been drawn at first,—and thus there would be $4\times3=12$ *chances favorable* to the drawing of *two prizes*, (§ 440.)

§ **445.** The Probability of the first of two events, or—if the first fail—of the second event, is equal to the probability of the first *plus* the probability of the second *multiplied into the improbability of the first.*

If one person is to be taken by lot from among *five* persons, A, B, C, &c., and another from among *six* other persons, F, G, H, &c., then the probability that the lot will fall on A, in the first case, or—if not on A—that it will fall on F, in the second case, is

$$\tfrac{1}{5}+\tfrac{1}{6}\times\tfrac{4}{5}=\tfrac{1}{5}+\tfrac{4}{30}=\tfrac{1}{3}.$$

The $\frac{1}{5}$ in this compound expression, is the probability that A will be taken out of the first company, and $\frac{1}{6}\times\frac{4}{5}$ is the probability that F will be taken out of the second, *but not* A *out of the first*, (§ 444.) The compound probability in question is the sum of these separate probabilities, (§ 443.)

Relative Probabilities.

§ **446.** The *relative* Probabilities of two contingent events, are expressed by the ratios of the probability of *each* to the *sum of the probabilities* of the two events.

EXAMPLE.

Let the probability that A will be allotted from one company be $\frac{1}{3}$, and the probability that B will be allotted from another company be $\frac{1}{5}$.

The sum of these probabilities is $\frac{1}{3}+\frac{1}{5}=\frac{8}{15}$, which is the probability that *one of the two*, A and B, will be the subject of the allotment, (§ 443); and the object now is to determine *what part of this combined probability is assignable to each.*

By *Partitive Proportion*, $\frac{1}{3}+\frac{1}{5} : \frac{1}{3} :: 1 : \frac{5}{8}$;

and, $\frac{1}{3}+\frac{1}{5} : \frac{1}{5} :: 1 : \frac{3}{8}$; (§ 229.) (§ 224.)

Hence, $\frac{5}{8}$ of the combined probability is assignable to A, and $\frac{3}{8}$ of it to B. In other words, it is more probable that the lot will fall on A than on B in the ratio of $\frac{5}{8}$ to $\frac{3}{8}$, or of 5 to 3.

Contingent Value of a Sum of Money.

§ **447.** When the payment of a Sum of money depends on the occurrence of a *contingent event*, the Value of the sum *at risk* is equal to the amount of that sum multiplied into the *probability of the event.*

EXAMPLE.

Suppose that 1 ticket is to be drawn from among 10 tickets, of which 2 are *prizes* of $100 each, and the rest are *blanks.*

The probability that the ticket to be drawn will be a *prize* is $\frac{2}{10}$ or $\frac{1}{5}$, since there are 2 chances in 10 in favor of a prize.

The *certainty* of drawing a prize would be worth $100; $\frac{1}{5}$ of this certainty, that is, a *probability* of $\frac{1}{5}$ of drawing a prize, is therefore worth

$$\$100\times\tfrac{1}{5}=\$20.$$

Thus the Value of the sum *at risk*, is equal to the amount of that sum multiplied into the probability of the event on which it depends.

On this Example some further remarks will be appropriate.—

A ticket in this *lottery* would be worth $20; and this value arises from an equivalence between opposite probabilities. The purchaser might lose $20, the probability of which is $\frac{4}{5}$; but he might gain $80, the probability of which is $\frac{1}{5}$; and these sums are equivalent when estimated according to the *probability* which attaches to each;

$$\$20\times\tfrac{4}{5}=\$80\times\tfrac{1}{5}.$$

There would be no *rational inducement*, therefore, to purchase a ticket at $20, any more than there would be to give $20 for $20. At *less* than 20 dollars, the purchase of a ticket would be a *prudent* pecuniary speculation; at more than 20, it would be an *imprudent* one.

Contingent events, in the long run, *conform to the laws of probability.* Hence a continued speculation in such a *lottery* as this, at less than $20 for a ticket, would produce a *gain*,—but at more than $20,—would produce a *loss*, to the speculator.

☞ The preceding propositions on Mathematical Probability, or the "Doctrine of Chances," are preliminary to a very brief exposition which we propose to make of the principles of *Life Annuities* and *Life Insurance*.

LIFE ANNUITIES.

§ **448.** A Life Annuity is a sum of money to be paid *annually* during the life of a person, called the *Annuitant*, but the payment is to cease at his death.

A Life Annuity, under the name of a *Pension*, is sometimes granted to an individual on account of past services to the country; and is sometimes purchased from a Company by the present payment of an equitable sum of money.

A *Temporary* Life Annuity is one which is limited to a given number of years, but is liable to fail at any time by the *death of the Annuitant*, or person on the continuance of whose life its payment depends.

Present Value of a Life Annuity.

§ **449.** The *Present Value* of a Life Annuity is estimated according to the *probabilities* of the annuitant's living *one*, *two*, *three*, &c., *years*,—continued to the utmost extent of life, or through the period by which the Annuity, if a *temporary* one, is limited.

The Probabilities that a person of any given age, will live one, two, three, &c., years, are obtained from an observed *rate of mortality*.

The following Table commences with 10,000 persons *at birth*, and shows the number who *die*, and the number who survive, for each year until all are dead.

This Table was formed from the registers of births and deaths in the city of Carlise, (England.) between the years 1779 and 1787. Its accuracy has been confirmed by observations at various other places, and by the experience of the oldest Life Annuity and Insurance Companies. Hence it is generally used by such Companies.

TABLE

Of mortality based upon observations at Carlisle, showing the rate of extinction of 10,000 *lives.*

AGE.	NUMBER OF SURVIVORS.	NUMBER OF DEATHS.	AGE.	NUMBER OF SURVIVORS.	NUMBER OF DEATHS.	AGE.	NUMBER OF SURVIVORS.	NUMBER OF DEATHS.
0	10,000	1539	35	5362	55	70	2401	124
1	8461	682	36	5307	56	71	2277	134
2	7779	505	37	5251	57	72	2143	146
3	7274	276	38	5194	58	73	1997	156
4	6998	201	39	5136	61	74	1841	166
5	6797	121	40	5075	66	75	1675	160
6	6676	82	41	5009	69	76	1515	156
7	6594	58	42	4940	71	77	1359	146
8	6536	43	43	4869	71	78	1213	132
9	6493	33	44	4798	71	79	1081	128
10	6460	29	45	4727	70	80	953	116
11	6431	31	46	4657	69	81	837	112
12	6400	32	47	4588	67	82	725	102
13	6368	33	48	4521	63	83	623	94
14	6335	35	49	4458	61	84	529	84
15	6300	39	50	4397	59	85	445	78
16	6261	42	51	4338	62	86	367	71
17	6219	43	52	4276	65	87	296	64
18	6176	43	53	4211	68	88	232	51
19	6133	43	54	4143	70	89	181	39
20	6090	43	55	4073	73	90	142	37
21	6047	42	56	4000	76	91	105	30
22	6005	42	57	3924	82	92	75	21
23	5963	42	58	3842	93	93	54	14
24	5921	42	59	3749	106	94	40	10
25	5879	43	60	3643	122	95	30	7
26	5836	43	61	3521	126	96	23	5
27	5793	45	62	3395	127	97	18	4
28	5748	50	63	3268	125	98	14	3
29	5698	56	64	3143	125	99	11	2
30	5642	57	65	3018	124	100	9	2
31	5585	57	66	2894	123	101	7	2
32	5528	56	67	2771	123	102	5	2
33	5472	55	68	2648	123	103	3	2
34	5417	55	69	2525	124	104	1	1

§ 450. The Probability—according to the preceding Table of Mortality—that a person of any *given age*, will live to any designated *higher age*, is expressed by the ratio of the number who attain the *higher* to the number who attain the *given* age.

For example, suppose it were required to express the probability that a person who is 30 years old, will attain the age of 60.

From the Table we find that of 5642 persons who attain the age of 30, but 3643 attain the age of 60.

The probability in question is therefore the ratio

$$\frac{3643}{5642}, \text{ (§ 440.)}$$

EXAMPLE.

In the Calculation of Life Annuities.

§ 451. If we had the value of an annuity of \$1 on a life at any given age, this value *multiplied by any other annuity* would produce the value of that annuity on the same life.

Let it be required then to calculate the Present Value of an Annuity of \$1 on the life of a person at the age of 100,—on the supposition that money will produce 5 per cent. at *compound* interest.

If the Annuitant live 1 year, \$1 will be paid at the end of the year.

The *present value* of \$1 payable in 1 year, is

$\$1 \div 1.05 = .95238'$. (359 ... 2,)

But there is only a *probability* of $\frac{7}{9}$ that he will live 1 year, (§ 450); the present value of the 1st payment on the annuity—subject to the *contingency of its failure* from the *decease* of the Annuitant—is therefore

$\frac{7}{9}$ of $.95238' = .74074'$. (§ 447).

If the Annuitant live 2 years, \$1 will be paid at the end of the 2 years.

The *present value* of \$1 payable in 2 years, is

$\$1 \div 1.1025 = .90702'$.

The probability that he will live 2 years, is $\frac{5}{9}$; hence the present value of the 2d payment—subject to the *contingency of its failure*—is

$\frac{5}{9}$ of $.90702 = .50390'$.

The present value of \$1 payable in 3 years, is

$\$1 \div 1.15762' = .86383'$.

The probability that the 3d payment will be received being $\frac{3}{9}$, the value of that payment is

$\frac{3}{9}$ of $.86383' = .28794'$.

The present value of $1 payable in 4 years, is

$\$1 \div 1.2155 = .82270'$.

The probability that the 4th payment will be received, being $\frac{1}{9}$, the value of that payment is

$\frac{1}{9}$ of $.82270 = 09141$.

There could be no more payments on this Annuity, according to our Table of Mortality; hence the entire Present Value of the annuity is,

$\$0.74074 + .50390 + .28794 + .09141 = \$1.62399$.

Having found the value of an Annuity of $1 on any given life, we may compute its value on all *younger lives*, and thus prepare a *Table of Life Annuities*, according to the following principle.

§ **452.** The Present Value of an Annuity of $1 on any given life, is equal to (the value of an annuity of $1 on a life 1 year *older* +1) × the present value of $1 payable in 1 year, × the *probability* that the given life will continue 1 year.

EXAMPLE.

To find the present value of an Annuity of $1 on a life at the age of 99, its value on a life at 100 having already been found to be $1.62399'. The required value is

$(\$1.62399+1) \times .95238 \times \frac{9}{11} = \$2.04466'$.

To demonstrate the correctness of this principle, we remark,

First. $1.62399 multiplied by .95238, the present value of $1 payable in 1 year, will produce the *present value* of the value $1.62399 of annuity *after the age of* 100. But the value thus obtained would depend on the Annuitants living 1 year, the probability of which is $\frac{9}{11}$. Hence, multiplying again by $\frac{9}{11}$, we find the *present value of the annuity estimated from the age of* 100 onwards.

Secondly. The $1 \times .95238$ is the present value of $1 payable in 1 year, and this multiplied by the probability $\frac{9}{11}$ of the Annuitant's living 1 year, produces the *present value of the annuity from the age of* 99 *to that of* 100.

Thirdly. The sum of the present values of the annuity for the two periods into which its duration is divided, as above, is the *entire present* value of the annuity; and this sum *results from the operation.*

From the value of an Annuity on a life of 99, may be found, in like manner, its value on a life of 98; and so on. In this way has been computed the following

TABLE

Showing the Present Value of an Annuity of $1, at 4 or 5 per cent., compound interest,—on a Single Life, according to the Carlisle Table of Mortality.

AGE.	4 P. C.	5 P. C.	AGE.	4 P. C.	5 P. C.	AGE.	4 P. C.	5 P. C.
1	16.556	13.995	34	16.219	14.260	67	7.701	7.227
2	17.728	14.983	35	16.041	14.127	68	7.380	6.941
3	18.717	15.824	36	15.856	13.987	69	7.049	6.643
4	19.233	16.271	37	15.666	13.843	70	6.709	6.336
5	19.592	16.590	38	15.471	13.695	71	6.358	6.015
6	19.747	16.735	39	15.272	13.542	72	6.026	5.711
7	19.790	16.790	40	15.074	13.390	73	5.725	5.435
8	19.766	16.786	41	14.883	13.245	74	5.458	5 190
9	19.693	16.742	42	14.694	13.101	75	5.239	4.989
10	19.585	16.669	42	14.505	12.957	76	5.024	4.792
11	19.460	16.581	44	14.308	12.806	77	4.825	4.609
12	19.336	16.494	45	14.104	12.648	78	4.622	4.422
13	19.210	16.406	46	13.889	12.480	79	4.394	4.210
14	19.082	16.316	47	13.662	12.301	80	4.183	4.015
15	18.956	16.227	48	13.419	12 107	81	3.953	3.799
16	18.837	16.144	49	13.153	11.892	82	3.746	3.606
17	18.723	16.066	50	12.869	11.660	83	3.534	3 406
18	18.608	15.987	51	12.566	11.410	84	3.329	3.211
19	18.488	15.904	52	12.258	11.154	85	3.115	3.009
20	18.363	15.817	53	11.945	10.892	86	2.928	2.830
21	18.233	15.726	54	11.627	10.624	87	2.776	2.685
22	18.095	15.628	55	11.300	10.347	88	2.683	2.597
23	17.951	15.525	56	10.966	10.063	89	2.577	2.495
24	17.801	15.417	57	10.625	9 771	90	2.416	2.339
25	17.645	15.303	58	10.286	9.478	91	2.398	2.321
26	17.486	15.187	59	9.963	9.199	92	2.492	2.412
27	17.320	15.065	60	9.663	8.940	93	2.600	2 518
28	17.154	14 942	61	9.398	8.712	94	2 650	2.569
29	16.997	14.827	62	9.137	8.487	95	2.674	2.596
30	16.852	14.723	63	8 872	8.258	96	2.628	2.555
31	16.705	14.617	64	8.593	8.016	97	2.492	2.428
32	16.552	14.506	65	8.307	7.765	98	2.332	2.278
33	16.390	14.387	66	8.010	7.503	99	2.087	2.045

Annuities on Joint Lives.

§ **453.** The Present Value of an Annuity which depends on the continuance of *both* of two lives, is estimated according to the *probabilities* that both lives will continue *one*, *two*, *three*, &c., years; and in like manner for an annuity which depends on *three* or more lives.

EXAMPLE.

Let it be required to calculate the Present Value of an Annuity of $1, which depends on the continuance of two lives, one at the age of 90 and the other at the age of 95,—at 5 per cent., Compound Interest.

If *both lives* continue 1 year, $1 will be paid at the end of the year.

The present value of $1 payable in 1 year, is

$$\$1 \div 1.05 = .95238'.$$

The probability that the *first* life will continue 1 year, is $\frac{105}{142}$, (§ 450); and the probability that the second will continue 1 year is $\frac{23}{30}$; then the probability that *both* lives will continue 1 year is

$$\tfrac{105}{142} \times \tfrac{23}{30}, \text{ (§444)}.$$

The present value, therefore of the 1*st payment* on the annuity,—subject to *the contingency of its failure* from the decease of one or both of the lives, is

$$\$0.95238 \times \tfrac{105}{142} \times \tfrac{23}{30} = \$0.5399'.$$

In the same way we might calculate the present values of the 2d, 3d, &c., payments, to the number of *nine*, which would bring the older life to the *limit of mortality*, when the annuity would necessarily cease.

The *sum* of all these present values would be the *entire present value* of the annuity.

Survivorship, or an Annuity on the Survivor of two or more Lives.

§ **454.** The Present Value of an Annuity which depends on the continuance of *either* of two lives, is estimated according to the *improbabilities* that both lives will fail before the end of *one*, *two*, *three*, &c., years; and in like manner for an annuity which depends on a survivorship among *three* or more lives.

EXAMPLE.

Let it be required to calculate the Present Value of an Annuity of $1, which is to continue so long as either of two lives, one at the age of 90, and the other at the age of 95, shall survive.

If *either* of the two lives continue 1 year, \$1 will be paid at the end of the year.

The present value of \$1 payable in 1 year, is

$$\$1 \div 1.05 = .95238'.$$

The *improbability* that the first life will continue to the end of the year, or the probability that it *will fail* before the end of the year, is $1-\frac{105}{142}$, (§ 442); and the probability that the second will fail before the end of the year, is $1-\frac{23}{30}$; then the *probability that both will fail* before the end of the year, is

$$(1-\tfrac{105}{142})\times(1-\tfrac{23}{30}), \text{ (§ 444).}$$

Consequently the Improbability that both the lives will fail before the end of the year, is

$$1-\overline{(1-\tfrac{105}{142})\times(1-\tfrac{23}{30})}, \text{ (§ 442).}$$

Now this improbability is the same as the *probability that the first payment on the annuity will be realized.*

Hence .95238' multiplied by said probability, will produce the *present* value of the first payment on the Annuity.

In like manner we might compute the present values of the 2d, 3d, &c., payments, to the number of *nine*,—when the older life must be dropped, and the Annuity thenceforth be considered with reference to the probabilities of the younger life's continuing to the same *limit of mortality.*

LIFE INSURANCE.

§ **455.** LIFE INSURANCE is an obligation assumed, usually by an Incorporated Company, in consideration of a *premium advanced*, to pay on the *decease* of the person on whose life the Insurance is effected, a certain sum to the one for whose benefit it is effected.

The Insurance may embrace the *whole life*, or be limited to a given number of years. In the latter case the obligation of the Company will not exist, unless the life insured shall fail within the given number of years.

The Premium for insurance is usually in the form of an *annual* payment,—the first payment being *in advance;* and its proportional amount is computed with reference to,

1. The *Expectation of Life* remaining to the insured,—by which is meant the *average* number of years of life which remain to him, according to an ascertained *rate of mortality;* and

2. The *rate* of Interest or profit at which the premiums paid may be augmented—to defray the expenses, meet the obligations, and secure the just interests of the Insurance Company.

Expectation of Life.

§ **456.** The *Expectation of life* at any given age, is equal to the *sum of* the probabilities that the persen will live over *one*, *two*, *three*, &c., years, increased by $\frac{1}{2}$.

EXAMPLE.

To compute the Expectation of Life of a person at the age of 100.

According to the Carlisle Table of Mortality, of 9 persons at the age of 100, only 7 live through the 1st succeeding year.

With respect to the 1st year, therefore, the 9 persons have an expectation, in the aggregate, of 7 years of life, which gives to each $\frac{7}{9}$ of a year; and $\frac{7}{9}$ is the probability that any one of the 9 will live 1 year, (§ 450).

Again; of the same 9 persons, but 5 live through the 2d year. Hence with respect to the 2d year, the 9 have an expectation of 5 years of life, which gives to each one of them $\frac{5}{9}$ of a year; and $\frac{5}{9}$ is the probability that any one of the 9 will live 2 years.

In like manner it will be found that each one of the 9 persons has an expectation of $\frac{3}{9}$ for the 3d, and of $\frac{1}{9}$ for the 4th year.

In the 5th year the last of the 9 persons dies; and if the probabilities in the case were only that the several lives would fail at the *terminations* of the successive years, we should have for the Expectation of Life of a person at the age of 100, the sum of the partial expectations found above; viz.

$$\frac{7}{9}+\frac{5}{9}+\frac{3}{9}+\frac{1}{9}.$$

But any one of the lives may fail *in the course of* any one of the five years; and the probability that it will fail in the *course of one of them*, if it do not at the *termination* of a year, amounts to a certainty. And since the probabilities of the life's failing are the same for every time in the year, we must suppose it to fail at the *middle* of the year. This supposition adds $\frac{1}{2}$ of a year to the Expectation found above.

We have then for the whole Expectation of Life at the age of 100,

$$\frac{7}{9}+\frac{5}{9}+\frac{3}{9}+\frac{1}{9}+\frac{1}{2}=\frac{41}{18}=2.277' \text{ years.}$$

In this way may be readily computed the following

TABLE

Showing the Expectation of Life, according to the Carlisle Table of Mortality.

AGE.	Expectation in years and 100ths.	AGE.	Expectation in years and 100ths.	AGE.	Expectation in years and 100ths.	AGE.	Expectation in years and 100ths.	AGE.	Expectation in years and 100ths.
	38.72	21	40.75	42	26.34	63	12.81	84	4.39
1	44.68	22	40.04	43	25.71	64	12.30	85	4.12
2	47.55	23	39.31	44	25.09	65	11.79	86	3.90
3	49.82	24	38.59	45	24.46	66	11.27	87	3.71
4	50.76	25	37.86	46	23.82	67	10.75	88	3.59
5	51.25	26	37.14	47	23.17	68	10.23	89	3.47
6	51.17	27	36.41	48	22.51	69	9.70	90	3.28
7	50.80	28	35.69	49	21.81	70	9.18	91	3.26
8	50.24	29	35.00	50	21.11	71	8.65	92	3.37
9	49.57	30	34.34	51	20.39	72	8.16	93	3.48
10	48.82	31	33.68	52	19.68	73	7.72	94	3.53
11	48.04	32	33.03	53	18.97	74	7.33	95	3.53
12	47.27	33	32.36	54	18.28	75	7.01	96	3.46
13	46.51	34	31.68	55	17.58	76	6.69	97	3.28
14	45.75	35	31.00	56	16.89	77	6.40	98	3.07
15	45.00	36	30.32	57	16.21	78	6.12	99	2.77
16	44.27	37	29.64	58	15.55	79	5.80	100	2.28
17	43.57	38	28.96	59	14.92	80	5.51	101	1.79
18	42.87	39	28.28	60	14.34	81	5.21	102	1.30
19	42.17	40	27.61	61	13.82	82	4.93	103	0.83
20	41.46	41	26.97	62	13.31	83	4.65	104	0.50

Rate of Interest to be assumed in Life Insurance.

§ **456.** The Rate at which the Interest of money may be estimated for the periods of time embraced by the transactions of Life Insurance, cannot be certainly determined.

According to the preceding Table, a person at the age of 21, has an expectation of living 40.75 years. The premium which he should contract to pay *annually* to insure a certain sum, say $1000, to be paid at his decease, would depend on the rate of interest or profit at which that premium might be invested, during the 40.75 years. This cannot be certainly foreseen.

The *higher the rate* of interest assumed, the lower will the computed premium be. If the assumed rate of interest should not be realized in the future experience of the Company, the premiums would fail to produce the means of paying the *sums insured;* and the Company would thus become *insolvent.*

In England, where the legal rate of interest is 5 per cent, it is considered unsafe to transact life insurance at more than 3 or 4 per cent. interest; in the United States, where money brings a higher profit, the rate assumed might probably be 5 per cent.

Present Value of a Sum insured on a Given Life.

§ 457. The Present Value of \$1, and thence of any other sum, *insured on a given life*, will be found by subtracting the present value of an Annuity of \$1 on the same life from the present value of a *perpetuity* of \$1, and dividing the remainder by a *unit* + the *value of the perpetuity.*

EXAMPLE.

To find the Present Value of \$1 payable at the end of the year in which a person now 50 years of age may die,—allowing interest at 5 per cent.

The present value of an Anuuity of \$1 on the given life, is

$$\$11.66.$$

The present value of a Perpetuity, or *perpetual annuity*, of \$1, is

$$\$1 \div .05 = \$20, \quad (\S\, 367).$$

Then the present value of \$1 to be paid at the end of the year in which the given life may fail, is

$$\frac{\$20-11.66}{1+20} = \frac{\$8.34}{21} = \$0.397143.$$

To demonstrate the correctness of this principle, we remark,

First. The present value of the Annuity subtracted from the present value of the Perpetuity, leaves the present value \$8.34 of a *perpetuity of* \$1, *commencing with the payment of* \$1 *at the end of the year in which the life may fail.*

Secondly. The divisor 1+20 is the present value of a *perpetuity* of \$1, commencing with the *present payment* of \$1.

Thirdly. The ratio between the *present values* of these two *perpetuities*, must be the same as the ratio between the present

value of *one* dollar payable at the end of the year in which the life may fail and *one* dollar.

Hence the *quotient* found, which is the value of this ratio, is the present value of \$1 *payable as supposed.*

The Present value of \$100 payable as in the Example would be

$$.397143\times100=\$39.7143;$$

of \$1000, \$397.143, and so on.

Premiums in Life Insurance.

§ **458.** The Annual Premium which, irrespective of either *expense* or *profit* to the Insurer, should be given for a Sum to be paid on the failing of a given life,—is equal to the *present value* of that sum, divided by a *unit* + the present value of an Annuity of \$1 on the same life.

EXAMPLE.

To find the annual Premium which, irrespective of expense or profit to the insurer, should be given for \$1000 to be paid at the decease of a person now 50 years of age,—allowing interest at 5 per cent.

The present value of \$1000 payable at the end of the year in which the given life may fail, as has been already found, (§ 457) is

$$\$397.143.$$

The Premiums *after the first* being payable in *one*, *two*, *three*, &c., years, their present value is the same as the present value of an equal Annuity on the life of the person.

The present value of an Annuity of \$1 on the life of a person at the age of 50, is \$11.66; and the present value of an annuity equal to the annual Premium, is therefore equal to

$$\$11.66\times \text{ the Premium.}$$

By adding the *first* Premium to the present value of the succeeding ones, we shall have the present value of *all the premiums*, which is equal to the present value of the *sum insured.*

Hence, the *Premium* + the *Premium* $\times 11.66=\$397.143$;
or, the *Premium* $\times(1+11.66)=\$397.143$;
which gives the *Premium* $=\$397.143\div12.66=\$31.369$.

Insurance on Joint Lives.

§ 459. In a *joint insurance* on two lives, the obligation of the Insurer is, to pay the sum insured on the decease of that life which fails the first of the two.

The Present Value of the Sum insured in this case, would be found from an Annuity on the joint continuance of the two lives, (§ 453), in the same manner as for an insurance on a single life, (§ 457); and from this Present Value the annual Premium may be computed as for a single life.

Temporary Life Insurance.

§ 460. In a *temporary* Life Insurance, the obligation of the Insurer is, to pay the Sum insured on the failing of the given life, *provided it shall fail within the period* comprehended by the insurance.

The Present Value of a temporary Life Insurance may be found from that of an insurance on the whole life—as in the following

EXAMPLE.

To find the Present Value of $1000 to be paid on the decease of a person now 50 years of age, provided he shall die within 5 years,—allowing interest at 5 per cent.

The present value of $1000 insured on the whole of the given life, as heretofore found, is

$$\$397.143.$$

The present value of $1000 insured on a life which is now 55 years old, found according to the same method, (§ 457), is

$$\$459.666.$$

$1000 insured on the whole of the given life would therefore be worth $459.666, at the end of the 5 years. The *present value* of this sum is

$$\$459.666 \div 1.27628 = \$360.161. \quad (\S\ 359....2)$$

The Probability that the given life will survive 5 years is $\frac{4000}{4397}$, (§ 450), and therefore the present value of $1000 insured on the whole of the given life, *if that life survive* 5 years, is

$$\$360.161 \times \tfrac{4000}{4397} = \$327.642.$$

Then the Present Value of $1000 to be paid on the decease of the given life, *if that life fail within* 5 years, must be

$$\$397.143 - \$327.642 = \$69.501.$$

§ **461.** The Annual Premium, irrespective of expense or profit to the Insurer, which should be paid for a *temporary* Life Insurance, may be found by dividing the present value of the Sum insured by a *unit* + the present value of an Annuity of $1 on the given life *for one year less than the period of insurance.*

Thus, to recur to the preceding Example, observe that the *first* Premium must be paid immediately, and that the four remaining premiums are equal to a temporary annuity of the same amount for 4 years.

The value of an Annuity of $1 on the given life 4 years hence, that is, at the age of 54, will be

$$\$10.624.$$

The present value of this sum payable at the end of 4 years, is

$$\$10.624 \div 1.2155 = \$8.74.$$

The Probability that the given life will survive 4 years is $\frac{4143}{4397}$; and hence the present value of the Annuity, subject to the contingency of its continuing 4 years, is

$$\$8.74 \times \tfrac{4143}{4397} = \$8.235.$$

The present value of an Annuity of $1 on the whole of the given life, is $11.66; cousequently the value of the *temporary* annuity on the given life for 4 years, is

$$\$11.66 - \$8.235 = \$3.425.$$

The Annual Premium is then the present value of the Sum insured, as found in the preceding Example, divided by 1+3.425, that is

$$\$69.501 \div 4.425 = \$15.706.$$

Different allowances for *expenses*, and *profits* on Capital employed, will be made by different Insurance Companies, and hence will arise differences in their *rates* or *premiums* for insurance.

Mutual Insurance.

§ **462.** In Mutual Life Insurance, the Funds of the Company consist wholly or chiefly of the Premiums paid; and any *surplus* which remains after paying expenses, and the *losses*, that is, the sums due on the *decease of lives insured*,—is distributed from time to time among the *policy holders.* By this method no injustice will result to the *insured from excessive premiums.*

The following are believed to be the *lowest rates* that have yet been adopted in Mutual Life Insurance.

TABLE

Of the Rates of Insurance charged by the "Kentucky Mutual Life Insurance Company."

Annual payment for Insurance upon $100.

For Life.		For Five Years.		For One Year.	
Age.	Premium.	Age.	Premium.	Age.	Premium.
14	$1.20	14	$0.58	14	$0.50
15	1.23	15	0.60	15	0.53
16	1.26	16	0.62	16	0.56
17	1.29	17	0.64	17	0.59
18	1.32	18	0.66	18	0.62
19	1.35	19	0.68	19	0.64
20	1.38	20	0.70	20	0.66
21	1.42	21	0.72	21	0.68
22	1.46	22	0.74	22	0.70
23	1.51	23	0.76	23	0.72
24	1.57	24	0.78	24	0.74
25	1.62	25	0.80	25	0.76
26	1.68	26	0.83	26	0.78
27	1.73	27	0.88	27	0.81
28	1.79	28	0.93	28	0.84
29	1.85	29	0.98	29	0.87
30	1.90	30	1.00	30	0.90
31	1.96	31	1.04	31	0.92
32	2.01	32	1.06	32	0.94
33	2.07	33	1.08	33	0.96
34	2.13	34	1.10	34	0.98
35	2.19	35	1.12	35	1.00
36	2.27	36	1.15	36	1.02
37	2.35	37	1.18	37	1.07
38	2.44	38	1.21	38	1.12
39	2.53	39	1.24	39	1.17
40	2.63	40	1.27	40	1.22
41	2.73	41	1.30	41	1.27
42	2.83	42	1.33	42	1.30
43	2.93	43	1.36	43	1.32
44	3.03	44	1.39	44	1.34
45	3.14	45	1.42	45	1.36
46	3.26	46	1.45	46	1.38
47	3.40	47	1.48	47	1.40
48	3.54	48	1.51	48	1.44
49	3.69	49	1.54	49	1.48
50	3.85	50	1.57	50	1.52
51	4.03	51	1.65	51	1.56
52	4.22	52	1.80	52	1.60
53	4.43	53	2.00	53	1.65
54	4.64	54	2.30	54	1.75
55	4.96	55	2.60	55	1.85
56	5.20	56	2.90	56	2.05
57	5.60	57	3.20	57	2.25
58	6.00	58	3.50	58	2.55
59	6.40	59	3.80	59	2.90
60	6.75	60	4.10	60	3.25

The rates of Premium are in the same proportion for larger amounts.